# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION

FOR THE

# CULTIVATION OF SCIENCE

**VOL. I.**

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# CONTENTS.

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Vol. I. No. I.


Saturday, February 6th, 1915 at 4-45 P.M., Dr. P. C. Roy C.I.E., D.SC., PH.D., Vice-President, in the Chair.

*Tautomeric changes in Organic Thiobodies through the agency of mercuric nitrite, mercuric chloride, cupric chloride, platinic chloride and monochloro acetic acid.*

By Dr. P. C. Ray, C.I.E, D.SC., PH.D.

Rai Bahadur Dr. Chuni Lal Bose, I.S.O., M.B., F.C.S., was in the Chair when Dr. P. C. Ray read the above paper.

Through the agency of Mercuric Nitrite, ethyl mercaptan yields a nitromercaptide of the formula

$$C_2 H_5 . S. Hg NO_2;$$

thiocarbamide yieids a compound of the formula

$$\begin{array}{l} NH \\ NH_2 \end{array} \!\!> CS. Hg NO_2 . HgS. HgO$$

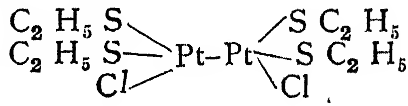
alkylthiocarbamide yields

$$\begin{array}{r} (C H_3) C_2 H_5 \\ NH_2 \end{array} \!\!> C S. Hg NO_2 . Hg S. HgO$$

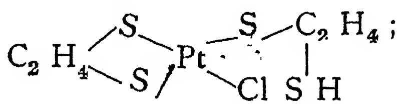
which is due to the presence of a potential mercapto group and thioacetamide yields

$$\begin{array}{l} \quad\quad N_5 \\ \quad\quad \| \\ CH_3 C. S. Hg NO_2 . HgS. HgO \end{array}$$

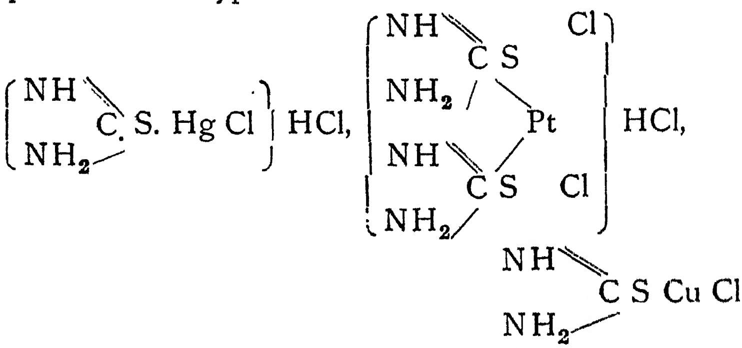
The same kind of changes have been found to take place through the agency of Cupric, Mercuric and Platinic Chlorides, with the formation of chloromercaptides. Ethyl mercaptan yields

$C_2H_5SHgCl$ and 

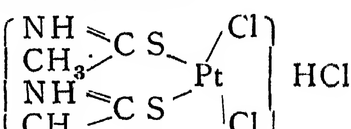
respectively with the corresponding chlorides. Thioglycol yields with platinic chloride, a chloromercaptide of the formula

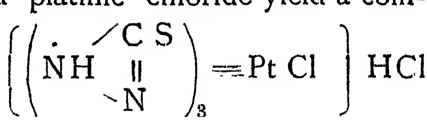
 thiocarbamide yields

with mercuric, platinic, and cupric chlorides, compounds of the types



Thioacetamide gives with platinic chloride



Thiosemicarbazide and platinic chloride yield a compound of the formula 

Thiocarbamide and monochloracetic acid form formamidine thiolactic acid

$$\begin{matrix} NH \\ NH_2 \end{matrix} \gg C.\ S.\ CH_2.\ CO\,OH$$

in aqueous solution though the hydrochloride of this is formed in acetone solution; mono-alkylthiocarbamide also yields corresponding substituted derivatives.

## *A New Type of Salts: Mercuric Chloro-iodide.*

By Rasik Lal Datta, M.Sc.

Mercuric chloride has been prepared in two ways (1) by heating in a sealed tube mercurous chloride and iodine (2) by the careful sublimation of a mixture of mercuric chloride and mercuric iodide. Like mercuric iodide it turns yellow and that at a definite temperature. Its pyridine compound was also prepared. The constitution of the salt could not however be definitely assigned at present, on which further work is in progress.

## *Experimental demonstration of Interference Fringes due to two point or linear sources nearly in line.*

By C. V. Raman, Esq., M.A.

In three papers published in the Philosophical Magazine for November 1906, January 1909 and May 1911, it has been shown by Mr. Raman that the diffraction-bands of the Fraunhofer class formed

by an obliquely-held rectangular aperture are unsymmetrical in character, both the width of the bands and the distribution of their intensity exhibiting this effect. Mr. Raman has since studied the case of two narrow rectilinear apertures parallel to each other in a plane on which the incident light fell very obliquely. The interference-fringes formed by the apertures at a great distance behind them or in the focal plane of the converging object-glass were exhibited at the meeting, and it was seen that they were also markedly unsymmetrical in character. The analogies and differences between this case and that of Newton's Diffusion-rings and the interference-rings of two point sources nearly in line were pointed out and explained. A full description with illustrative photograph, measurements and a mathematical discussion will appear in a special Bulletin.

## *On the Correlation of the Kamthi Beds.*

By Hem Chandra Das-Gupta, M.A., F.G.S.

Hitherto a two-fold classification of the Gondwana System has been generally accepted and these two groups are distinctly marked off from each other by very characteristic palaeobotanical considerations (1). In a small book on Indian geology by Mr. Vredenburg (2) the Gondwana System has been grouped into three main divisions—the lower, the middle and the upper. This three-fold division led me to study the question about the correlation of some of the Gondwana beds and the present note embodies the results I have arrived at.

The first attempt to establish a three-fold division of the Gondwanas was due to the late Dr. Feistmantel (3). His classification is given below and as my remarks apply only to the lower and the middle Gondwanas, the sub-division of the upper Gondwanas has been omitted.

| | |
|---|---|
| Upper Gondwana. | |
| Middle Gondwana | Transitional beds.<br>Panchet division.<br>Damuda division. |
| Lower Gondwana | Karharbari beds with fossils<br>Talchir shales with fossils<br>Talchir boulder bed without fossils. |

Dr. Feistmantel correlated the lower Gondwanas with the Carboniferous-Permian and the middle with the Trias. This classification has also been practically

(1) A manual of the Geology of India, by R.D. Oldham, p. 155.

(2) A Summary of the Geology of India, (second edition) p. 50.

(3) Pal. Ind., Ser. XII, Vol. IV, p. XXV.

followed by Frech and Nœtling (4) the only main difference being in the matter of correlation, as according to the authors of Lethæa Geognostica the lower Gondwanas correspond to the kohlenführende Dyas and also in the absence of any mention of the 'transitional beds'. Mr. Vredenburg's three-fold division, however, differs considerably from the groupings above referred to, as the following table will show :—

| | | |
|---|---|---|
| Upper Gondwana. | | |
| Middle Gondwana | Maleri.<br>Kamthi.<br>Panchet. | |
| Lower Gondwana | Damuda. | Raniganj.<br>Ironstone shales.<br>Barakar. |
| | Talchir. | Karharbari.<br>Talchir. |

So far as our present knowledge is concerned there cannot be any difference of opinion regarding the propriety of uniting the Talchir and the Damuda series of rocks under one main division—the lower Gondwanas, and my remarks will be confined towards the members united under the middle Gondwanas by Mr. Vredenburg with special reference to the correlation of the Panchet and the Kamthi series of rocks, because the relative position of the Panchet and the Kamthi beds, as shown above, appears to me to be of doubtful value.

For an account of the Panchet beds we are indebted to the late Dr. Meddlicott (5) and these beds contain, besides some vertebrate and invertebrate remains,

(4) Leth. Geogn. Pal. Vol. II, pp. 608-609.

(5) Mem. Geol. Surv. India, Vol. III, p.p 126-137.

some plant fossils which will be mentioned in the sequel. Our first detailed knowledge about the Kamthi beds was derived from the observations of the late Dr. Blanford (6). Besides the occurrence in the Nagpur area, the Kamthi beds have also been met with chiefly in the Wardha valley coal-field (7) and in the Godavery district (8). The well-known Kamthi flora will be enumerated later on.

We will now proceed to consider the chronological relation between the Kamthi and the Panchet beds. The Panchets are known to overlie the Raniganj beds but no section has as yet been recorded indicative of the relationship between the Panchet and the Kamthi series (9). In the Nagpur area the Kamthi beds have been found overlying the Talchirs while in the Wardha valley and in the Godavery-Pranhita valley they have been found associated with the Barakars and in places the Kamthis have been found overlapping the Barakars (10) while evidences are also adduced pointing out to the possibility of a post-Barakar and pre-Kamthi denudation (11). This is all that is known about the relationship between

(6) Mem. G.S.I., Vol. IX, pp. 305 et. seq.

(7) Mem. G.S.I., Vol. XIII, pp. 66-81.

(8) Mem., G.S.I., Vol. XVI., pp. 208-211 and Mem. G.S.I., Vol. XVIII, pp. 250 et. seq.

(9) In his small treatise Mr. Vredenburg says "one observes sometimes a gradual passage from the Panchet to the Kamthi stage," (p. 58). I am, unfortunately, unable to follow this statement unless it is due to the fact that Mr. Vredenburg uses Kamthi and Pachmari as synonymous (p. 57).

(10) Mem. G.S.I., Vol. XIII, p. 67.

(11) Ibid. pp. 65-66.

the Damudas and the Kamthis in the field and from this we can only conclude that the Kamthi stage is younger than the Barakar stage, but no light is thrown about the mutual relationship between the Kamthi and the Panchet beds.

Stratigraphical evidence failing we have to apply to the organic contents. Mr. Vredenburg has correlated the Kamthi beds with the Rhaetic on theground that *Danæopsis Hughesi* Feistm. has been met with in these beds (12). We are however unable to follow this statement for reasons given below.

According to the late Dr. Feistmantel (13) *Danæopsis Hughesi* is found in his Transitional beds, including under this epithet a group of peculiar beds with both lower and upper Gondwana fossils and found in the South Rewa Basin. Hughes found these beds to be more extensive than what was estimated by Dr. Feistmantel and these beds were styled as Supra-Barakars (14). It may be observed here that in course of a description of the Auranga coal-field flora, some beds have also been designated as Transitional Beds (15). These beds have no *Danæopsis Hughesi* and include chiefly the very common lower Gondwana fossils and on the scanty material available nobody can seriously think of correlating these two Transitional Series of rocks.

The inclusion of the Transitional beds with the Kamthi beds seems to be originally suggested by

(12) op. cit. p. 59.
(13) Pal. Ind., Ser. XII, Vol. IV, p. 6.
(14) Mem., G.S.I., Vol. XXI, p. 207.
(15) Pal. Ind., Ser. XII, Vol. IV, pt. 2, p. 11.

Hughes (16), though he himself was not very quite sure of it and thus the term Supra-Barakar, above referred to, was introduced instead of Transitional, but not Kamthi. This correlation appears to be doubtful and for this I shall, first of all, enumerate the plants that have been recorded from the Kamthi beds as well as from the Supra-Barakars.

| Kamthi fossils (17) | Supra-Barakar fossils (18) |
| --- | --- |
| 1. *Phyllotheca indica* | 1. *Schizoneura* (?) |
| 2. *Taeniopteris danoeides* | 2. *Asplenium whitbiense* |
| 3. ,, *Feddeni* | 3. *Danæopsis Hughesi* |
| 4. ,, *cf. Mc. Clellandi* | 4. *Glossopteris indica* |
| 5. *Glossopteris indica* | 5. ,, *ampla* |
| 6. ,, *ampla* | 6. *Sphenopteris polymorpha* |
| 7. ,, *Browniana* | 7. *Thinnfeldia cf. odontopteroides* |
| 8. ,, *angustifolia* | 8. *Noeggerathiopsis hislopi* |
| 9. ,, *stricta* | |
| 10. *Gangamopteris cyclopteroides* | |
| 11. *Noeggerathiopsis hislopi.* | |
| 12. *Cardiocarpus (?) sp.* | |

Comparison of these two lists brings out a few features which are remarkable. These features are:—

(i) The presence of *Gangamopteris cyclopteroides* in the Kamthi beds and its absence from the Supra-Barakar.

(ii) The presence of *Asplenium Whitbiense* and of *Thinnfeldia cf. odontopteroides* in the Supra-Barakar and the absence of these two genera from the Kamthi.

(16) Mem. G.S.I. Vol. XXI, p. 208.

(17) A manual of the Geology of India by R. D. Oldham p. 169; The Glossopteris flora by Arber., p. 206; Pal. Ind. New. Ser., Vol. II, Mem. I, p. 3.

(18) Pal. Ind. Ser. XII, Vol. IV.

(iii) The smaller number of species of *Glossopteris* in the Supra-Barakar beds than in the Kamthi and the absence of any new species in the former.

It is now known that *Gangamopteris* is confined to the Palaeozoic rocks (19), *Asplenium whitbiense* is Mesozoic (20) and *Thinnfeldia odontopteroides* is upper Triasic or Rhaetic (21). Thus the association of *Gangamopteris* with *Asplenium whitbiense* and with *Thinnfeldia odontopteroides* is not to be naturally expected and this is also corroborated by the evidence of the fossils cited above. About the third peculiarity attention will be drawn a little later on.

Though these two series of fossils differ from each other as stated above there are however three species which are common to both. These are *Glossopteris indica*, *Glossopteris ampla*, and *Noeggerathiopsis hislopi*. These three are, however, among the most cosmopolitan of the Glossopteris flora, the last being found even as far as Siberia (22).

The most important feature of the Supra-Barakars is, however, the presence of *Danæopsis Hughesi*. Besides being found in the Rewa basin, this species has also been met with in South Africa, China and Tonkin. In South Africa *Danæopsis Hughesi* has been obtained from the Burghersdorp beds or Upper Beaufort Series. These fossils have been described

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(19) Seward : Fossil plant. Vol. II, p. 513.

(20) Ibid p. 344.

(21) Ibid p. 538.

(22) Mem. du Com. Geol. St. Petersburg. New Ser. Livr. 86 (1912).

by Prof. Seward (23) and, besides *D. Hughesi*, they include :—

1. *Schizoneura sp.*
2. *Taeniopteris carruthersi.*
3. *Glossopteris sp.*
4. *Odontopteris Browni.*
5. *Strobilites laxus.*
6. *Pterophyllum sp. cf. Tietzii.*
7. *Stigmatodendron dubium.*
8. *Thinnfeldia sphenopteroides.*

In Tonkin *Danæopis cf. Hughesi* Fstm. has been found in the island of Sommet Buisson. A rich flora has been found from this region. The fossils have been described by Zeiller (24) and the Rhætic age has been assigned to them.

The knowledge of *Danæopsis Hughesi*, as it occurs in China, is extremely meagre (25). This fossil has been found in the gorge near the village of San-schi-lipa ; no other fossil has been found associated with it while the beds underlying the *Danæopsis*-bearing beds have a *Glossopteris*-facies with *Cordaites*-like leaf which may be *Cordaites* or *Noeggerathiopsis hislopi.*

A comparison of the flora of the *Danæopsis-Hughesi*-bearing beds, as found in these areas, so wide remote, with the Kamthi fossils shows that the fossils found in these beds are all distinguishable

(23) Q. J. G. S. 1908. pp. 83-105; The Geology of Cape Colony by Rogers and Toit, p. 206.

(24) Etudes des Gites Min. France Paris, 1903.

(25) Denkschr. d. Kais. Akad. Wiss. Vol. LXX, 1901, pp. 144-145.

from the Kamthi fossils by (*a*) the absence of *Gangamopteris*, (*b*) the presence of many mesozoic genera and (*c*) a very limited occurrence of *Glossopteris*. From these considerations it becomes quite clear that the Kamthi beds are much older than the beds furnishing *D. Hughesi* (26).

The next point for discussion is the relative age of the Kamthi and the Panchet beds. For a solution of this question an appeal should be made to the floral evidence. The Kamthi flora has been given above and the following list includes the Panchet plants (27):—

1. *Schizoneura Gondwanensis.*
2. *Tæniopteris stenoneura.*
3. *Glossopteris indica.*
4. ,, *ampla.*
5. ,, *angustifolia.*
6. *Thinnfeldia cf. odontopteroides.*
7. *Cyclopteris (?) pachyrhaca.*
8. *Pecopteris concinna.*

---

(26). Very recently *Danaeopsis Hughesi* has been found by Zalessky from the upper Permian of the Petschora basin. The associates include *Gangamopteris rossica*, a species founded on leaves agreeing in shape and in the absence of a definite midrib with other forms of the genus, but distinguished by the rarity of anastomosis between the secondary veins, a feature regarded as due in part to imperfect preservation. The species may be identical with that described by Schmalhausen as *Zamiopteris glossopteroides*, which Zeiller compares with *Lesleya*. It is, however, interesting and suggestive that a few cross-connections occur between the veins, and it is by no means unlikely that Zalessky is correct in adopting the generic name *Gangamopteris*. The almost complete absence of anastomosis in some *Glossopteris*-fronds from India and South Africa affords a parallel case.

(Seward: Antarctic fossil plants 1914. p. 36).

(27) A manual of the Geology of India by R.D. Oldham p. 171; The Glossopteris flora by Arber.

A comparison of this with the Kamthi flora also brings out clearly the presence of some mesozoic plants, the absence of *Gangamopteris* and the smaller number of the species of *Glossopteris*. This last point requires a little elucidation.

A table is given below showing the distribution of the species of *Glossopteris* in the different Gondwana beds and it is quite clear that so far as the development of *Glossopteris* is concerned the genus begins from the Talchir, attains its maximum in the Raniganj and then begins to decline till it reaches the Jubbulpore stage where, according to Feistmantel (28), it is represented by a single species *Gl. cf. indica* and that no new species appeared after the Raniganj series.

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(28). Pal. Ind. Ser. XI. Vol. II. p. 90.

| Species. | Talchir. | Karhar-bari. | Barakar. | Ironstone Shale. | Raniganj. | Kamthi. | Panchet. |
|---|---|---|---|---|---|---|---|
| 1. *Glossopteris indica* | + | + | + | + | + | + | + |
| 2. ,, *decipiens* | | + | | | | | |
| 3. ,, *ampla* | | + | + | + | + | + | + |
| 4. ,, *Browniana* | | | + | ... | + | + | |
| 5. ,, *intermedia* | | | + | ... | + | | |
| 6. ,, *angustifolia* | | | + | + | + | + | + |
| 7. ,, *retifera* | | | | + | + | | |
| 8. ,, *conspicua* | | | | + | + | | |
| 9. ,, *tortuosa* | | | | | + | | |
| 10. ,, *orbicularis* | | | | | + | | |
| 11. ,, *formosa* | | | | | + | | |
| 12. ,, *divergens* | | | | | + | | |
| 13. ,, *stricta* | | | | | + | + | |
| 14. ,, *longicaulis* | | + | | | | | |

This list leads one to suspect that the Kamthi flora is older than the Panchet flora and this suspicion is further strengthened by a comparison of all the fossil plants found in the different series just referred to. The following table will exhibit this quite clearly :—

| Species. | Talchir. | Karhar-bari. | Barakar. | Ironstone shale. | Raniganj. | Kamthi. | Panchet. |
|---|---|---|---|---|---|---|---|
| 1. *Phyllotheca indica* | | | | | + | + | |
| 2. „ *robusta* | | | | | + | | |
| 3. „ *Griesbachi* | | | + | | | | |
| 4. *Schizoneura Wardi* | | + | | | | | |
| 5. „ *gondwánensis* | | | + | ... | + | ... | + |
| 6. *Actinopteris bengale-nsis* | | | | | + | | |
| 7. *Sphenophyllum speciosum* | | | + | ... | + | | |
| 8. *Tæniopteris danæ-oides* | | | + | + | + | + | |
| 9. „ *Roylei* (29) | | | | | + | | |
| 10. „ *Feddeni* | | | + | ... | + | + | |
| 11. „ *cf. Mc Clellandi* | | | | | + | + | |
| 12. „ *cf. stenoneura* | | | | | | | + |
| 13. *Dictyopteridium sporiferum* | + | ... | ... | ... | + | | |
| 14. *Palaeovittaria Kurzi* | | | | | + | | |
| 15. *Glossopteris angustifolia* | | + | + | + | + | + | + |
| 16. „ *indica* | + | + | + | + | + | + | + |
| 17. „ *ampla* | | + | + | + | + | + | + |

(29) In his Glossopteris flora Arber mentions two plants, *Cladophlebis Roylei* and *Cladophlebis sp.* as occurring in the Damuda beds (pp. 142-144). One of these, viz. *Cladophlebis sp.* is extremely fragmentary and no reliance can be put on it; the other was named *Pecopteris Lindleyana* by Royle and *Alethopteris Lindleyana* by Feistmantel while the arguments that have led Arber to refer this plant to *Cladophlebis* do not seem to be very conclusive. (Geol. Mag. 1901, pp. 548-549).

| Species. | Talchir. | Karhar-bari. | Barakar. | Ironstone shale. | Raniganj. | Kamthi. | Panchet. |
|---|---|---|---|---|---|---|---|
| 18. *Glossopteris decipiens* | | + | | | | | |
| 19. ,, *Browniana* | | | + | ... | + | + | |
| 20. ,, *intermedia* | | | + | ... | + | | |
| 21. ,, *retifera* | | | | + | + | | |
| 22. ,, *conspicua* | | | | + | + | | |
| 23. ,, *divergens* | | | | | + | | |
| 24. ,, *stricta* | | | | | + | + | |
| 25. ,, *formosa* | | | | | + | | |
| 26. ,, *orbicularis* | | | | | + | | |
| 27. ,, *tortuosa* | | | | | + | | |
| 28. ,, *longicaulis* | | + | | | | | |
| 29. *Gangamopteris cyclopteroides* | + | + | + | + | + | + | |
| 30. ,, *angusftiolia* | + | | | | | | |
| 31. ,, *Whittiana* | | | | | + | | |
| 32. ,, (?) *buriadica* | | + | | | | | |
| 33. *Neuropteridium validum* | | + | | | | | |
| 34. *Sphenopteris polymorpha* | | | + | ... | + | | |
| 35. ,, *Hughesi* | | | | + | + | | |
| 36. *Belemnopteris Wood-Masoniana* | | | | | + | | |
| 37. *Cyclopteris dichotoma* | | | + | | | | |
| 38. ,, *pachyrhacis* | | | | | | | + |
| 39. *Pecopteris phegopteroides* | | | | | + | | |

| Species. | Talchir. | Karhar-bari. | Barakar. | Ironstane shale. | Raniganj. | Kamthi. | Panchet. |
|---|---|---|---|---|---|---|---|
| 40. *Pecopteris concinna* | | | | | | | + |
| 41. *Merianopteris major* | | | | | + | | |
| 42. *Thinnfeldia cf. odontopteroides* | | | | | | | + |
| 43. *Noeggerathiopsis Hislopi* | + | + | + | + | + | + | |
| 44. „ *Whittiana* | | + | | | | | |
| 45. „ (?) *lacerata* | | + | ... | ... | + | | |
| 46. „ (?) *Stoliczkana* | | + | | | | | |
| 47. *Cardiocarpus indicus* | | + | | | | | |
| 48. „ (?) *Milleri* | | + | | | | | |
| 49. *Pterophyllum Balli* | | | + | | | | |
| 50. *Rhipidopsis ginkgoides* | | | + | | | | |
| 51. „ *densinervis* | | | | | + | | |
| 52. *Ottokaria bengalensis* | | + | | | | | |
| 53. *Voltzia heterophylla* | | + | ... | ... | + | | |

A comparison of this list leaves no room to doubt that the Kamthi flora is older than the Panchet flora but younger than the Raniganj flora as a whole.

The gradual passage of the Raniganj to the overlying Panchet may be used as an argument against accepting the Kamthi beds as being intermediate between the Raniganj and the Panchet. But a little consideration will show that this obstacle is not really so difficult as it appears to be.

While describing the Kamthi beds of the Wardha Valley coalfield, Hughes spoke of a Kamthi-Barakar unconformity due to a pre-Kamthi denudation (30). From this it is not unreasonable to think that while deposition in the Damuda basin was going on after the Barakars had been deposited, the conditions in the Wardha Valley and in the other areas where the Kamthi beds are developed were such as to render any fresh deposit impossible. In these areas the denuding agencies were in operation and this period of denudation practically coincided with the period of deposition of the whole of the ironstone shales and a great part of the Raniganj group and it is only when the uppermost members of this last group were being deposited that a migration of a part of the decaying Raniganj flora took place to the regions where the Kamthi beds are now found.

Mr. Vredenburg has put the Panchet, the Kamthi and the Maleri beds under the middle Gondwanas. We have given above our reasons for holding that the Kamthi beds are older than the Panchet. *Glosso-*

(30) Mem. Geol. Surv. Ind., Vol. XIII. pp. 65-66.

*pteris* has been found all throughout these formations showing its period of initial stage, maximum development and practical extinction and it seems that this is an extremely important palaeontological point which should not be lost sight of in any scheme regarding the classification of the Gondwanas. Hence I think it advisable to follow the author of the Manual of Indian Geology and to unite all the *Glossopteris*-bearing beds under one group—the lower Gondwanas, the lower Gondwanas consisting chiefly of the following series of rocks :—

| | |
|---|---|
| Panchet | Panchet |
| Raniganj (including the Kamthi beds towards the top) | Damuda |
| Ironstone shale | |
| Barakar | |
| Karharbari | Talchir |
| Talchir | |

It may be pointed out here that, while describing the Kamthi beds, the late Dr. Blanford was also led to formulate an opinion similar to that sketched above on the material then available (31) and this note of mine may be looked upon as an elucidation of the view suggested by the late illustrious deceased.

There now only remain the Maleri beds placed by Mr. Vredenburg at the top of his middle Gondwanas and considering the rich suite of vertebrate remains in these beds together with the complete absence of plant fossils, a distinction of these beds (and along with them of the beds at Tiki and at Denwa) into a separate group—the middle Gondwanas—might be desirable.

(31) Mem.. Geol. Surv. Ind. Vol. IX. pp. 329-330.

copper oxide obtained is 0·1913. From this it can be calculated that the percentage of copper present in the substance is 30·06.

*Experiment II* :—

0·200 grms substance taken, which is decomposed with sulphuric acid and residue dissolved in dilute hydrochloric acid as before, from which copper is precipitated with sodium thiosulphate and estimated as sulphide. The amount of copper sulphide found in 0·2000 grms. of the substance is 0·0776 grms. from which copper present is 31·00 per cent.

*Experiment III* :—

In this experiment done exactly in the above way, the substance taken is 0·2016 grms. CuS obtained is 0·0784 grms. Hence Cu present is 31·04 per cent.

Estimation of Iron.

*Experiment No. I* :—

0·514 grms. substance taken, decomposed by ignition and the residue dissolved in HCl, Cu is eliminated by $H_2S$ as sulphide and from the filtrate, iron is precipitated as hydrate and estimated as oxide in accordance with the usual method, which is actually found in this case to be 0·1015 grms whence iron is 13·80 per cent present.

*Experiment II* :—

In this experiment done exactly in the above way from 0·4046 grms. of the substance, 0·0786 grms. of $Fe_2O_3$ is obtained from which Fe present is 13·58 per cent.

*Experiment III* :—

0·4698 grms. of the substance taken which is decomposed with strong $H_2SO_4$ and the residue dissolved in dilute HCl and made up to 200 cc. The solution is divided into two equal parts from each of which iron is precipitated as hydrate and estimated as oxide.

The amount of $Fe_2O_3$ obtained from the 1st solution is 0·0495 grms. from which Fe present is 14·78 per cent.

The amount of $Fe_2O_3$ obtained from the 2nd solution is 0·0483 grms. from which Fe present is 14 38 per cent.

The average of the two experiments is 14·58 per cent.

Estimation of ammonia.

*Experiment I* :—

0·3716 grms. of the substance taken, which is distilled with caustic potash and the distillate collected into 10cc of $\frac{N}{2}$ $H_2SO_4$ diluted with distilled water. When the distillation is finished, the amount of $H_2SO_4$ which has not been neutralized is estimated by tritration with $\frac{N}{5}$ caustic soda. The amount of caustic soda solution required for the purpose in this experiment is 5·80 cc. Hence ammonia present in the substance is 17·56 per cent.

*Experiment II* :—

0·8138 grms. of the substance taken of which the distillate is collected as before into 20 cc. of $\frac{N}{2}$ $H_2SO_4$ and the amount of $\frac{N}{5}$ caustic soda solution required for the unneutralized acid is 12·8 cc. From this ammonia present in the substance is 15·53 per cent.

*Experiment III* :—

0·8760 grms. of the substance taken which is distilled as before into 20 cc. of $\frac{N}{2}$ $H_2SO_4$ and the unneutralized acid required 8·4 cc. of $\frac{N}{5}$ NaOH which gives 15·00 per cent. of ammonia to be present in the substance.

Estimation of total Nitrogen.

*Experiment I* :—

0·0988 grms. of the substance taken, from which the total nitrogen is set free by Kjeldal's method and is collected into 10 cc. of $\frac{N}{2}$ $H_2SO_4$. The amount of unneutralized acid present is determined by tritration with $\frac{N}{5}$ KOH of which 12·7 cc. is required in the present case. Hence total Nitrogen present is 34·85 per cent.

*Experiment II* :—

Another experiment similarly done by taking 0·100 grms. of the substance and collecting the resultant ammonia into 10 cc. of $\frac{N}{2}$ $H_2SO_4$. The amount of $\frac{N}{5}$ KOH required for the neutralization of the unneutralized acid in this case is 12·50 cc. from which the amount of total nitrogen present is 35·00 per cent.

*Experiment III* :—

The percentage of nitrogen is determined by Dumas method and is found to be 35·20 per cent.

Combustion analysis.

*Experiment I* :—

0·1598 grms. of substance taken, weight of which after combustion is 0·0934 grms., *i.e.*, the residue is 58·44 per cent of the original substance.

Wt. of caustic potash tube + $CO_2$ = 56·3206
Weight of KOH tube = 56·2212

$CO_2$ present = 0·0994
which gives 16·96 per cent carbon present.

wt. of $CaCl_2$ tube + $H_2O$ = 50·5002
wt. of $CaCl_2$ tube = 50·4472

$H_2O$ present = 0·0530
which gives 3·68 per cent $H_2$ present.

*Experiment II* :—

0·1004 grm. substance taken, weight of which after combustion is 0·0592 grms. which is 58·96 per cent.

wt. of KOH tube + $CO_2$ = 56·2212
wt. of KOH tube = 56·1560

$CO_2$ present = 0·0652
which gives 17·50 per cent carbon.

wt. of $CaCl_2$ tube + $H_2$ O = 56·4244
wt. of $CaCl_2$ tube = 56·3898

$H_2O$ present = 0·0346
which gives 3·82 per cent of $H_2$.

Percentage of residue after ignition.

In connexion with combustion analysis just described, I have noted that the percentage of residue after combustion in two experiments to be respectively 58·44 per cent and 58·96 per cent.

Similar result can be obtained by igniting the substance over a blow-pipe. For example I took 1·5326 grms. of the substance which after ignition gave 0·895 grm. of residue which gives 58·40 per cent.

The result of another similar experiment is 58·22 per cent.

Theoretical formula of the substance.

It will be seen from the following table that the results obtained from the analysis correspond with the theoretical percentages as calculated from the formula $Cu_2$ Fe $C_6N_6$ 4 $NH_3$. Now a substance having the above formula if strongly ignited will leave a residue consisting of CuO and $Fe_2O_3$. If we start with two molecules of the substance, *viz.*, $2Cu_2$ Fe $C_6$ $N_6$, 4 $NH_3$ the residue after ignition will be 4 CuO + $Fe_2O_3$. Molecuar weight of two molecules of substance is 814·2.

Molecular weight of 4 CuO + $Fe_2O_3$ is 478·2.

$$\therefore \frac{\text{wt. of residue after ignition} \times 100}{\text{wt. of substance taken}} = \frac{478{\cdot}2 \times 100}{814{\cdot}2}$$

= 58·73 per cent.

It will be seen from the annexed table that this theoretical percentage of residue also corresponds closely with the result actually obtained.

| | Experiment I. | Experiment II. | Experiment III. | Average of previous experiments. | Calculated from theoritical formula. |
|---|---|---|---|---|---|
| Percentage of Cu ... | 30·06 | 31·00 | 31·04 | 30·70 | 31·24 |
| do. of Fe ... | 13·80 | 13·58 | 14·58 | 13·98 | 13·73 |
| do. of $NH_3$ ... | 17 08 | 15·53 | 15·00 | 15·87 | 16·70 |
| do. of total N ... | 34·85 | 35·00 | 35·20 | 35·01 | 34·39 |
| do. of C ... | 16·96 | 17·50 | ... | 17·23 | 17·68 |
| do. of $H_2$ ... | 3·68 | 3·82 | ... | 3·75 | 2·94 |
| Percentage of residue after ignition. | 58·44 | 58·96 | 58·40 | 58·60 | 58·73 |

A newly discovered compound or not?

Searching carefully through the literature, it appears that none has noticed previously to me that an ammoniacal solution of copper ferricyanide when kept for sometime gives a crystalline precipitate of a definite chemical compound. However some of the previous workers have described compounds, having analogous composition, which requires comparison with the one obtained by me.

The following occurs in Gmellin's book on Chemistry Vol. VIII.

"4 $NH_3$, $C_6N_3$ Fe $Cu_2$ (Cyano ferrure de cûvre biammoniacal). Ferrocyanide of copper digested with aqueous ammonia diminish in bulk and becomes green and crystalline, but if the ammonia be poured off and water added the combined ammonia dissolves out and the red colour is restored. This experiment may be repeated *ad lebitum*; the decanted ammonia is pale green containing but a small quantity of copper in solution and when mixed with water in closed glass vessels, deposits an orange yellow substance (Vauquelin Ann Chem Phys, 9, 120, also schow 25,60)."

It will be easily seen that the substance described above possesses physical characteristics entirely different from those of the substance prepared by me and hence is a distinct substance. The above also holds true regarding another substance described by Gmellin to which he ascribes the formula "4 $NH_3$, $C_6N_3$ Fe $Cu_2$ + Ag."

J. Messner, a German chemist carried on a research regarding ferrocyanide compounds and his researches have been published in Ziet Anorg Chemie 8, 363—393. A summary of the article has been published in

the Journal of the Chemical Society Abstracts 1895, part I, pages 405 to 407.

The following is an extract from the above summary. "$Cu_2 (Fe C_6N_6), 8 NH_3$ is obtained by adding a solution of ammonium ferrocyanide in strong ammonia to a solution of cuprous chloride in the strongest ammonia and allowing the mixture to remain for a few hours. It crystalizes in beautiful black, lustrous prisms, is very soluble in ammonia and is not decomposed by cold absolute alcohol. The crsytals on exposure to air quickly decompose into ammonia, copper ferrocyanide and the compound, $(Cu_2 Fe C_6N_6), 4 NH_3$."

It will be seen from the above description that the substance obtained by J. Messner which occurs as a decomposed product, must be an amorphous substance, hence a different substance from that obtained by the writer which is a crystalline substance, though both these substances possess identical chemical formula *viz.*, $Cu_2 Fe C_6N_6, 4 NH_3$.

Above I have tried to give all the references I have been able to find out from the literature about the substances possessing analogous Chemical formula with that obtained by me and as I have shown that all these substances obtained by other workers possess different physical characteristics from those of substane obtained by me, I may conclude that the substance described in this paper is a chemical compound which has not been previously obtained by other chemists.

In conclusion I beg to thank the authorities of the Indian Association for the Cultivation of Science for their kindly allowing me to work in the laboratory of the Association.

## *Experimental Demonstration of Combinational Vibrations.*

By C. V. Raman M.A.

In Bulletin No. 11 of this Association and the Physical Review for January 1915, the author has shown how two tuning-forks simultaneously varying the tension of a string stretched between them, may maintain vibrations whose frequencies are related jointly to those of the two forks. By using electrically maintained forks, the vibrations of the string thus maintained may be readily projected on the screen. This was shown at the meeting, and the various frequencies of vibration and their corresponding forms became very evident to the audience.

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The Chairman vacated the Chair and Dr. B. L. Chaudhury occupied it.

## *Synthesis of the Salts of the Sulphonium Series with two Sulphur Atoms.*

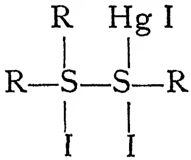
By Dr. Ray, C.I.E., D.Sc., Ph.D.

It is found that when mercuric nitrite acts upon the mercaptans (*e. g.*, methyl and ethyl mercaptans) a class of crystalline bodies are obtained. Thus :

$$Et\,S\,H + Hg\,(NO_2)_2 = Et\,S\,Hg\,NO_2 + HNO_2$$

These nitromercaptides, when treated with an alkyl iodide, give rise to a series of compounds of the Disulphonium type, having the empirical formula

$R_2 S_2 Hg I_2 R I$. They are really disulphonium compounds and are formulated thus :



where R represents an alkyl radicle.

It may be mentioned here that Hilditch and Smiles obtained the same salt by a different method, although they regarded it as a derivative of the monosulphide. (T., 1907, *91*, 1397).

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Dr. B. L. Chaudhury vacated the Chair and Dr. P. C. Ray, occupied it again.

*On the occurrence of some gases in magnesite from Salem.*

By JNANENDRA CHANDRA GHOSH and others.

The presence of gases in rocks and minerals has been extensively investigated by Ramsay, Travers, Tilden, Guitor, etc. Helium has been found in radioactive minerals, hydrogen, carbon monoxide, etc.

In the course of an estimation of nitrogen by Duma's method, it was noticed that if Salem magnisite be substituted for $NaHCO_3$ the apparatus could not be completely freed from gases insoluble in KOH, at the initial stages of the experiment. This led to a suspecion, that magnesite on heating gives off besides $CO_2$, a gas insoluble in lye.

Further experiments to determine the nature of this residual gas were thus carried out :—A piece of magnesite was freed from all sources of extraneous contamination by striking off the superficial parts with a porcelain pestle, and then ground to powder in a

porcelain mortar. The combustion tube was subjected to a preliminary heating for an hour in a current of $CO_2$ to drive off all gases adsorbed on its surface. It was then filled with magesite and the air inside chased off by a current of $CO_2$ obtained from a $NaHCO_3$ tube attached to it. The gas generated by heating magnesite was next collected over lye. The residual gas was found to be partly soluble in ammoniacal cuprous chloride solution ; the insoluble part exploded with oxygen, but the reaction products did not give any test of carbon dioxide. The gas therefore is probably a mixture of hydrogen and carbon monoxide.

Again on dissolving magnesite in hydrochloric acid, a gas is obtained which is not soluble in KOH solution. The nature of this gas is also under investigation.

The presence of hydrogen and carbon monoxide may be explained by assuming—Firstly that they remain occluded in the mineral ; Secondly that a chemical reaction takes place between carbon dioxide, moisture, and the carbonaceous matter that may be present in this sedimentary rock thus

$$CO_2 + C \rightleftharpoons 2\,CO.$$

$$H_2O + C \rightleftharpoons H_2 + CO.$$

Experiments are in progress to determine which of these hypothesis is the correct one.

An analysis of the sample of magnesite by Dains, will prove interesting,—

Silica—0·29 % Iron oxide and Alumina—0·3 %
MgO—47·35 % $CO_2$—51·44 % Combined water—0·27 %

PROCEEDINGS

OF THE

# INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. I. No. 4.

Wednesday, December 22nd, 1916, at 5 P.M., C. V. Raman, Vice-President, in the Chair.

## *On the Ancient Hindu conception of Ether.*

By MANINDRA NATH BANERJEE, F.C.S.

Preliminary note.

*Ether in the Vedas*—

*Yaska* mentions sixteen synonyms of *ether* in the *Vedas*. They are :—

अम्बरम् (*ambaram*), वियत् (*biyat*), व्योम (*byoma*) बर्हिः (*barhih*), धन्व (*dhanva*), अन्तरिक्षम् (*antariksam*), आकाशम् (*ākasam*), आपः (*apah*), पृथिवी (*prithivi*), भूः (*bhuh*), स्वयम्भूः (*svayambhuh*), अध्वा (*adhva*), पुष्करम् (*puskaram*), सगरः (*sagarah*), समुद्रः (*samudrah*), अध्वरम् (*adhvaram*)*. The explanation of each of these terms as rendered by some of the oldest vedic commentators has been fully given in the original paper of which this is a mere abstract. But it may be interesting to bring forward the term '*adhvaram*' allied and akin to the Greek '*aether.*'

* निरुक्तम् Nighantü—I,3.

The Sanskrit term *adhvaram* means '*upper air.*' The Greek term '*aether*' means '*upper air.*' From the similarity in meaning and construction we are inclined to think *adhvar* and *aether* to be allied terms. But after its use in the *Vedas* (Rig-veda 8,3,23,3,), its occurrence in any other branch of Sanskrit Literature as a synonym of '*ether*' has not yet been, so far as our investigation is concerned, discovered and for this reason, it seems to us, there is no mention of such a term in the famous lexicon *amarkosa* along with the synonyms of *akasam.*

Again the idea of a fifth material element like *ether* in the comparative ancient literature is, as far as our investigation is concerned, nowhere found. The *Chow*, the famous Chinese ancient literature of about 2,000 years B.C. in its 4th Book on science and allied subjects does not mention anything about *ether* as it describes the characteristic qualities of five elements, *viz.*, earth, water, fire, wood and metal. The Babylonian, the Phoenician and the Egyptian literature also do not give us the idea of a fifth element. While in the earlier Greek Literature we have also the idea of four elements only. So we fail to recognise in the ancient literature, except in the *Vedas* and their allied branches, the conception of a fifth element like *ether*. Plato, in his dialogues—"*Timaeus*," makes the mention for the first time of the term *ether* and Aristotle, quite originally though, deals with it in his IV Book of Physics by an elaborate argument on the "*Plenum* or *Vacuum*, *i.e.*, space surrounding the universe." Thus here for the first time we see the idea of a fifth element introduced into European Literature.

One of the vedic commentators* explains the term '*adhva*' from which the term *adhvar* is said to be derived, saying that *it is that through which the sun constantly travels, i.e., it is one which allows passage to the sun.* We are apt by our prejudices to credit the belief that in the vedic times the people had no clear knowledge of Astronomy as they considered the sun to revolve round the earth and thus interpret the meaning of '*adhvar*' as rendered by the above commentator to be allowing passage to the moving sun in his usual revolutions round the earth. Such interpretations, however, cannot be accepted in the face of the definite mention in the Vedas—"**स वा एष न कदाचनास्तमेति**" (*Sa ba esa na kadacanastameti*) meaning that '*the sun never sets*'. Dr. Martin Hauq commenting on this passage observes—"This passage is of considerable interest containing the denial of the existence of sunrise and sunset. The vedic author supposes it (the sun) to remain always in its position in the sky * * * * ." The same view may be supported from an observation of Dr. Monier Williams in his 'Hindu Wisdom' where he says—"We may close the subject of the Brahmanas by paying a tribute of respect to the acuteness of the Hindu mind which seems to have made some shrewd astronomical guesses more than 2000 years before the birth of Copernicus." Thus from a consideration of all these assertions and various other arguments and evidences the meaning of *adhva* as giving passage to the sun may be interpreted to express as *transmitting the sun's rays* which is clearly supported by the explana-

* *Vide—Nirukta.* Bibliothica Indica (Asiatic Society of Bengal).

tions of other synonyms of ether such as, आकाशम् (*ākasam*), पुष्करम् (*puskaram*), व्योम (*byoma*), वियत् (*biyat*), etc.

In using the commentaries on the Vedas we have not only followed 'SAYANACARYA,' the great vedic commentator who is, unfortunately, not much thought of by such critics as Vincent Smith, but we have also tried to closely and freely adopt the views of such earlier vedic commentators as *Yaska* himself, *Skandasvami*, *Bhavasvami*, *Rahadeva*, *Srinivasa*, *Madhavadeva* (not *Sayana-Madhava* of later times), *Ubatabhatta*, *Bhaskaramisra*, *Bharatasvami*, as well as those of the Nirukta commentaries of *Skandasvami* and Nighantu commentaries of *Ksirasvami* and *Anantacarya* not excepting the vedic commentary of the great author of Vedadipa, viz., *Mahidhara*.

Thus following, one could see through the *Vedas* a very ancient conception of "ETHER", *as an all-pervading, all permeable, highly rarified substance, the means of transmission of light rays, a fifth state of material existence to which everything is reducible, and which was considered to be the primordial matter having motion and energy (sometimes identified with electricity itself) though it can never combine with any other body*, and such a view of '*the ether*' permeates through the whole length of Sanskrit Literature as is evidenced by the well-known expression in our BHAGAVAT GITA—

"यथा सर्व्वगतं सौक्ष्म्यादाकाशं नोपलिप्यते
सर्व्वत्रावस्थितो देहे तथात्मा नोपलिप्यते" (१३-३३)

[' 'Yatha sarvagatam sauksmyādākasam nopalipyate Sarvatrābasthito dehe tathātmā nopalipyate"—13. 33.], which has been rendered by a renowned English scholar* thus—"As the *omnipresent* ether is not affected by reason of its *subtlity,* so seated everywhere in the body the self is not affected."

## Ether in the Vedangas and the Darsanas.

After the vedic period we have tried to give an account of the conception of ETHER in the Vedangas as *Manusmriti* and other *Granthas.* Then we have attempted to project the views on ETHER of the various schools of Indian Philosophy—*viz.,* I—The Vedānta system of Vyasa, II—The Yoga system of Pātanjala, III—The Sankhya system of Kapila, IV—The Mimāmsā system of Jaimini, V—The Nyaya system of Gotama, and VI—The Nyaya-Vaisesika system of Kanāda. The whole of this literature from the days of Manu up to later times endorses fully the earliest vedic conception of ETHER and not a line of this could be construed to mean by any one to be a mere metaphysical working hypothesis of more or less of the character of fantastical dreams of the ancient Hindu philosophers.

## Ether in the comparative literature of the ancient people other than Indians.

Next we proceed to describe the Chinese, the Egyptian, the Babylonian and the Phoenician conception of the elements and try to shew that the idea of a fifth element like that of *ether,* is altogether wanting in them. The same is the case with the Greek schools

* Mrs. Annie Besant.

of Thales, Anaximander, Heraclitus, *etc.*, until in Plato's dialogues (in Timaeus) we find the mention of 'Ether' or 'upper air', a full exposition of which is dealt with by Aristotle in his Physical auscultation, Book IV. *According to Aristotle every material body must have an etherial system before or it might be reducible to ether.*

### Views of later philosophers.

After Aristotle natural philosophers theoretically supposed space to be filled with some rarified residues of vapours or gases. And this view even corresponds with Kant's, Laplace's and other theories which aim at explaining the unity and plan in the creation of the heavenly bodies. The same theory also explains the uniformity of chemical composition of the entire universe which has been practically supported by the spectrospic demonstrations of the later periods and which offers a means of interchange between heavenly bodies through the agency of *Ether.* Later on, the idea of subjecting the conception of ether as a rarified gas to experimental investigation and measurement troubled many scientists. But, however correct this idea might be, as it alone could direct the mind to the right direction and lead to reliable results, it has to be given up owing to certain practical impossibilities that came in the methods of measuring pressures with any degree of accuracy under tenths of a millimetre of mercury.

### Views of modern Scientists.

In the days of Galileo, Newton and Lavoisier, the natural science seemed to be somewhat sceptical about

the conception of ether as a rarified gas. But later on, Lord Kelvin from various considerations attributed ether some weight and came to the conclusion that a cubic metre of ether must weigh less than 0·000 000 000 000 000 1 gram while a cubic metre of hydrogen weighs 90 grams under the atmospheric pressure. Proceeding on this idea and on the facts of the discovery of the natural classification of chemical elements and of the inert group of gases, as well as of radium and radio-active properties of some elements, Mendeleeff was led to formulate a purely theoretical conception of ether, thus—*That it is an elementary gas like helium or argon, incapable of chemical combination, which he termed* X *and assigned to it a position in the periodic table high above in the Zero group of elements. That the particles and atoms of this lightest element* X, *capable of moving freely everywhere throughout the universe, have an atomic weight nearly one millionth that of hydrogen and travel with a velocity of about 2,250 kilometres per second.*

Sir James Dewar in his Presidential address of the British Association at its Belfast meeting in 1902, lent a thought of his to the idea that the highest regions of the atmosphere which are the seat of the Aurora Borealis must be considered to be the province of hydrogen and of the argon and its analogues. This, some of the leaders of contemporary sciences supposed to be only a few steps from the yet more distant regions of space and from the necessity of recognising the existence of a still lighter gas capable of permeating and filling space and thus giving a tangible reality to the conception of the ether.

## ANCIENT HINDU CONCEPTION OF ETHER COMPARED TO THAT OF MODERN SCIENCE.

But, however, valuable and important the various theoretical speculations of a large number of thoughtful scientists might be regarding the conception of ether, it must be said that modern science has not been able to proceed a step further beyond the views promulgated in the earliest statement concerning ether in the Vedic and allied branches of ancient Hindu Literature.

P. Larousse, as we learn in his 'Dictionnaire complet' defines ether as an imponderable, elastic fluid filling space and forming the source of light, heat, electricity, etc.' But, "the modern thoughtful man of science," as Mendeleeff observes, "is obliged to admit in the ether, the properties of a substance (fluid), while at the same time, in order to explain in some way the transmission of energy through space by its motion the ether is assumed to be an all-pervading 'medium'," —a conception which vie equally with that mentioned in the earliest Sanskrit Literature by the Hindu Risis of yore; while the latest attempt of the modern scientists, such as, Mendeleeff, Dewar, J. J. Thomson, Lodge, and others to identify *ether* with the *primordial matter* had its clear support from the views long expressed by the Hindu savants which fill nearly the whole range of their literature, as even in 'Surya Siddhãnta' we find—

"मनसः खं ततो वायुरग्निरापोधराक्रमात्
गुणैक वृद्धया पञ्चैव महाभूतानि जज्ञिरे" ॥ १२, २२ ॥ *

* Cf. Manu I, 19 and 20; also I, 75 to 78,

["Manasah kham tato bayuragnirapodharākramāt
Guṇaika bridhya pancaiba mahabhūtani jagñire"–12,23],
meaning—'from mind by degrees came forth *the ether* therefrom *the air*, *the fire*, *the water* and *the earth.* The five great elements thus are formed by the gradual increase of one additional property in each.'

## Conclusion.

A forgotten chapter in the history of physical science consists in the total ignoring of the important facts concerning the ancient Hindu conception of matter. Such ignoring of historical facts is due to the imperfect notion of the ancient Hindu conception of elements, the actual meaning of which the modern scientific world could not either thoroughly grasp or hold out being, as it believes, incongruous to present views on the subject. But whatever incongruity may be noticed in the conception of elements of some of the civilised races of the ancient world, we may, without any fear of contradiction, say, after a careful study and comparison of the various ancient doctrines relating to the subject, that the Hindu conception stands apart as significantly sublime in its essence as accurate in its scientific acumen as could be expected of so high and perfectly scientific a race (as even some foreign savants consider the ancient Hindus to have been), who contributed so much to the illuminating civilisation of the ancient times on earth. But quite ignorant of this led a Kopp or Wheywell or a Cajori or Ball or even a master-mind as Ernst von Meyer to belittle or ignore the ancient Hindu conception of elements, until lately at the instance of that noble minded citizen of France—the world renowned

Berthelot—attention of the European savants was drawn, to some extent, to the brilliant essay on Hindu Chemistry by our most revered countryman, Dr. P. C. Ray, which led Sir Henry Roscoe to give a complimentary note on it in his first volume of Chemistry. Yet, this is but a pioneer work only. The scientific world of modern days is still in the dark about the ancient ideas of matter, it, therefore, behoves us to project them by degrees after a thorough and searching enquiry into the various branches of our Sanskrit Literature—the well of scientific facts undefiled, which now remains unexplored. The faithful expounding of our ancient literature by our own countrymen, thoroughly conversant with modern sciences, could alone remove doubts and controversies on many points of Hindu-Oriental research which, owing to ignorance of the actual Hindu traditions and a true knowledge of Hindu religion, Hindu heredity of life, etc., has hitherto been a cypher to most of the foreign scholars.

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## *Action of Strychnine on the Dying Heart*

By Prof. N. C. Bhattacharyya, M.A., P. Dass Gupta, B.Sc., and N. N. Mitra, B.Sc. of the Presidency College, Calcutta.

A white rabbit was anaesthetised with ether; the chest was then opened and the heart exposed. It was not beating then. After opening the pericardium the volume of the heart increased considerably and within half a minnte the heart began to beat, at first slowly then normally; after ten minutes the heart began to sink; the beats became weaker and weaker; the ven

tricles could not follow the auricular contractions. The number of beats at this period was as follows :—

| No. of Ventricular beats per minute | No. of Auricular beats per minute |
| --- | --- |
| 26 | 139 |

Now the effect of strychnine on the heart was tried; and the great sensitiveness of the preparation was first noticed. The sinking heart seems to be more susceptible to drugs than any other preparation. The animal was lying in a supine position; the chest cavity formed a sort of basin in which the heart was lying. The basin could contain about an ounce of normal saline and the lower half of the heart remained submerged in the solution. The effect of strychnine was now tried. A 0.5 per cent. solution of the drug was ready. A drop of this was mixed with an ounce of normal saline. The chest cavity was then emptied of any previous fluid and the strychnine solution was then poured on the heart. After one minute the beats were counted. In the succeeding observations the number of drops of strychnine per ounce of normal saline was gradually increased. The most interesting thing noticed was that the strychnine behaved differently with the auricles and the ventricles. It increased the ventricular beats but decreased the auricular contractions, both in number and force. It tends in the case of sinking heart to remove the arhythmic condition of the heart and bring it to a condition of rhythmicity. As will be seen from the table below, it was found that with a sufficient large dose of strychnine the heart began to beat rhythmically, the ventricles following all the auricular beats.

When the dose of strychnine was increased beyond a certain limit the heart stopped. But from this condition even, the heart was revived several times by washing it with normal saline. And when the heart recovered from the effect of strychnine the same arhythmic condition was again produced. The more the strychnine was washed away the greater was the discrepancy between the auricular and ventricular beats. When strychnine was again supplied to the heart, rhythmic condition was again produced; the ventricular contractions increased and the auricular contractions decreased in number.

Now the question is, whether the strychnine acts by stimulating the nerve centres in the heart or by stimulating differently the auricular and ventricular muscle fibres. If the last view is true it would suggest that there is some difference in these fibres either in structure or in chemical composition; but—this does not seem to be warranted by our present knowledge of the subject. But there seems to be a serious draw-back about the nervous-control-theory. After opening the chest of the animal no artificial respiration was started. So the animal was really asphyxiated during the period of experiment. The peculiar strychnine effect could be obtained even after two hours. It is doubtful if the nerve cells would remain alive, so long after respiration was stopped. In the present case the rabbit's heart lived for over two and half hours. Probably the peculiar climatic condition of the tropic helped the heart to last longer. The atmosphere here at Calcutta during rainy season is saturated with water vapour; evaporation of water is

very slow. The temperature of the room where the experiment was performed was 30.5°C.

## TABLE.

*Observations on Rabbit's heart "in situ"; chest opened.*

| Ventricular beats per minute. | Auricular beats per minute. | |
|---|---|---|
| 26 | 139 | (i) The heart was washed with normal saline. |
| 21 | 43 | (ii) A drop of strychnine (0.5% sol.) mixed with an ounce of normal saline was allowed to fall on the heart. Each observation is taken after an interval of one minute. |
| 28 | 45 | |
| 17 | 103 | (iii) The heart was again washed with normal saline. |
| 24 | 59 | |
| 21 | 53 | |
| 21 | 44 | |
| 28 | 106 | (iv) Strychnine was added. |
| 21 | 57 | |
| 28 | 46 | |
| 24 | 45 | |
| 21 | 45 | (v) Stronger solution of strychnine added. |
| 24 | 38 | |
| 21 | 43 | |
| 19 | 38 | |
| 21 | 21 | |

| | | |
|---|---|---|
| 12 | 12 | (vi) More strychnine added to the basin. |
| 2 | 2 | |
| 0 | 0 | |
| 10 | 10 | (vii) The heart was repeatedly washed with normal saline. |
| 30 | 30 | |
| 24 | 44 | |
| 28 | 76 | |
| 28 | 88 | |
| 26 | 92 | |
| 30 | 84 | (viii) Strychnine was again added. |
| 28 | 46 | |
| 34 | 34 | |

## Summary.

1. In Calcutta during the hot part of the rainy season an exposed rabbit's heart lasts for over two hours.

2. Such a heart preparation is gradually dying. It forms an exceedingly sensitive preparation for studying the action of drugs; changes in the heart can be easily followed by the eye.

3. If strychnine be applied to a sinking heart which has lost its rhythmicity, it improves the ventricular beats both in number and force and restrains the auricular contractility by decreasing the number of auricular contractions.

In conclusion, our best thanks are due to Professor S. C. Mahalanobis, for his kind guidance during the work.

Mr. Narendra Mohun Bose of the Physiological Laboratory, Presidency College, observed that Prof. Bhattacharya wanted to establish in his paper the two following points from his observation on the action of strychnine in normal saline upon the exposed heart of a rabbit. These are :—

(i) The mammalian heart can beat for a long time after the chest wall is opened, provided it is washed with normal saline, or better with a weak solution of strychnine in normal saline.

(ii) The action of strychnine on the auricles and ventricles is not the same. It promotes the ventricular contraction but decreases the auricular contraction.

The first point is really quite interesting, although it has been noted in Leonard Hill's "Further Advances in Physiology" p. 58, that the mammalian hearts can be revived many days after death. I do not remember to have seen in any book that such hearts can be revived, when the chest wall is opened, without perfusion of normal saline but simply by washing with Ringer's fluid or normal saline.

As regards the second point, I am rather chary in accepting it. The observations upon which this is based are, I suppose, capable of a better explanation. Prof. Bhattacharya noticed after removing the chest wall of a deeply anaesthetised rabbit—

(*a*) that the heart was lying still ;

(*b*) that after it was washed several times with normal saline solution it began to beat but there was on rhythm between the auricular and the ventricular beats—the

former being much more frequent than the latter ;

(*c*) when it was flushed with weak strychnine solution in normal saline, the number of auricular beats became very much less and that of the ventricular beats greater than before but the rhythmicity was not established ;

(*d*) when strong strychnine solution in normal saline was used rhythmicity was established, but the frequency of the auricular beats became much less than before.

All these phenomena can be explained if we assume that the strychnine is a stimulant of muscular activity in small doses, but acts as a protoplasmic poison in higher doses.

(*a*) The quiescent state of the heart that was noticed when the chest wall was removed is certainly due to the overdose of ether or chloroform which was used for anaesthetising the animal (vide Further Advances in Physiology—pp. 4-5).

(*b*) When the heart was washed with normal saline the osmotic pressure of the anaesthetic in the cardiac cells diminished and the heart recommenced its beating. The arhythmicity of beats that was noticed between the auricles and the ventricles must certainly be due to some block either in the A-V Bundle of His or in the A-V node, or in the ventricular musculature for otherwise the ventricles would have

taken up the rhythm of the auricles as they do in the normal heart.

(*c*) When the heart was washed with solution of strychnine, this block was partially removed as the auricular contractions became more forcible owing to the stimulant action of the drug. As a result of this the number of the ventricular beats increased, but the cause of the simultaneous diminution of the auricular beats is to be found in the poisonous action of the drug in consequence of which the cardiac muscle cells were gradually dying away.

(*d*) With the stronger solution of strychnine the contraction became still more powerful, so that the block was removed and the rhythmicity was established. But at the same time the poisonous effect of strychnine became more manifest and consequently the auricular beats became much slower.

In conclusion Mr. Bose said :—The theory suggested by Prof. Bhattacharya as an explanation of his observations may hold good, but the explanation suggested above is more probable than the one put forward by Prof. Bhattacharya.

The Chairman observed that the study of the curves recorded by the rabbit's heart under the condition described by the Professor might be of great interest and that electro-cardiograms might also be taken and studied.

## On the Crystalline Limestone from Daltonganj.

By Hem Chandra Das Gupta, M.A., F.G.S.

### *Introduction.*

In the maps of the Daltonganj coalfield published by Hughes to accompany his memoir on that area [1], mention has been made of the existence of a band of crystalline limestone lying just along the boundary between the Talchirs and the Archaeans; but as Hughes' main purpose was to describe the coal measures nothing has been said about the gneisses and the associated crystalline limestone. A second report on the Daltangonj coalfield appeared later on [2] in which Mr. LaTouche recorded the results he obtained from boring operations in the area. There is a map accompanying this paper and the band of the crystalline limestone is marked on it also. There is no description of the limestone but a study of his map shows that the crystalline limestone lies (i) along the faulted boundary between the Talchirs and the Archaeans and (ii) within a small zone which has been considerably deformed, as indicated by a difference of dip of 20° within a distance of quarter of a mile. About a year ago I had an opportunity of visiting this band of crystalline limestone and the results of my study are recorded in this note.

*Associated rocks: Granite-gneiss. Pyroxene-granulite.*

The band of crystalline limestone is rather narrow and extends over a mile. At some places the limestone is succeeded by a granite gneiss while at other

(1) Mem. Geol. Surv. Ind. Vol. VIII pp. 325-346.
(2) Rec. Geol. Surv. Ind. Vol. XXIV. pp. 141-153.

places it is associated with a garnetiferous rock, the latter rock, in its own turn, being followed by the granite-gneiss. The granite-gneiss is of the type usually met with in the Bengal Archaean area and there is nothing peculiar about it. The felspar includes a potash-bearing one, microcline. The garnetiferous rock just mentioned may be described as a pyroxene-granulite. It is highly basic with a specific gravity of about 3·3. In one place the pyroxene-granulite was observed to be associated with crystalline limestone while at another place no such association could be seen and specimens, collected from these two localities, appear very different especially when examined under the microscope. In both rocks the chief constitutents are garnet, green non-pleochroic pyroxene, a very small amount of secondary hornblende, basic plagioslase felspar and a very highly refracting substance doubtfully identified as zircon. The difference in the two rocks is found, however, chiefly in the different types of pyroxene. In the granulite not associated with the crystalline limestone the pyroxene patches are fairly big, while in the granulite associated with the limestone the pyroxene crystals appear to be drawn out, showing that the latter have undergone much crushing. The garnet of the latter rock also appears to be somewhat broken, but the evidence is not conclusive.

*Crystalline limestone: Ophicalcite.*

The limestone is thoroughly crystalline, granular and varies in colour, being grayish but always with a greenish tinge. Specimens of the rock were collected from three different localities, namely from the two

extremities of the exposure as also from the place where it lies in association with the pyroxene-granulite above referred to. All these rocks stain beautifully red with Lemberg's solution so that the rock is mainly composed of calcite. Besides calcite the other important mineral is serpentine, occuring generally in round and ovoid patches with some small amounts of muscovite here and there. The rock can, I think, unhesitatingly be described as an ophicalcite.

### *Ophicalcite : Previous Indian records.*

Ophicalcite or serpentinous limestone has already been recorded from many Indian localities, *e.g.*, from Western India [3] from Hazaribagh [4] and Chhindwara [5]. The occurrences in the last area have been very thoroughly described by Dr. Fermor and here besides the three different classes of crystalline limestone, calciphyre has also been obtained.

### *Ophicalcite : Origin of Serpentine.*

Serpentine is one of the commonest secondary products of magnesium-bearing silicates, there being quite a number of minerals from which it may be derived and they include olivine, pyroxene, amphibole and garnet. There is no restriction whatever as to the type of the magnesium-silicate from which the serpentine of ophicalcite may be obtained. Zirkel [6] has described an ophicalcite in which the serpentine has been found to be derived from olivine. An

(3) Mem. Geol. Surv. Ind. Vol. VI. p. 321.
(4) Mem. Geol. Surv. Ind. Vol. XXIV. p. 41.
(5) Rec. Geol. Surv. Ind. Vol. XXXIII pp. 162-220,
(6) Neu. Jahrb XXI pp. 828-832, 1870.

ophicalcite has been described by Prof. Kemp [7] the serpentine of which has, nearly in all cases, been derived from colourless pyroxene, but in several instances the unchanged core of the serpentine has been found to be an isotropic mineral of a high refractive index. This substance, according to Prof. Kemp, may very probably be a garnet. Merrill [8] has described an ophicalcite in which the serpentine is an alteration or a metasomatic product after a mineral of the pyroxene group. Two types of ophicalcite have been recorded from the Anglesey area and the serpentine of one of them is supposed to have been infused from without [9].

*Daltonganj ophicalcite : Evidence of origin.*

Any theory about the origin of ophicalcite must satisfactorily account for the origin of calcite and of serpentine, the chief constituents of the rock. In order to decide this question I had a number of slides prepared and I was fortunate enough to come across a few which I hope throw some light on this genetic question. One of the slides thus examined shows a fairly big white pyroxene undergoing alteration into calcite and serpentine. The same feature is also observed in another slide but not so markedly. There is another section which, besides calcite, serpentine and a few patches of muscovite, contains some non-pleachroic bright patches of high refractive index and which appear to me to be nothing but garnet.

(7) Bull. Geol. Soc. Amer. Vol. VI pp. 221-262, 1895.
(8) Amer. Journ. Sci. 3rd Ser. Vol. XXXVII pp. 189-191, 1889.
(9) Rep. Brit. Assoc. Adv. Sci. 1888 p. 409.

*Alteration of Pyroxene.*

It is well known that pyroxene alters into a number of minerals and among these alteration processes may be mentioned the simultaneous production of calcite and serpentine. An alteration like this has been described by Dr. Fermor from the Chhindwara area [10] the equation, representing the change according to Van Hise, being :—

$$3\,CaMgSi_2O_6 + 3\,CO_2 + 2\,H_2O = H_4Mg_3Si_2O_9\,^{(11)} + 3\,CaCO_3 + 4\,SiO_2$$

*Alteration of garnet.*

There is a large number of minerals into which garnet may be changed and a list of these alteration products is to be found in the monograph of Van Hise on metamorphism. One of the commoner products is serpentine, the change being supposed to take place according to the following equation :—

$$Mg_3Al_2Si_3O_{12} + 5\,H_2O = H_4Mg_3Si_2O_9 + 2\,Al(OH)_3 + SiO_2.$$

*Optical anomaly of garnet.*

Mention has been made in a previous paragraph of the existence of garnet in a slide. This slide contains only a few patches of unaltered garnet and an examination of these shows that some patches are strongly doubly refracting though there is at least one patch which is practically isotropic. Optical

(10) Op. Cit. p. 171.

(11) D. Fogy has tried to prove that the formula of serpentine should be $Si_4O_{12}(MgOH)_6H_2$. (Sitzb. d. Math-Nat. Kl. K. Akad. Wiss Bd. CXV. Abt. I. pp. 1081-1085, 1906).

anomaly of garnet has been known for a very long time and there has been a good deal of investigation regarding the cause of the anomaly. In this connection mention may be made of an augite-garnet rock described by Von Federow [12]. This rock is of eruptive origin with a tendency to metamorphism and, as a result of this metamorphism, garnet has been changed to epidote and augite to chlorite and secondary hornblende with the separation of calcite and quartz. Thin sections of this rock examined under microscope show that there are two different types of garnet one of which is brown and the other colourless. The brown substance is optically normal but the colourless one shows anomalous optical properties as a rule. The chemical composition of these two garnets is very nearly the same but there is a difference in their specific gravity. As a result of his investigations into the cause of this anomaly in the behaviour of garnet Von Federow supposes the phenomenon to be certainly due to some changes in the molecular volume of the mineral——'und zwar die relative Aenderung des Molekularvolumens'. According to him 'die Ursache kann allein eine solche sein, welche von den mechanischen, bei der Deformation der Form entstehenden Kräften abhängig ist.'

*Optical anomaly of garnet in the ophicalcite of Daltonganj.*

From the above considerations it is quite clear that the optical anomaly of garnet may be due to a variety of causes and in the same rock there can occur garnet

(12) Zeitschr. f. Krystal. Vol. XXVIII pp. 276-290, 1897.

both isotropic and anisotropic. The zone where the Daltonganj ophicalcite occurs is one which has undergone some disturbance, though the disturbance might have been quite local in character. Isotropic substances when under considerable stress in any one direction become anisotropic and accordingly there is no wonder if at least a few of the garnets which are in the process of alteration show some anomalous character under the crossed nicols.

*Ophicalcite: Suggested origin.*

It appears to me from the study I have made of this crystalline limestone that we may suppose it to be derived from the alteration of a pyroxene-garnet rock. Under this hypothesis the calcite is derived from the alteration of pyroxene and serpentine mainly from garnet, but also to a small extent from pyroxene.

*Presence of muscovite.*

From the equations given above regarding the alterations of augite and garnet it will appear that secondary silica and gibbsite are also produced. These substances have not been observed in the sections examined but they, according to Van Hise, may be dissolved and transported elsewhere so their absence is no proof that these substances were not actually formed during the alteration. Mention has been made of the presence of muscovite in some of the slides and it is just possible that at least a part of this secondary gibbsite and silica has been changed into muscovite according to the following equation:—

$$6\,Al(OH)_3 + 6\,SiO_2 + K_2CO_3 = 2H_2KAl_3Si_3O_{12} + CO_2 + 7H_2O$$

The potassium-carbonate required for this change might have been derived from the potash-felspar observed in the granite-gneiss and mentioned before.

*Alteration from pyroxene-granulite.*

I think that there is no reasonable ground for doubting that the ophicalcite is a secondary rock and that it has been derived from a previously existing rock consisting of pyroxene and garnet. The occurrence of pyroxene-granulite in immediate association with the crytalline limestone has been noted above and it has been observed that the pyroxene affords evidence of being much broken. Besides pyroxene and garnet the rock contains basic plagioclase felspar and this is also liable to be changed to calcite according to the following equation :—

$$CaAl_2Si_2O_8 + CO_2 = CaCO_3 + 2SiO_2 + Al_2O_3;$$

the alumina, as suggested by Dr. Fermor [13], may be subsequently supposed to be removed in solution as an alkaline aluminate. From these considerations and further from the fact that no other pyroxene-garnet bearing rock lies in the neighbourhood, the derivation of this ophicalcite from the pyroxene-granulite may be suggested as a plausible hypothesis.

*Cementation.*

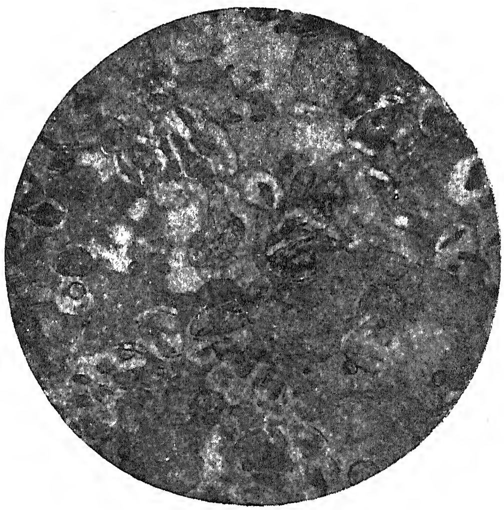
There is however one difficulty which presents itself regarding this hypothesis. The pyroxene of this pyroxene-granulite is green coloured and occurs in small grains while the pyroxene referred to in the first slide examined is rather large and white, though in this slide pyroxenes of smaller size are also present.

(13) Op. Cit. p. 194.

If we suppose, however, that the individual small grains of pyroxene of pyroxene-granulite were cemented prior to re-crystallization so that the enlargement is due to cementation this difficulty may be overcome while decolouration has very often been noted as a result of metamorphism.

## *Summary.*

The purpose of this short note is to put on record that (1) the crystalline limestone of Daltonganj is in the main a serpentinous limestone or ophicalcite; (2) this ophicalcite is derived from a pre-existing pyroxene-garnet bearing rock and (3) this pyroxene garnet bearing rock may be the pyroxene granulite associated with the ophicalcite.



× 22

Ophicalcite showing the ovoid patches of *garnet* changing to *Serpentine*.

# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION

FOR THE

# CULTIVATION OF SCIENCE

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# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. II. No. 1.

Saturday, the 26th February, 1916 at 5 P.M., C. V. Raman, Esq., M.A., Vice-President, in the chair.

## *On the Maintenance of Vibrations by a Periodic Field of Force.*

### Part I : Experimental.

By Ashutosh Dey.

The effect of a periodic field of force on the motion of a body subject to its influence has already been discussed by Mr. C. V. Raman in Bulletin No. 11.* One of the results of outstanding importance noticed was the series of special relations between the frequency of the field and that of the steady vibration possible under its action. It was shown that the motion is capable of being maintained when its frequency is either equal to, or is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ or $\frac{1}{6}$ of the frequency of the field, that is, any submultiple of it, but not when the frequency has any intermediate value. The experimental work and theory published

* And also in the Phil. Mag. January, 1915,—"On Motion in a Periodic Field of Force."

in that paper related to the motion of a system with only one degree of freedom, the period of free vibration of which is determined entirely by the field*. Recently, when experimenting with the electrically maintained vibrations of wires, certain interesting effects have been noticed which may be classed with the phenomena referred to above, but which merit separate discussion, in view of the fact that the system in this case has a series of free periods of its own, quite independently of the field. These will now be described.

*Experimental Method and Results.*

The present investigation relates to the vibrations of a steel wire about two meters in length, stretched vertically under an adjustable tension, and subject to the transverse periodic force exerted by an electro-magnet placed near some selected point on it. The electro-magnet is excited by an intermittent current from a fork-interruptor of frequency which in this experiment is generally 60 per second. The forced vibrations (having the same frequency as the intermittent current), which are usually excited in the first instance, when the tension of the wire is adjusted for resonance, are of the same form as those described by Klinkert in a paper on electrically maintained vibrations†. It is noticed, however, that when the

* The vibrations studied in that paper were those of the armature of a synchronous motor of the attracted-iron type when not in rotation, under the influence of the magnetic field due to an intermittent current.

† G. Klinkert, Annalen Der Physik, vol. 65 (1898). For a practical application in acoustics of this class of vibrations, see Science Abstracts (1916), page 433.

tension is such that the wire vibrates in two, three or larger number of segments, and the electro-magnet is not too far away from the wire, *the motion of the usual type first set up is unstable*, and gradually changes form, the nodes ceasing to be points of rest, and the frequency of vibration changes to a value which is a submultiple of the frequency of the fork. For instance, if the wire initially divides up into two segments and vibrates with a frequency of 60, its centre which at first is a node gradually acquires a very considerable motion, and the frequency of the vibration alters to 30. Similarly, if the wire initially vibrates in 3 segments, the frequency changes to 20 when the instability sets in; when the initial vibration is in four segments, the frequency changes to either 30 or 15, according as the instability does or does not result in a movement of the centre of the wire, and so on.

The rate at which the instability sets in and results in a change of type depends upon the position of the electro-magnet, its distance from the wire, and the strength of the intermittent current which excites it. Generally speaking, the rate of increase of the motion at the nodes is quite small, and it may take some minutes for the change to develop to the fullest extent. The gradual alteration of the form of the vibration may thus be closely studied, and this fact adds considerably to the interest of the experiment from the acoustical point of view. If the distance of the electro-magnet from the wire and strength of the exciting current be suitably proportioned, the vibration with the altered frequency finally reaches a steady

state, the amplitude of vibration then attaining its maximum. If however, the electro-magnet be too near the wire, or if the exciting current be too strong in proportion to the distance, the motion continues to increase in amplitude till the wire finally comes up against the pole of the magnet. This occurs most frequently when the tension is small and the wire divides up initially into a considerable number of segments.

Theory (as will be shown below) indicates that the ordinary forced vibration which is excited when the tension of the wire is adjusted for resonance is not at all essential to enable a vibration having a frequency equal to a submultiple of the frequency of the field to be set up and maintained. This has been tested in the following way :—Two electro-magnets are placed opposite different points on the wire, one or the other of which would be excited at pleasure. The first being placed opposite a point distant about $\frac{1}{8}$th of the length from one end and excited, the tension of the wire is carefully adjusted for resonance so that it vibrates in two, three or larger number of segments as desired. The second electro-magnet is placed exactly opposite a node of this forced oscillation, so that, in accordance with the well-known principle, it is incapable of maintaning a forced vibration of the ordinary kind, when fed with intermittent current. It is observed that when the second electro-magnet alone is excited, the *wire remains practically at rest. But this state of rest is unsatble*, and gradually a vibration develops and attains a large amplitude, its frequency being a submultiple of the frequency of the field.

Investigation by the method of vibration-curves shows that in the motion thus excited, the components having the same frequency as the field or any multiple thereof are practically or entirely absent. To enable the frequency of the field to be compared with that of the motion set up by it. the vibration-curves of some selected point on the wire and of a small style attached to the fork-interruptor are simultaneously recorded on photographic paper.* This may be done either at some stage in the progressive change of vibration-type, or when the motion reaches a final steady state. Sixteen records obtained in the course of the work are reproduced in Plates I and II.

They represent cases in which the frequency of the vibration was either equal to, or $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, or $\frac{1}{5}$, or $\frac{1}{6}$ of the frequency of the field. Most of the records were secured at a fairly early stage of the progressive change of the vibration-form. Each record consists of two curves, the upper one represents the vibration of the wire and the lower one represents the time-curve obtained from the fork-interruptor as mentioned. The following table is in explanation of records reproduced in Plates I and II.

* A method based on the optical composition of the vibrations of the fork and of a selected point on the wire, could no doubt be used for the same purpose as an alternative.

| No. | Type. | Point excited. | Point observed. | |
|---|---|---|---|---|
| 1 | 1st | $\frac{l}{8}$ | $\frac{7l}{8}$ | Group I. |
| 2 | 2nd | ,, | ,, | |
| 4 | 3rd | ,, | ,, | |
| 5 | ,, | ,, | ,, | |
| 10 | 4th | ,, | ,, | |
| 11 | 5th | ,, | ,, | |
| 12 | ,, | ,, | ,, | |
| 13 | ,, | ,, | ,, | |
| 14 | 6th | ,, | ,, | |
| 15 | 6th | at a | later stage | |
| 3 | 2nd | $\frac{l}{2}$ | $\frac{7l}{8}$ | Group II. |
| 6 | 3rd | $\frac{l}{3}$ | $\frac{7l}{8}$ | |
| 8 | 2nd | $\frac{l}{8}$ | $\frac{l}{2}$ | Group III. |
| 7 | 3rd | ,, | ,, | |
| 9 | 4th | ,, | ,, | |
| 16 | 5th | ,, | ,, | |

## Part II: Mathematical Appendix.

By C. V. Raman, M.A.

---

### *Theory of the Experiments.*

The attractive force of the electro-magnet in the experiments described in Part I, is exercised on a very small region of the wire, which may practically be treated as a mathematical point. The essential feature of the case which enters into the explanation of the phenomena noticed above, is that this attractive force is not a simple function of the time but depends also on the position, at the particular epoch, of the point on the wire with reference to the pole of the electro-magnet. In other words, the expression for the maintaining force is not independent of the form of the maintained motion. For the present purpose, we may write it as a product of two functions, one of which involves only the time, and the other is determined by the position of the wire in the field. Thus

$$\text{Force} = \mathrm{F}(y_0) f(t)$$
$$= \mathrm{F}(y_0) \sum_{n=0}^{n=\infty} a_n \cos\left(\frac{2\pi r n t}{\mathrm{T}} - e_n\right)$$

where $\mathrm{T}/r$ is the periodic time of the field and $y_0$ is the displacement of the wire at the point $x_0$ (opposite the pole of the electro-magnet) from its position of equilibrium. $y_0$ being positive when measured towards the pole, $\mathrm{F}(y_0)$ increases with $y_0$ and may be taken to be unity when $y_0=0$. We may expand $\mathrm{F}(y_0)$ by Taylor's theorem and write it in the form $(1+by_0+cy_0^2+\text{ etc.})$. If the force varies inversely as some power of the distance between the

pole and the wire*, it may readily be shown that the constants $b$, $c$, &c. are all positive. The complete expression for the force which may be assumed to act at the point $x_0$ of the wire, is thus

$$(1 + by_0 + cy_0^2 + \text{etc.})\ \Sigma\, a_n \cos\left(\frac{2n\pi rt}{T} - e_n\right)$$

We may now consider first, the ***ordinary forced vibration.*** This may be obtained by the method of successive approximations. To begin with, we may neglect the quantities $by_0$, $cy_0^2$ &c., in the expression for the maintaining force, which then assumes the simple form $\Sigma\, a_n \cos\,(2\pi rnt/T - e_n)$.

Since the forced vibration is of negligible amplitude except when the period of the field is more or less nearly equal to one of the free periods of the wire, the harmonic components in the motion may be determined, term by term, from the corresponding components of the impressed force. The forced vibration may therefore be written as :—

$$\sum_{n=1}^{n=\infty} a_n\, k_n \sin\frac{n\pi x}{\alpha} \sin\frac{n\pi x_0}{\alpha} \cos\left(\frac{2n\pi rt}{T} - e_n - e'_n\right)$$

where $\alpha$ is the length of the string or of each vibrating segment, and $k_n$, $e'_n$ are quantities, which, in respect of each harmonic, may be expressed in terms of the natural and impressed frequencies of vibration and of the decrement of the free vibrations. If $x_0$ is equal to $\alpha$ or any multiple of it, the forced vibration becomes negligibly small, the periodic force having an inappreciable effect when applied at a node.

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* From the measurements made by Klinkert over a limited range, it would appear that the attractive force on the wire varies inversely as the square root of the distance.

An interesting example in which the formula given above may be applied, is that of a single impulse acting at the point $x_0$, once in each period of vibration. The coefficient $a_n$ is then the same for all the harmonics and $e_n = 0$ for all values of $n$. It may readily be shown that if the period of the forced vibration in this case is somewhat greater or somewhat less than the period of free vibration of the string, the form of the maintained vibration is practically the same as that of a string plucked at the point $x_0$. For, the phase-constants $e'_n$ are then practically all equal to *zero* and $\pi$ respectively.

Further, $k_n$ is then practically independent of the dissipation of energy (whatever this may be due to) and is inversely proportional to the difference between the squares of the natural and impressed frequencies. For different harmonics, $k_n$ is proportional to $1/n^2$ and the expression for the forced vibration is then of the form

$$\pm \sum_{1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi x}{\alpha} \sin \frac{n\pi x_0}{\alpha} \cos \frac{2n\pi rt}{T}$$

and is thus similar to that of a string plucked at $x_0$ in the same direction as the periodic impulses or in the opposite direction, according as the natural frequency is greater or less than the frequency of the impulses* If the periodic force instead of being impulsive, has a finite constant value during a part fraction $2\beta$ of the period and *zero* at other times, the maintained

* It is assumed of course that the free periods of the wire, form a harmonic series. This may be subject to modification if the wire is imperfectly flexible or yields at the ends.

vibration in the two extreme cases assumes the form

$$\pm\sum_{1}^{\infty}\frac{1}{n^3}\sin\frac{n\pi x}{\alpha}\sin\frac{n\pi x_0}{\alpha}\sin\frac{2n\pi r\beta}{T}\cdot\cos\frac{2n\pi rt}{T}.$$

If $\beta$ be small, this is practically of the same type as the expression for a plucked string in respect of the first few harmonics, but would differ from it in respect to the harmonics of higher order.

The next step is to introduce a correction in the expression for the impressed force on account of the neglected terms $by_0, cy_0^2$ etc. On substitution of the value of $y_0$ first found in these terms and simplifying the product F $(y_0) f(t)$, it is seen that the correction results only in alterations of the amplitudes and phases of the harmonic components of the impressed force, but no new terms are introduced of which the frequency is not the same as that of the field or a multiple thereof. This shows that the corrections cannot by themselves, result in an alteration of the frequency of the forced vibration, so long as we assume, in the first instance, that $y_0$ has the same frequency as the field of force. They may however result in the impressed force (and therefore also the maintained motion) including such partial components as are absent in the field itself.

A consequence of the preceeding formulæ which is of particular importance is that, when the impressed force is of an impulsive character, the corrections $by_0, cy_0^2$ &c. when introduced, cannot result in any alterations in the *relative* amplitudes and phases of the components of the maintained vibration. For, the product F $(y_0) f(t)$ is *zero* at all times except at the particular instant in each period at which the

impulse acts, and as these epochs are fixed, any change in F $(y_0)$ can only result in the amplitudes of *all* the components of the impressed force, being increased or decreased in the same proportion, their phases remaining unaltered. The non-uniformity of the field may thus affect the amplitude of the vibration but cannot alter its form, it being assumed, of course, that the amplitudes are not so large as to alter the free periods. This peculiarity of the action of a non-uniform impulsive field is the explanation of certain interesting observations described but not explained by Klinkert in the paper referred to above. Klinkert experimented with two wires, both electro-magnetically maintained, one of which was self-acting and the other was worked by a current on a separate circuit, rendered intermittent by the vibration of the first wire. The vibration-curves of the two wires showed a marked dissimilarity, a special feature of interest being the fact that the vibration of the second wire, when at its maximum, was practically similar to that of a plucked string. In view of what has been said above, this result will be readily understood. The magnetic field is of appreciable strength, only during a small fraction of the period and may thus be regarded as of an impulsive character. When there is an appreciable difference between the natural and impressed frequencies of vibration, the form of the motion approximates to that of a plucked string, and this is what is actually observed when the exciting current is rendered intermittent by an independent interruptor. It is when the natural frequency is somewhat greater than the

impressed frequency, that the vibrations of the largest amplitude and those that show the closest similarity to the vibrations of a plucked string are obtained. For the vibrations are then nearly in the same phase as the impulses, and as an increase in the amplitude brings the position of the wire at which the impulses act, closer to the electro-magnet and therefore still further increases the magnitude of the impulses, a vibration of large amplitude may be maintained in spite of the difference between the natural and impressed frequencies of vibration. The increase of natural frequency due to a large amplitude would also tend to encourage the assumption of this form of vibration and to make it stable. The conditions are, however, entirely different when the vibrating wire is a self-acting interruptor which determines the period and character of its own excitation, and a detailed mathematical theory of the vibration-forms obtained with it, must be reserved for separate consideration.

We may now pass on to consider the cases in which the frequency of the vibration is not the same as the frequency of the field, but is a submultiple of it. To fix our ideas, we may assume the free vibration of the wire, when it divides up into $r$ segments to have nearly the same period as the field, that is $T/r$. The period of vibration of the wire as a whole, is therefore T. Experiment shows that the forced vibration having the period $T/r$ may be unstable, giving place to a vibration in the period T. To explain this result, we may examine the effect, according to our equations, of superposing a small vibration of period T upon

the ordinary forced vibration, if any, of period $T/r$. If $by_0$, $cy_0^2$ etc. be neglected, there is no component in the impressed force having the period T, and the initial disturbance assumed would die away in the ordinary course. It is not possible therefore to obtain the phenomena illustrated in Plates I and II with uniform fields of force. With non-uniform fields, the additional terms $by_0, cy_0^2$ etc. have to be taken into account, and it may readily be shown on expanding the product $F(y_0) f(t)$ in a series of sines, that there would be a term of period T in the expression which would, under certain circumstances, be capable of magnifying the assumed disturbance continually, till it assumes a large amplitude. For example we may take $r=2$, and the initial disturbance to be, say, $\gamma \sin \frac{2\pi t}{T}$. The product $by_0 a_1 \cos\left(\frac{4\pi t}{T} - e_1\right)$ would containa term $ba_1 \gamma \sin \frac{2\pi t}{T} \cos\left(\frac{4\pi t}{T} - e_1\right)$ which on being expanded is seen to include a component $ba_1 \gamma \sin e_1 \cos \frac{2\pi t}{T}$. This is proportional to the assumed disturbance, has the same period, and has a phase in advance of it by 90°. It would therefore tend to magnify the assumed disturbance of period T, till the latter reaches a considerable amplitude. An explanation of the phenomena is thus possible for the case $r=2$, in which no part is played by the component of $y_0$ having the same frequency as the field. For the cases in which $r=3$ or 4 &c., we have to proceed to a higher degree of approximation by taking into account not only the assumed disturbance of frequency $1/T$, but also other subsidiary compo-

nents whose frequencies are multiples of $1/T$ and play a part in the magnification or maintenance of the vibration of that frequency. If in the distribution of the field F $(y_0)$, only the first correction term $by_0$ is taken account of, the analysis proceeds practically on the same lines as that contained in Bulletin No. 11, except that, instead of the equations of motion for one degree of freedom, the general formula for the normal co-ordinates in the forced vibrations of the wire would have to be used. The same general result would be obtained, that the components in the motion having the same frequencies as the field or any multiple of it would not play any part in the maintenance of the motion of the kind now considered. We have already seen how this indication of theory may be verified experimentally. When however the correction terms of higher order, that is $cy_0^2$ &c., are considered, some modification of this general statement, might become necessary.

## Summary and Conclusion.

The paper considers experimentally and theoretically a case of vibrations maintained by a *non-uniform* periodic field of force which is of some practical importance. It is shown that when a wire divides up into two or more segments and vibrates under the transverse attraction of an electro-magnet, the motion which has the same frequency as the field, may be rendered unstable by the non-uniformity of the field and then passes over into one, the frequency of which is a submultiple of the frequency of the field. Photographic records illustrating the first six cases of the kind are presented with the paper. It is also shown

that a motion of this type may be set up and maintained even where the attracted point on the wire is a node and the ordinary forced vibration is therefore absent. The effects of the non-uniformity and of the periodic variation of the field on the ordinary forced vibration, are also considered in detail and the mathematical theory of certain effects noticed by Klinkert is set out.

## *On the Application of the Physical Property $\frac{\text{Atomic volume}}{\text{Density}}$ of the elements in determining the Degree of Chemical Affinity in simple chemical combinations.*

BY MANINDRA NATH BANERJEE.

*(Preliminary Note).*

Various physical properties of the chemical elements, such as *atomic volume*, melting points, extensibility, thermal expansion, conductivity for heat and electricity, heat of formation of oxides and chlorides, magnetic and diamagnetic properties, refraction equivalents, (*vide*—on these properties, Lothar Meyer. *Modern Theories of Chemistry*, 1883, p. 144 ff), "Hardness of the free elements" (Rydberg, *Zeitscr phys.* 33,353, 1900) "change of volume on Fusion" (M. *chem.* Topler, *Wied. Ann.* 53,343,1894). "viscosity of salts in aqueous solution (Jul. Wagner. *Zeitscr. phys. chem.* 5,49,1890), "Colour of Ions" (Carey Lea, Sill. *Am. Journ* [3], 49,343,1894), "Ionic Mobility" (Bredig, *Zeitshr. Phys. Chem.* 13,289,1894). "Compressibility of solid elements" (T. W. Richards,

*Zeitschr. phys. chem.* 61, 183, 1908) *etc.*, have been shewn to be more or less marked periodic character in reference to atomic weight and to add to this we may shew that the $\frac{\text{Atomic volume}}{\text{Density.}}$ of the elements is also a periodic function of atomic weights. We have already shewn in a preliminary note "On the Relationship of Atomic Volumes and the specific gravities of the elements" (*vide* Proc. of the Indian Association for the cultivation of Science, September, 1915) that $\frac{\text{Atomic volume}}{\text{Density}}$ number has a marked relationship with the *chemical activity* or the *chemical affinity* of the elements. Hence the periodicity of *chemical affinity* with reference to atomic weights is quite apparent. This attaches much importance to the $\frac{\text{Atomic volume}}{\text{Density.}}$ number of the elements, the proper investigation of which we are now engaged in. One of the results of such an investigation is the application of it in determining the degree of chemical affinity in simple chemical combinations. In doing this, we have to use two formulæ which, for the present, may be taken as empirical ones. These are—

$$(1)\quad \frac{x\,\frac{A}{D^2}\;y\,\frac{A'}{D_1^{\,2}}}{xy\,(\text{mol. wt. of the comp}).}\qquad\qquad (2)\quad \frac{xy\left(x\,\frac{A}{D^2}\;y\,\frac{A'}{D_1^{\,2}}\right)}{\text{mol. wt. of the comp.}},$$

where $A/D^2$ and $\frac{A'}{D_1^{\,2}}$ are the $\frac{\text{Atomic volume}}{\text{Density.}}$ of the component elements and $x$ and $y$ their number of atoms respectively. Of these the former is meant for combination between two component elements of opposite character as electropositive and electronegative ones while the latter is meant for combinations between elements of the same character,

*i.e.* between two electronegative or two electro-positive ones. The numerical values of these are considered as tangents of some angles which may be found in a Logarithmic table. These angles are all plotted in a quadrant, those obtained from the first formula *positively* (*against* the direction of the hands of a watch), while those obtained from the second formula *negatively* (*with* the direction of the hands of a watch). Thus the latter angles have their complimentary angles. These complimentary angles, together with the former set of angles are arranged in a table in their decreasing order and their corresponding numerical values against them as also putting the solid, liquid and gaseous compounds separately. Thus we have a complete table which illustrates very clearly the degree and character of chemical affinity exerted in the combinations (for the tables *vide* our paper on "ATOMIC IMPACT"—Annual Report, Indian Association for the Cultivation of Science, 1913). The subject is still under investigation and when it is complete a detailed description of these may be conveniently published.

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# On the Alteration of Pyrite occurring in Steatite.

By Sures Chandra Datta, M.Sc.

## Introduction.

With the kind permission of Mr. Coggin Brown, Assistant Superintendent of the Geological Survey of India and sometimes Professor of Geology at the Presidency College, Calcutta, I had an opportunity of accompanying a party of Presidency College students to the Pindari Glacier (lat. 30°-15½′; long. 80°-2′) in charge of Professor Das Gupta and the specimens that are described in this short note were obtained during this trip in the month of June and July, 1913.

## Collection of Specimens.

On the day the party was proceeding beyond Bageswar, 26 miles North of Almorah, I came across a specimen of steatite with pseudomorphs of limonite after pyrite, embedded in it. These pseudomorphs, it must be mentioned, almost always contain kernels of unaltered pyrite within. The next day on the way to Loharkhet, 49 miles north of Almorah, another specimen of steatite with several limonite pseudomorphs, was collected by me and on careful examination it was observed that the occurrence of pseudomorphs of limonite after pyrite in steatite was very common in this latter area.

## Description of Steatite.

The rocks between Bageswar and Loharkhet are quartzite, steatite and limestone. There is a considerable number of landslips in this part, especially on the way to Loharkhet, exposing sometimes the shin-

ing steatite schists on the mountain sides. In some specimens of quartzite near Bageswar, flakes of talc occur. The limestone is magnesian. Steatite is more or less white, shining with characteristic soapy feel and is very impure—the impurity, chiefly carbonate, sometimes attaining a considerable proportion and even in some cases the amount of carbonate—dolomite—found mixed up with it, has been estimated to be about 65 per cent. Ferruginous impurity is comparatively speaking very insignificant and in some specimens, it has been found to be about 1·5 per cent. (estimated as $Fe_2O_3$). In some parts of steatite there are individuals, and in other parts aggregates, of limonite pseudomorphs. From these pseudomorphs originate cracks more or less in parallel directions *i.e.*, parallel to the foliation of the schists. The cracks are filled up with talc and quartz and there is no limonite in them. It is well known that the—change of pyrite to limonite is accompanied with an increase in volume (1), and it is not unlikely that these cracks were formed, during the process of this alteration, the cracks being younger than the original pyrite crystals. The talc flakes filling up the cracks are also evidently younger than the cracks and were very likely formed from solution of dolomite and quartz. The talc flakes filling up these cracks have no regularity in their arrangement while the talc flakes occurring in steatite itself but not in the cracks appear to have a parallel banded course and running also roughly parallel to the direction of the cracks just mentioned.

(1) Monographs of the United States Geological Survey, Vol. XLVII, p. 215.

Besides the dolomite and talc flakes mentioned above, patches of quartz have also been observed in the rock. The dolomite and quartz have both signs of pressure. The films of talc mentioned above intervene between these two and here, as a matter of fact, the proportion of quartz and dolomite is such that talc can be formed with the subsequent introduction of moisture, for it is well known that talc is in general a mineral of the zone of weathering (1). In the case of talc occurring in the cracks, dolomite and quartz were transported in solution; while the formation of talc occurring in the rock was due to dolomite and quartz originally present in the rock and a subsequent introduction of moisture. Sometimes chlorite with a slight green pleochroism and straight extinction has been found to be associated with this steatite rock. This chlorite decomposes at the ends into grains of dolomite and limonite, with seggregations of quartz which have inclusions of very fine fibres of chlorite in the same state of decomposition and in the same way. Decomposition always begins from the ends and not from the sides. Possibly this chlorite is an intermediate stage between the dolomite of the schist and iron ores on the one hand and the original rock from which these minerals have gradually and subsequently been developed, on the other. This characteristic decomposition of chlorite into dolomite, quartz and limonite is an indication, says Rosenbusch, of the zone of weathering (2).

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(1) Monographs of the United States Geological Survey, Vol. XLVII, p. 350-351.

(2) Ibid, p. 347.

*Description of Limonite.*

Limonites occurring in steatite have been examined. As mentioned before they are always pseudomorphs after pyrite and these pseudomorphs are sometimes single individuals and sometimes aggregates—the aggregates being sometimes elongated in the direction parallel to the foliation of steatitic schists. Rock is more or less loose round these pseudomorphs within short distances and these less compact portions of the rock are coloured brown, the colour fading with the distance from the central pyrite. Forms present in the pseudomorphs are cubes and combinations of cubes and pyritohedra but pyritohedra are not present alone. Whenever pyritohedra occur they occur always in small size with a tendency to pass into striations—these striations being parallel to the edges of cube and pyritohedron. It has already been said that almost all the pseudomorphs have remnants of pyrite in them. There is no carbonate of iron in limonite and no dolomite included in pyrite. There are grains of quartz in pyrite and in limonite. Possibly these quartz grains are the same as the quartz of the quartzite in steatite but they got included in pyrites when these pyrite crystals were formed. The pseudomorphs do not contain gold or copper. Limonite has not been formed by infiltration as there is no break between pyrite and limonite. This suggests that the deposition of limonite has taken place simultaneously with the solution of pyrite. In limonite there are portions quite light and portions which are quite dark coloured. The dark coloured portions are more or less circular in section and are gradually merging into

portions quite light, there being no sharp line of demarcation between these two. This structure had possibly been developed by the nature in which pyrite crystals have been attacked, to be mentioned below, by the circulating solution and by the pressure consequent on the increase of volume when limonite has been formed from pyrite. Limonite does not develop in the zone of anamorphism (1). Moisture and Oxygen are necessary for its formation from pyrite (2). Limonite is formed in the belt of weathering (3). Crystals of pyrite of higher symmetry form where pressure is great (4). Cubes with striations and combinations of cubes and pyritohedra almost reduced to striations are cases of higher symmetry. However, formation of limonite indicates the stage of the zone of weathering, whereas the formation of pyrite denotes the stage of deep-seated zone. The nature of the occurrence of pyrite crystals and the formation of their aggregates suggest the possibility that ferruginous matter formerly present in the original rock which is linked with chlorite, as mentioned before, was transported by some circulating solution and concentrated at number of centres in the rock itself—these centres being chemically active and the whole process taking place in the deep-seated region *i.e.* in the zone of anamorphism These pyrites, when brought up within the zone of weathering, began to

(1) Monographs of United States Geological Survey, Vol. XLVII, p. 233.

(2) Ibid, p. 216.

(3) Ibid, p. 216.

(4) Ibid, p. 215.

change into limonite by the bicarbonate solution derived from the solution of enormous quantities of carbonate in the form of dolomite in steatite. This alteration is well known (1). It has also been proved experimentally (2). During this change sulphuric acid is produced which has loosened the steatite round altered pyrite crystals.

### *Planes of Chemical Alteration in Pyrite.*

The way in which the bicarbonate solution attacks pyrite has been found to be interesting. As far as I am aware there has been no mention of this phenomenon in the current literature. Rosenbusch (3) says that when pyrite changes to limonite the alteration begins from periphery towards the interior. The solution no doubt acts from the periphery but alteration progresses inwards through planes parallel to (100) and (111)—the former plane being altered with greater energy and more easily as evidenced from the fact that in the region of contact between limonite and pyrite and in the boundary of the latter, minute alteration products of limonite parallel to (100) and (111) do occur—those parallel to (100) being more numerous and also greater in length. The alteration products sometimes occur in zig zag lines which when observed under the microscope very carefully appear to be but parallel to the traces of (100) and (111) *i.e.*, can be split up into traces of (100) and (111)—the former being greater in number and in length. In a

(1) A text Book of Mineralogy by E.S. Dana, p. 301.

(2) Econ. Geol. ii, pp. 14-23, 1907.

(3) Rosenbusch mikr Physio Bd. I, pt. 2, p. 9, 1905.

slide of pyrite there occur two parallel rods of limonite, parallel to striation, parallel to the edges of cube and pyritohedron. These parallel thick bands of limonite are crossed by a similar band at right angles to both; while there are also very fine alteration products parallel to minute traces of (111) in these rods and at their junctions or corners at the crossing. There are indistinct cleavages in pyrite, parallel to (100) and (111) (1). Thus it is seen in this case of pyrite, the physical property of cleavage and the chemical property of alteration are in the same plane.

*Summary.*

It has been the main purpose of this note to establish :—

(1) That alteration of pyrite to limonite within steatite schists is attended with increase in volume giving rise to cracks round pyrite crystals and this is partially responsible for the production of a peculiar structure within limonite.

(2) That the materials necessary for the production of talc were accumulated in two different ways, *viz.*, by transportation and in situ—transportation being very local in character in as much as it was effected from one part of the rock to another near about.

(3) That chlorite fibres decompose at the ends and not at the sides and that this chlorite, as mentioned before, is linked with the

(1) Rock minerals by Iddings, p. 523, 1906.

original rock which, at its some period of geological history, gave rise to pyrite crystals of higher symmetry than that of pyritohedron, in deep-seated zone, from the ferruginous matter present in the rock itself in some form or other.

(4) That pyrite crystals alter to limonites being attacked in planes parallel to (100) and (111)—the former plane being more easily and energetically attacked.

In conclusion I wish to thank Mr. H. C. Das Gupta, M.A, F.G.S. for the facilities given to me to work in the Geological Laboratory, Presidency College and also for helping me with some suggestions while preparing this note and I also wish to thank my old teacher Mr. Vredenburg, A.R.S.M., A.R.C.S. of the Geological Survey of India for having kindly gone through the manuscript.

Ripon College, Calcutta,
February, 1916.

# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. II. No. 2.


Saturday, the 29th, July, 1916 at 5 P.M. The Hon'ble Justice Sir Asutosh Mukerjee, KT., Saraswati, Shastra-Vachaspati, C.S.I., M.A., D.L., D.SC., F.R.S.E., F.R.A.S., &c., Vice-President, in the chair.

## *On the Wolf-notes of the Violin and Cello: How are they caused?*


BY C. V. RAMAN, M.A.


It has long been known that on all musical instruments belonging to the violin family there is a particular note which it is difficult to elicit in a satisfactory manner by bowing. This is called the "wolf-note," and when it is sounded the body of the instrument is set in an unusual degree; and it appears to have been realized that the difficulty of maintaining the note steadily is due in some way to the sympathetic resonance of the instrument.* In a recent paper,† G. W. White has published some interesting experimental work on the subject, confirming this view. The most striking effect noticed is the *cyclical* variation in

* Guillemin,—"The Application of Physical Forces," 1877.

† G. W. White,—Proc. Camb. Phil. Soc., June, 1915.

the intensity of the note when the instrument is force to speak at this point. White suggests as an explanation of these fluctuations of intensity that they are due to beats which accompany the forced vibration impressed on the resonator when the impressed pitch approaches the natural pitch of the system. The correctness of this suggestion seems open to serious criticism. For, the beats which are produced when a periodic force acts on a resonator are of brief duration, being merely due to the superposition of its forced and free oscillations, and when, as in the present case, the resonator freely communicates its energy to the atmosphere and the force itself is applied in a progressive manner and not suddenly, such beats should be wholly negligible in importance, and should, moreover, vanish entirely when the impressed pitch coincides with the natural pitch. In the present case the feature is the *persistency* of the fluctuations of intensity and their markedness over a not inconsiderable range; and it is evident that an explanation of the effect has to be sought for on lines different from those indicated by White. I had occasion to examine this point when preparing my monograph on the "Mechanical Theory of the Vibrations of Bowed Strings," which will shortly be published, and the conclusions I arrived at have since been confirmed by me experimentally.

From the mechanical theory, it appears that when the pressure with which the bow is applied is less than a certain critical value, proportionate to the rate of dissipation of energy from the vibrating string, the bow is incapable of maintaining the ordinary mode of vibration in which the fundamental is dominant, and

the mode of vibration should progressively alter into one in which the octave is the predominant harmonic‡. In the particular case in which the frequency of free oscillation of the string coincides very nearly with that of the bridge of the violin and associated masses, the mode of vibration of the string is *initially* of the well-known type in which the fnndamental is dominant. But the vibrations of the string excite those of the instrument, and the vibrations of the latter increase in amplitude, the rate of dissipation of energy increases continually till it outstrips the critical limit, beyond which the bow fails to maintain the usual type of vibration. As a result of this, the mode of vibration of the string progressively alters to a type in which the fundamental is subordinaie to the octave in importance. The vibration of the belly then begins to decrease in amplitude, but, as may be expected, this follows the change in the vibrational form of the string by a considerable interval. The decrease in the amplitude of the vibrations of the belly results in a falling off of the rate of dissipation of energy, and, when this is again below the critical limit, the string regains its original form of vibration, passing successively through similar stages, but in the reverse order. This is then followed by an increase in the vibrations of the belly, and the cycle repeats itself indefinitely. The period of each cycle is approximately twice the time in which the vibrations of the belly would decrease from the minimum, if the bow were suddenly removed.

‡ Compare with the observations of Helmholtz,—'Sensations of Tone,' English Translation by Ellis, p. 85.

The foregoing indications of theory are amply confirmed by the photographs of the simultaneous vibration-curves of the belly and string of a cello at the wolf-note pitch. It is seen that the form of vibration of the string alters cyclically in the manner predicted by the theory, and that the corresponding changes in the vibration-curve of the belly *follow* those of the string by an interval of about quarter of a cycle. That the two sets of changes are dynamically interconnected in the manner described is further confirmed by the prominence of the octave in both curves at the epochs of minimum amplitude. The explanation of the cyclical changes given above is also in accordance with the observed fact that they disappear and are replaced by a steady vibration when the ratio of the pressure to the velocity of bowing is either sufficiently increased or sufficiently reduced. In the former case the string vibrates in its normal mode, and in the latter case the fundamental disappears altogether and the string divides up into two segments.

*Effect of Muting on the "Wolf-note."*

Since the pitch of the wolf-note coincides with that of a point of maximum resonance of the belly, we should expect to find that by loading the bridge or other mobile part of the body of the instrument important effects are produced. This is readily shown by putting a mute on the bridge. The pitch of the wolf-note then falls immediately by a considerable interval. On the particular 'cello' I use, a load of 17 grammes fixed at the highest point of the bridge lowers the wolf-note pitch from 176 to 160 vibrations per second. A larger load of 40·4 grammes depresses

it further to 137 vibrations per second, and also causes two new but comparatively feeble resonance-points to appear at 100 and 184 respectively, without any attendant cyclical phenomena. An ordinary brass mute has a very similar effect.

*The Formation of Violin-tone and its Alteration by a Mute.*

The positions of the frequencies of maximum resonance of the bridge and associated parts of the belly for the notes over the whole range of the scale are undoubtedly of the highest importance in determining the character of violin-tone, and the explanation of the effect of a mute on the tone of the instrument is chiefly to be sought for in the effect of the loads applied on the frequencies of the principal free modes of vibration of the bridge and associated parts of the belly. The observations of P. H. Edwards on the effect of the mute* are evidently capable of explanation on the basis of the lowering of the frequencies of maximum resonance by the loading of the bridge. But a more detailed understanding of the dynamics of the problem requires further theoretical and experimental investigation. Recently, I have secured an extensive series of photographs showing the effect on the motion of the bridge in its own plane produced by fixing a load on it at one or other of a variety of positions. The close parallelism between the effect of loading, as shown by these photographic curves and

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* P. H. Edwards, Physical Review, January, 1911.

† Giltay and De Hass, Proc, Roy. Soc. Amsterdam, January 1910. See also E. H. Barton and T. F. Ebblewhite, Phil. Mag., September 1910, and C. V. Raman, Phil. Mag., May 1911.

as observed by the ear, seems to show that the motion of the bridge in its own plane determines the quality of violin-tone to a far greater extent than might be supposed from the work of Giltay and De Haas†. A detailed discussion of this and other problems relating to the physics of bowed instruments is reserved for a separate communication.

Several other interesting effects have since been noticed, of which the following is a summary :—

*(a)* Cyclical forms of vibration of the G-string and belly of a 'cello may also be obtained when the vibrating length is double that required for production of the wolf-note, that is, when the frequency is half that of the wolf-note. In this case, when the pressure of the bow is sufficient to maintain a steady vibration, the second harmonic in the motion of the belly is strongly re-inforced. When the pressure is less than that required for a steady vibration, cyclical changes occur, the principal fluctuations in the motion, both of the string and the belly, being in the amplitude of the second harmonic. In this, as in all other cases, the cyclical changes disappear and give place to a steady vibration, when the bow is applied at a point sufficiently removed from the end of the string. In this particular case, a large, *almost soundless* vibration may be obtained by applying the bow rather lightly and rapidly at a point distant one-fifth or more of the length from the end; the octave is then very weak in the vibration of the string, but may be restored, along with the tone of the instrument, by increasing the pressure of the bow.

*(b)* The 'cello has another marked point of resonance at 360 vibrations per second. The pitch of this is also lowered by loading the bridge.

*(c)* When the vibrating length of the G-string or D-string of the 'cello is about a fourth of the maximum or less, cyclical forms of vibration may be obtained at almost any pitch desired, by applying the bow with a moderate pressure rather close in the bridge.

*(d)* As the frequency of vibration is gradually increased from a value below to one above the wolf-note frequency, the phase of the principal component in the "small" motion at the end of the string, that is also of the transverse horizontal motion of the bridge, undergoes a change of approximately 180°. This is in accordance with theory.

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## *Reversion of Fertile Regions into Sterility in Phanerogamic Plants.*

By Surendra Chandra Banarjee, M.A, B.Sc.

The species "Crotolaria sericea, Retz." of the sub-order Papilionaceae under Leguminosae, normally flowers in the cold season, generally between November and February. After flowering and fruiting, the plant generally dies away; thus it is an annual herb, although some individuals attain a tall and robust body.

In this plant the sexual reproductive region, or the bract-leaf-region is very well marked and quite distinct from the vegetative, or foliage-leaf-region (Fig. 1). Generally, plants are busy in their early life in building up their body, i. e., adding to and strengthening their frame work. After the vegetative processes are completed, i.e., after the body of the individual has attained the normal dimensions, the development of the sexual organs begins, and is generally localised, i.e. these organs are developed in some definite part of the plant body. This part of the plant body together with the organs of reproduction is known as the bract-leaf-region, or the Inflorescence. The inflorescence is thus the upper or concluding part of the plant body, and the organs of reproduction, namely the flowers, are the ultimate ends of the various branches of the aerial part of the plant body.

There are facts to prove that flowers are modified shoots. The phenomenon observed in the present case is a proof that inflorescence is a modified form of the branch-system in the foliage-leaf-region. Some of

the branches of the inflorescence became sterile and produced foliage leaves towards their tips, while bearing the usual bracts lower down, and others produced a few flowers. Thus sterile or foliage leaves were borne on branches similar to those which normally bear bract-leaves in the axils of which fertile shoots i.e. flowers are produced. So that, the fertile region reverted to sterility, i. e. inflorescence reconverted itself into vegetative region.

The phenomenon was observed on the 24th May 1916, in a seedling which grew up by the side of its old parent which had flowered in the last winter. The seedling brought forth its inflorescence branches while itself had attained a height much shorter than what is normally attained by others in the flowering time in winter.

Normally, the seedlings of this species grow after the rains when their subterranean part, i.e. their root system is busy in collecting food materials, from the moist soil for building up the vegetative body. The seedling under observation rose from a seed in May when the superficial soil was very dry and consequently the root system could not procure the requisite quantity of food materials. Although there was a very scanty supply of food from the soil, the aerial environments of the seedling favoured growth; for the seedling grew up by the side of a tank and in a bush of pine apple and other plants (Fig. 3). Thus it had around it an atmosphere of water vapour from the tank, rendered very hot by the abnormal solar heat of May, 1916. Thus, hot-house conditions prevailed round about the seedling. On account of this insufficient

food supply and forced growth, the plant very soon finished up its vegetative functions in order to take up the reproductive functions and consequently attained a very short stature—the universal sequence of events in a plant life being reproduction following vegetation. But in this case the reproductive activity did not produce satisfactory results, as only a few flowers, and only, perhaps, one or two pods were produced (Fig. 2). So, in order to economise the scanty supply of food at its disposal, the plant again converted its fertile region into vegetative region, apparently with the object of propagating itself by vegetative means; for, much less energy is spent on the production of foliage leaves than on fertile ones. Hence, the season must have had an influence on the reversion.

The seedling in question lived till it flowered in the succeeding winter (end of November, 1916) and thus it behaved like a perennial.

A similar reversion of fertile regions into sterility was observed in the case of a mango inflorescence. A mango tree which is on the bank of a tank bore its inflorescence twigs in the last winter season—when mango trees are normally in flower. The inflorescence twigs, however, curiously changed into vegetative shoots—foliage leaves, with their charactertistic colour and texture, came out from the twig where flowers were expected. Unfortunately no photo was taken at that stage. The probable explanation in that case might have been injury caused by ants, as that individual was full of ants at the flowering time.

A similar reversion is also illustrated by a "Zalacca edulis *Reinw,*" a formidable Burmese palm with no

stem and with leaves 16-20 feet long, the petioles of which are armed with very sharp thorns. The inflorescence of this comes out from its base near tne ground curving forwards and downwards. One of the branches of the inflorescence has been recorded to have directly ended in a new plant with roots (vide plate 222-223, Plantae Asiaticae Rariores Vol. III. by Nathaniel Wallich, M. & Ph. D. and published in 1830.) (Fig. 4, reduced from the plate quoted above and which can be seen in the Herbarium, Royal Botanic Garden, Sibpore.)

## *On the Zonal Distribution of* PLACENTICERAS TAMULICUM, Kossmat.

BY HEM CHANDRA DAS-GUPTA, M.A., F.G.S.

In the year 1912 I had an opportunity of visiting a part of the well-known cretaceous rocks of Southern India in charge of a party of students from the Presidency College, Calcutta. The geology of the area has been very fully described by the late Dr. Blanford[1], while the fossils, in the first place, were thoroughly described by Dr. Stoliczka[2]. Ever since a fairly large volume of literature has appeared dealing with the geological features of the area.

The *ammonites* found in the beds include the genus *Placenticeras* represented by 3 species viz :—*Pl. tamulicum* Kossmat, *Pl. syratale* Morton and *Pl. warthi* Kossmat. A fourth species, collected by Mr. P. N. Bose from the Bagh beds of India, has been

(1) Mem. Geol. Surv. Ind., Vol. IV, pp. 1-217.

(2) Pal. Ind. Ser. I, III, V, VI, VIII, Vols. I-IV.

described and named *P. Mintoi* by Mr. Vredenburg.(3) This genus has also been recorded from many places outside India.

An account of the zonal distribution of the species belonging to this genus was published by Böhm (4) in 1898 and by Mr. Vredenburg in 1908(5). Two additional species have been fully described by Sommermier from the lower cretaceous rocks of Northern Peru(6). Accordingly, as known at present, the zonal distribution of the species of *Placenticeras* is substantially the same as that published by Mr. Vredenburg and to his list the two species of Sommermier, just referred to, have only to be added.

My observations in the Southern Indian creteceous rocks have shown, however, that a slight change in this table is necessary and the reasons for such an alteration are recorded in this short note.

Shillagoody is a small village situated within the Ariyalur area(7). The place is known to be fossiliferous and the following fossils have hitherto been recorded from these beds :—

1. *Pentacrinus* sp.
2. *Axinæa sub-planata* Stol.
3. *Anomalocardia (Scapharca) ponticerina* d'Orb.
4. *Vola quinquecostata* Sow.
5. *Plicatula striato-costata* Stol.

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(3) Rec. Geol. Surv. Ind., Vol. 36, pp. 109-121.

(4) Zeitschr. d. deutsch. geol. Gesselsch., Vol L. (1898), p. 200.

(5) Op. cit., p. 120.

(6) N. J. f. Min. Geol. u. Pal. Beilage-Band., xxx (1910), pp. 330-336.

(7) Mem. Geol. Surv. Ind., Vol. iv., p. 131.

6. *Plicatula instabilis* Stol.
7. *Cerithium (Sandbergeria) Trichinopolitense* Forb.
8. *Chemnitzia sp. indet.*
9. *Trochactæon truncatus* Stol.
10. *Nautilus sublævigatus* d'Orb.

Some of the fossils mentioned in the foregoing list were obtained by the Presidency College party but the find includes also the following :—

1. *Hemiaster tuberosus* Stol.
2. *Hemiaster cristatus* Stol.
3. *Terebratula subrotunda* Sow.
4. *Veniella obtruncata* Stol.
5. *Leptomaria indica* Forbes.
6. *Protocardium hillanum* Sow.
7. *Trigonia Brahminica* Forbes.
8. *Ostrea cf. acutirostris* Nilss.

Besides, one specimen of *Placenticeras* has been collected in the Shillagoody beds in association with the fossils just mentioned and a comparison with the known species of the genus at once proves its identity with *Placenticeras tamulicum* Kossmat [(8)].

As far as we know at present *Placenticeras tamulium* Kossmat is confined to the upper part of the Trichinopoly (=lower senonian) beds and, as a matter of fact, a special stress has been laid upon the occurrence of this fossil in assigning the Trichinopoli beds

(8) Kossmat—Untersuchungen über die Südindische Kreidformatian, pp. 78-80. Pl. VIII, fig. 1.

to the lower senonian stage [9]. Accordingly the presence of this fossil at Shillagoody shows that either (i) at Shillagoody there exists a small Trichinopoly inlier; or (ii) *Placenticeras tamulicum* Kossmat has a wider vertical range than what is known at present.

A study of the fossils found in association with *Placenticeras tamulicum*, shows that nearly all of them have been recorded from the Ariyalur (=campanian) beds. According to Stoliczka *Protocardium hillanum* has been recorded only from the Trichinopoly group. This species is, however, of very wide geological limit and in Europe the species has been found almost everywhere 'in cenomanian, turonian and senonian deposits'[10]. Accordingly the occurrence of *Protocardium hillanum* in the Ariyalur beds is not inconsistent with the facts known about its distribution.

From these considerations it is quite clear that no Trichinopoly bed is present at Shillagoody. The

(9). 'Von grosser stratigraphischer Wichtigkeit sind in dieser Fauna *Schlænbachia Dravidica* Kossm and *Placenticeras tamulicum*, (Blanf) Kossmat, denn beide sind mit sehr bezeichnend Leitformen des untersenonen nahe verwandt, ja sogar specifisch nur schwer von den ausländischen Repräsentaten der gleichen Formengruppe trennbar.' (Kossmat—op. cit. pp 198-199).

(10). Pal. Ind., Ser. VI, Vol. III, p. 220. Doubts have, however, been expressed regarding the identification of this species as will appear from the following:—

Specimens from the Trichinopoli group of Southern India were identified with *Protocardium hillanum* by Forbes and Stoliczka, who stated that they were unable to draw any line of separation between the Indian and European examples. The concentric ribbing is coarser in most of the Indian forms, and in some the smooth inner portions of the posterior area, is relatively larger than in specimens from Blackdown (Woods. Monogr. Pal. Soc. LXII, 1908, p. 200).

palæontological evidences, on the other hand, all point to the conclusion that, as shown in Blanford's map, the area belongs to the Áriyalur group. Thus we have to arrive at the second alternative of assigning a wider range to *Placenticeras tamulicum* Kossmat *i.e.*, from the lower senonian to campanian It may be pointed out here that this is the only species of *Placenticeras* that is found throughout the whole of the senonian, the other senonian species of the genus being confined either to the lower or to the upper division.

# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. II. No. 3.

Saturday, the 23rd September, 1916, at 5 P.M.
C. V. Raman, Esq., M.A., Vice-President, in the chair.

## *On Discontinuous Wave-Motion.*

### Part II.

By

C. V. RAMAN, M.A., and ASHUTOSH DEY.

In an important paper on the theory of discontinuous wave propagation,* Harnack has given an elegant general formula expressing the mode of vibration of a string, whose configuration is completely determined by a finite number of discontinuous changes of velocity travelling over it. As an illustration of his result, Harnack has discussed in detail the cases, in which the form of vibration is determined by one, and by two such changes of velocity respectively. The analysis indicates that the case of a single discontinuity is identical with that of the principal mode of vibration of a bowed string, and in a previous communication

* A Harnack, *Mathematatische Annalen*, Vol. 29, Page 486.

from this laboratory,† it has been shown how this mathematical result may be confirmed experimentally. The general case of two discontinuities considered by Harnack covers a considerable and interesting variety of forms of vibration, and the method described in the previous paper has now been successfully extended so as to obtain an experimental confirmation of Harnack's results in some of these cases also.

*Experimental Method.*

If $C_1$ and $C_2$ represent two discontinuous changes of velocity travelling on a string of finite length which completely determine its motion, the velocity-diagram of the string must in general consist of three parallel straight lines as shown in Fig. 1(*a*) or Fig. 1(*b*). Each of the two outer lines passes through one of the fixed ends of the string and is separated from the intermediate line by a discontinuity.

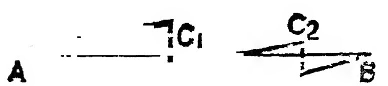



Fig. 1(*a*).

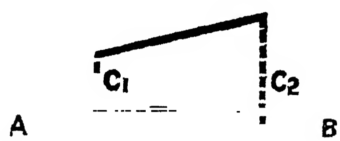



Fig. 1(*b*).

In Fig. 1 (*a*), the discontinuities are of the same sign, and in Fig. 1(*b*), they are of opposite signs. A velocity-distribution similar to that shown in either of the figures would be obtained if a string has initially a uniform angular velocity about a point in its own line, and if in the course of this motion, the two points A and B, are suddenly fixed, either simultaneously or

† C. V. Raman, M. A. and S. Appaswamaiyar, 'on Discontinuous Wave-Motion.' Phil. Mag., Jan., 1916.

else successively at an interval less than that necessary for an impulse to travel from A to B or *vice-versa.* If the point about which the string has initially a uniform angular velocity, lies within A B, we have the case shown in Fig. 1(*a*). If it lies in B A produced, the velocity-distribution is similar to that shown in Fig. 1(*b*). The photographic records presented with this paper (Plate I) refer to a case in which the string has initially a uniform angular velocity about a point lying outside the two fixed stops A and B; and the discontinuous changes of velocity, which determine the form of vibration, are therefore of opposite signs.

The method by which the records are obtained is similar to that described in the previous paper, but with certain necessary modifications. The tension and initial motion of the string are, as before, secured by a weight attached to its free end which is allowed to swing down in the manner of a Pendulum. The stops A and B are placed approximately in a vertical line below the point of suspension of the string. As soon as the string impinges on the lower stop B, the weight swings inwards, and the end B is thus effectually fixed. The upper stop A, however, presents some difficulty as it is practically in the same line as that joining B with the point of suspension, and does not therefore, except at the first impact, effectually fix the string at A. To avoid this difficulty, a small cylinder of soft iron is fixed to the string midway between A and its point of suspension, and an electro-magnet is provided which when the string reaches the vertical position carrying the cylinder with it, draws

the latter inwards and then holds it. The stop at A is thus rendered completely effective. The initial motion at any point on the string between A and B and its subsequent vibration are photographically recorded on sensitive paper contained in a dark slide which moves downwards behind an illuminated slit set across the string.*

When the position of the stops A and B is such that the string impinges upon both simultaneously, the impulses cross one another midway between A and B, and the resultant vibration is then necessarily symmetrical. The first six records shown in the plate refer to the motion at different points when this condition is practically attained. The last record however shows a different case in which the string is fixed at the stop A an appreciable interval after it is fixed at B. The condition is attained by drawing A, a little out of the straight line joining B and the point of suspension. The discontinuities cross elsewhere than at the centre of the string twice in each period of vibration, and the vibration curves are then necessarily asymmetrical. This is evident from the record shown.

*Theory.*

The theoretical form of the vibration curves at any specified point may be deduced from the velocity-diagram. For, the velocity at any given point on the string is unaffected by the motion of the discontinuities except when one of them actually passes over it.

* For facility of work, it is arranged that the weight and string are released electromagnetically. Simultaneously an auxiliary pendulum is released which after an adjustable interval of time breaks a contact and releases the photographic slide.

Successive velocities and the intervals for which they subsist are thus known, and the vibration-curve which represents the resulting displacements may be plotted from these values without difficulty. The records shown in the plate are found to be completely in agreement with the results thus obtained. The special feature of interest is that the vibration-curves are seen to be intermediate in form between the two-step zig-zags of a bowed string and those characteristic of a plucked string* The reason for this is not far to seek. When one of the discontinuities is zero, we have, as already seen, the case of the bowed strings.

When $C_1 = -C_2$, the motion reduces exactly to that of a string plucked at the point at which the discontinuities cross. The cases actually recorded in these experiments are those in which $C_1$ and $C_2$ are unequal but of opposite signs and are thus intermediate between the two extreme types referred to above.

The cases in which $C_1$ and $C_2$ are of the same sign are also of interest in connection with the theory of the special forms of vibration of a bowed string obtained at the "wolf-note" Pitch, and also under other conditions when the vibration-curves assume the form of four-step zig-zags. Experiments are being undertaken to reproduce these special forms of vibration by the method indicated in this paper.

### *Summary and Conclusion.*

In these experiments, the characteristic vibration-forms produced by the motion of two unequal discon-

* Krigar-Menzel and Raps, *Sitzungberichte* of the Berlin Academy, 1893, Page 509.

tinuous changes of velocity of opposite signs have been observed and recorded photographically. Some are of the symmetrical type and the others are asymmetrical. The results are in full agreement with the mathemetical theory first given by Harnack. The vibration curves are found to be intermediate in form between those charactaristic of bowed and of plucked strings. The cases in which the discontinuities are of the same sign are also of special acoustical interest and will be studied separately.

# *On the Method of distinguishing between Calcite and Aragonite by staining by Aniline Black.*

BY SURES CHANDRA DATTA, M.SC.

Cobalt nitrate is generally employed to distinguish between calcite and aragonite while iron sulphate, silver nitrate, kongoroth and alizarin may also be used for the purpose [1]; but the effect of aniline black (benzo azurin) as a stain on calcite and aragonite with a view to distinguish between them has not been studied as yet [2]. The application of this stain for the purpose of distinguishing between calcite and aragonite is very simple and the details of an experiment are given below :—

Powdered calcite and aragonite are taken in two separate test tubes; very dilute sulphuric acid is poured into each and boiled; then solution of aniline black is added and minerals are boiled in the solution of sulphuric acid and aniline black for sometime; liquid is drained off and the minerals are washed several times in cold boiling water when the powders of calcite and aragonite are observed to have violet and blue stains on them respectively. Sulphuric acid may, however, be replaced by many other acids and salts but still a difference in the nature of the stain will be observed and the purpose of this short note is to put on record these observations. It may be observed, however, that the intensity of colour assumed by calcite is not the same in all cases, in some cases the colour is very deep, in others lighter. This remark is

(1) Dœlter's works.

(2) *Ibid.*

applicable also to the case of aragonite. In some cases after washing with boiling water, two or three times, the stain, assumed by calcite powder, is deeper than that of aragonite, in other cases it is the reverse. When the minerals are boiled only in solution of aniline black in water there is also a stain on the minerals and the intensity of this stain varies with the treatment with acids and salts as stated before. Following is a complete list of experiments—the experiments being conducted on the method outlined above.

| The solution of acid or salt in which calcite and aragonite powders are separately boiled before the addition of aniline black solution in water. | Stain on calcite. | Stain on aragonite. | REMARK. |
|---|---|---|---|
| The minerals boiled in the solution of aniline black in water only. | Violet ... | Blue ... | $\alpha$; $x$ |
| Sulphuric acid ... | Light violet ... | Do. ... | $\alpha$; $x$ |
| Nitric acid ... | Ditto ... | Do. ... | $\alpha$; $x$ |
| Hydrochloric acid ... | Very light violet | Do. ... | $\alpha$; $x$ |
| Phosphoric acid ... | Practically white *i.e.*, *nil*. | Bluish tinge ... | $\alpha$; $x$ |
| Oxalic acid ... | Deep violet ... | Deep blue ... | $\alpha$; $x$ |
| Tartaric acic ... | Light violet ... | Do. ... | $\alpha$; $x$ |
| Citric acid ... | Do. ... | *Nil* | $\alpha$; $y$ |
| Acetic acid ... | Do. ... | Blue ... | $\alpha$; $x$ |
| Formic Acid ... | *Nil* (nearly) ... | Do. ... | $\alpha$; $x$ |
| Lactic acid ... | Light shade of violet. | Light blue ... | $\alpha$; $x$ |

| The solution of acid or salt in which calcite and aragonite powders are separately boiled before the addition of aniline black solution in water. | Stain on calcite. | Stain on aragonite. | REMARK. |
|---|---|---|---|
| Picric acid ... | Light violet ... | Blue .. | $\alpha$; $x$ |
| Succinic acid ... | Light violet (on washing once with cold water.) | Practically white *i.e.*, *nil* (on washing once with cold water.) | $\beta$; $y$ |
| Salicylic acid ... | Violet ... | Blue ... | $r$; $x$ |
| Malic acid ... | Light violet ... | *Nil* (nearly) ... | $\beta$; $y$ |
| Malonic acid ... | Do. ... | *Nil* (nearly) ... | $\beta$; $y$ |
| Meconic acid ... | Violet ... | Bluish ... | $\alpha$; $y$ |
| Phtalic acid ... | Light violet ... | Blue ... | $\alpha$; $x$ |
| Uric acid ... | Dull violet ... | Rich blue ... | $\alpha$; $y$ |
| Gallic acid ... | Very light shade of violet nearly white (on washing once with cold water.) | Shade of Blue (on washing once with cold water). | $\alpha$; $x$ |
| Benzoic acid ... | Violet ... | Blue ... | $\alpha$; $x$ |
| Ammon-sulphate ... | *Nil* | Do. ... | $\alpha$; $x$ |
| Ammon-chloride ... | Light shade of violet. | Light shade of blue. | $r$; $y$ |
| Ammon-nitrate ... | Light violet ... | Blue ... | $\alpha$; $x$ |
| Ammon oxalate ... | Rich violet ... | Do. ... | $\alpha$; $y$ |
| Ammon tartrate ... | Light violet ... | Light shade of blue. | $\beta$: $y$ |
| Ammon citrate ... | Light shade of violet (on washing once with cold water.) | *Nil* (on washing once with cold water). | $r$; $y$ |
| Ammon acetate ... | *Nil* | Blue ... | $\alpha$; $x$ |
| Ammon benzoate ... | Light violet ... | Do. ... | $\alpha$; $x$ |
| Ammon phosphate ... | Violet ... | Do. ... | $\alpha$; $x$ |

$\alpha$—Very easily distinguishable after 2nd or 3rd washing.

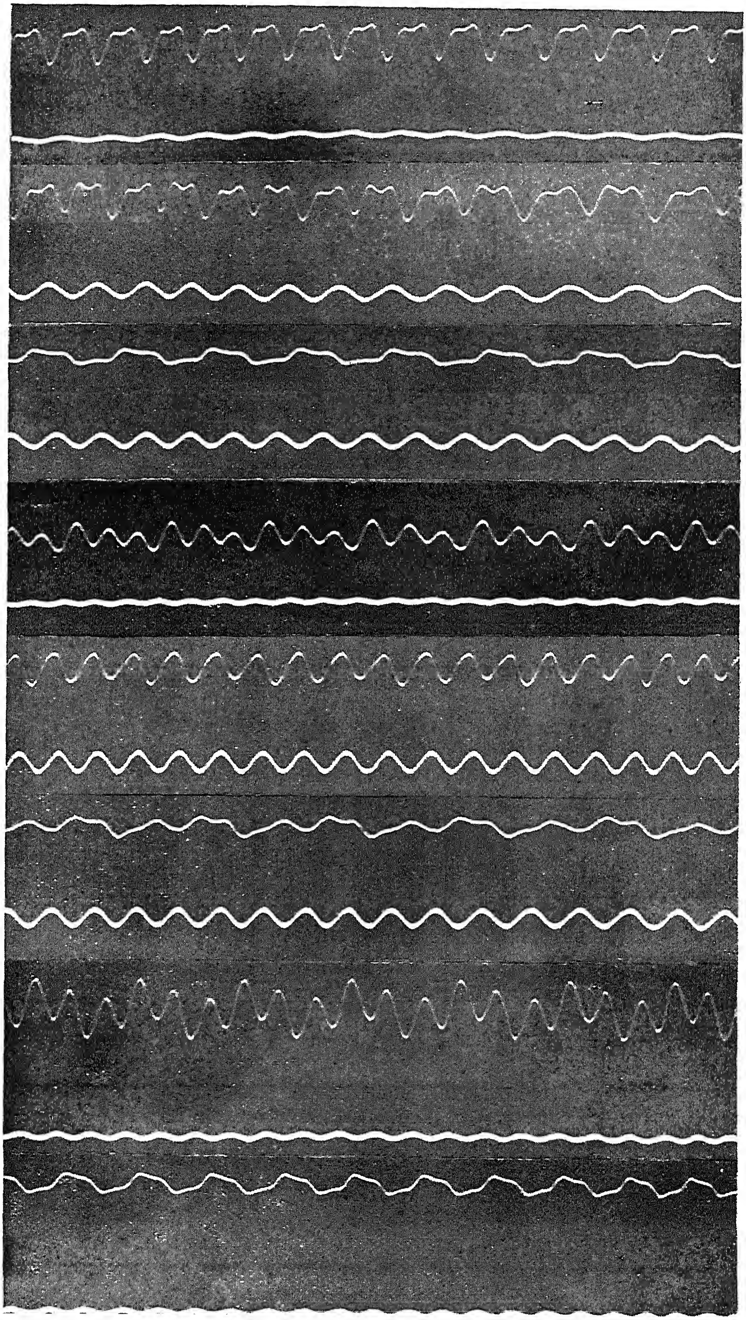
$\beta$—Easily distinguishable after 2nd or 3rd washing.

$\gamma$—Distinguishable with difficulty.

$x$—Colour on aragonite is deeper than that on calcite.

$y$—Colour on calcite is deeper than that on aragonite.

*N.B.* The stains on the powders of calcite and aragonite fixed by the method outlined above, are better distinguished under water than when they are dry. The intensity of stain on either of the minerals, calcite ,and aragonite, is not the same always, same acid or salt being used. But under the same conditions, with the same acid or salt there is always a difference between the stains on the minerals calcite and aragonite—the stained minerals being washed first in cold water when the intensity remains great though also there is great difference between the stains on calcite and aragonite, but this intensity decreases somewhat in some cases, but in others colour practically disappears on washing with boiling water afterwards.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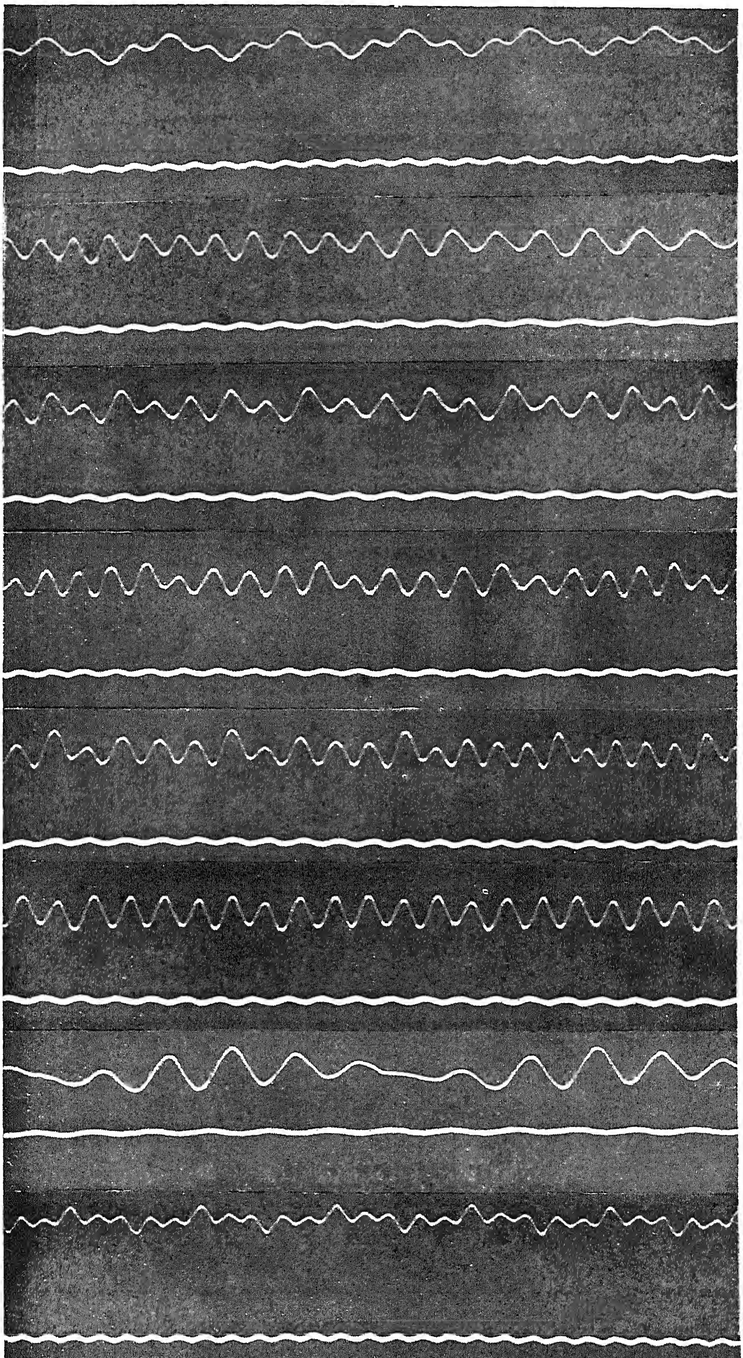

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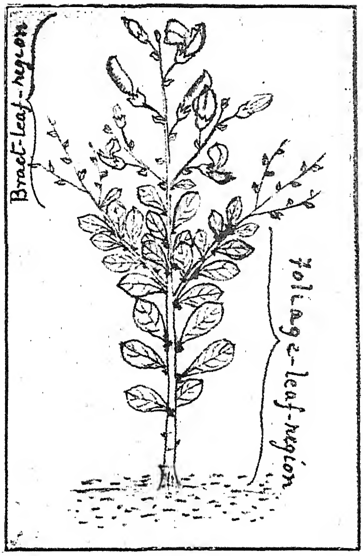



Fig. 1.

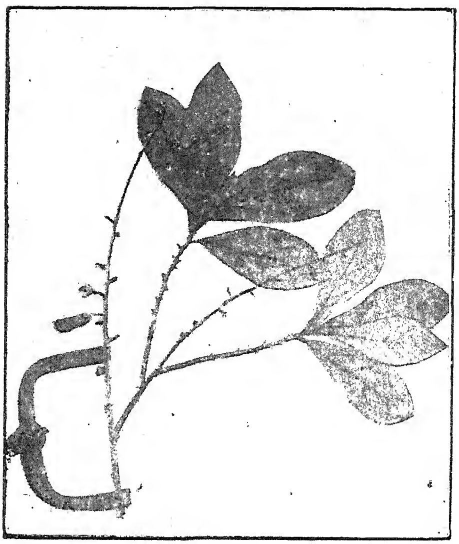


Fig. 2.

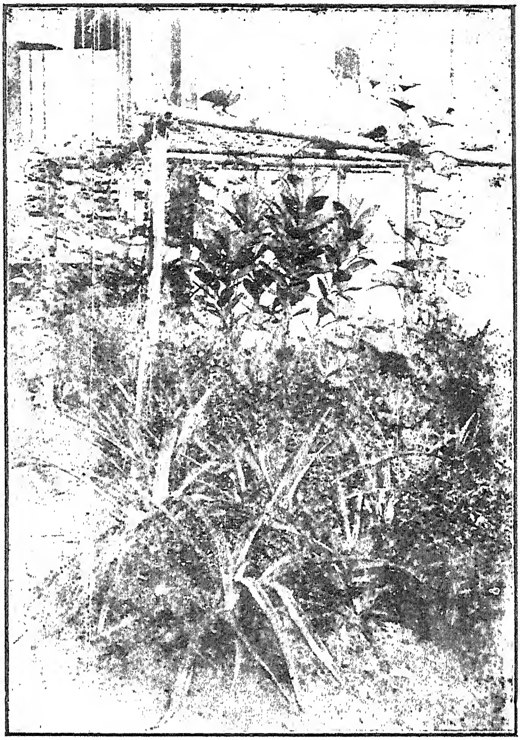


Fig. 3.

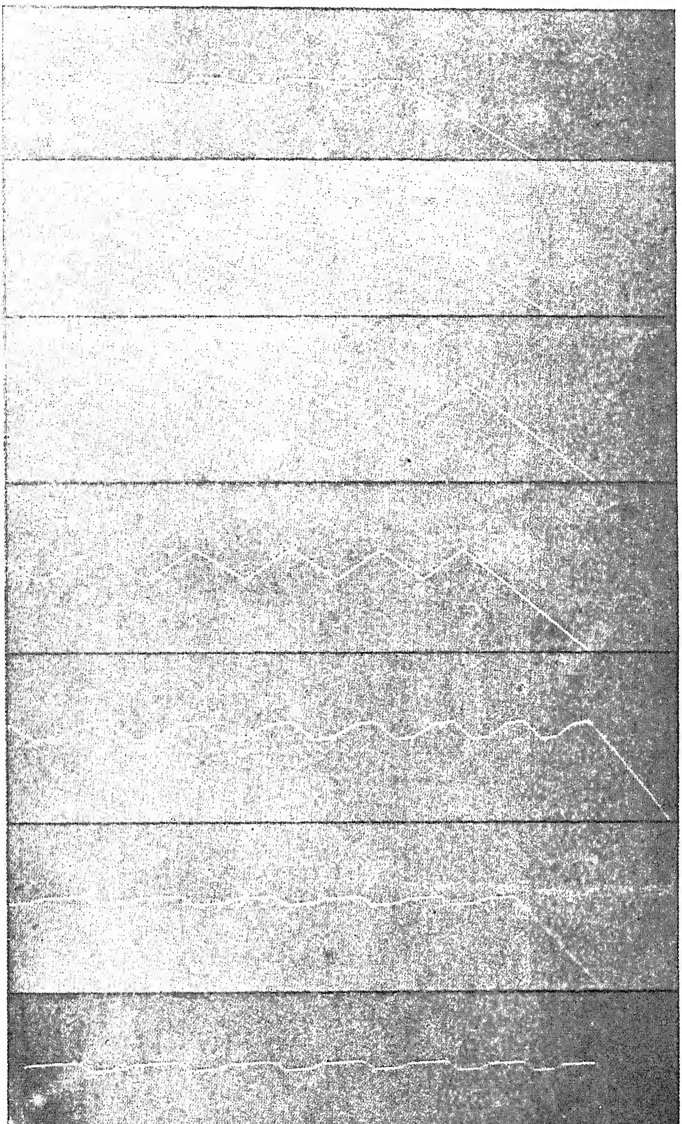
Banerjee. PLATE III,


**Fig. 4.**

*Photo by* B. Maitra.

Placenticeras tamulicum Kossmat.

Observed at



$l/9$

$l/4$

$2l/5$

$l/2$

$3l/4$

$8l/9$

$1l/9$

Photographs of vibration-curves showing the initial motion and the subsequent vibration with discontinuous changes of velocity.

# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION

FOR THE

# CULTIVATION OF SCIENCE

**VOL. III.**

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1917.

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## PLATES.

# PROCEEDINGS

OF THE

# INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. III. No. 1.

Saturday, February 24th, 1917 at 5 P.M. Dr. B. L. Chaudhury, D.Sc., F.L.S., F.R.S.E., Vice-President, in the chair.

## *Notes on a Free living Amœba of a new Species.*

BY RAI BAHADUR, DR. GOPAL CHANDRA CHATTERJEE, M.B.

Introduction.—In routine bacteriological exanimation of samples of potable water, I accidentally found a protozoa of identically the same species as the one I found previously in the intestinal contents of a diarrhœa case. It formed a subject of a paper I recently published. This stimulated me to make a study of protozœ fauna of potable waters with a view to 1stly isolation in pure state protozœally speaking of some of the easily culturable types and study them in all their phases in test-tube cultures so that it may serve to illuminate some of the obscure points in the life history of some of the allied parasitic forms, 2ndly to find out any particular species or group of species in any sample of water, presence of which may indicate possible fæcal contamination.

I entirely depended on cultural methods for separation and study of the protozoe. By these means, I have succeeded in isolating several types. One of these is an amœba which showed well marked characters by which it can be easily identified and differentiated from the previously known ones.

Description of the amœba.—It first came under my observation in ordinary peptone bouillion culture of a sample of water. In this it grew luxuriantly, a single drop of the broth showing millions of them. When examined under high power without staining. these were seen as small globular bodies floating about in the fluid. They showed very little amœboid movements. When examined carefully, the circular bodies showed change of shape and occasionally a slight protrusion of pseudopodia.

When stained by Giemsa stain, they show a body about 3 to 5 $\mu$ diameter. In the centre is situated a voluminous nucleus ; this shows a cluster of chromatic dots distributed throughout the nucleus without showing any differentiation of central caryosome and peripheral chromatic layer. No definite nuclear membrane can be made out in many specimens. The chromatic dots forming the nucleus are distributed throughout the protoplasm of the cell, so that a differentiated nucleus is not clearly seen. The protoplasm of the cell shows no vacuole no pulsating vacuole can be made out. It is uniformly stained.

Method of division—The dividing stage has been clearly followed by me in plate culture underneath the microscope. As a rule one amœba divides into two. The process takes from begining to complete

separation not more than 5 to 10 minutes. In stained preparation, no definite karyokinetic figure can be made out.

In some specimens, a large number of big-sized amœba are found which show the nucleus divided into 5 to 8 parts and distributed through several parts of the body of the amœba. This seems to suggest a process of schizogony but actual division into several amœbæ has not been observed by me.

Cyst or spore formation.—In several ordinary types of culture amœbæ found in water, they show a well marked tendency to form cyst on the third or fourth day of culture irrespective of supply of nutrition or of water—in fact this cyst formation is the rule. In this amœbæ I failed to find any cyst even after 10 days of culture in solid medium, After this time, they die out and disappear from culture whereas in the case of ordinary amœbæ they can be seen in solid medium even after two months though most of them have transformed themselves to the cystic stage.

Flagellate condition.—Under certain condition, which I could not make out, the majority of the individual amœba in culture transform themselves suddenly to actively moving flagellate condition. These when fixed and stained by Iron-Hematoxylin show a small oval body about $3\mu$ length, showing clear protoplasm and a nucleus situated at one pole of the body. The nucleus show clear differentiation of a central caryosome and a clear zone outside it. In front of the nucleus are seen originating two flagella without the intervention of any basal granules.

Cultural characters.--This amœba shows a remarkable property of growing luxuriantly in ordinary agar used for cultivating bacteria and also in broth. In Frosch's medium, it also grows luxuriantly. In Musgrave's medium it grows scantly. On the surface of potato, the amœba grows very luxuriantly. When a film is made from potato culture, the amobœ are seen in various sizes, some being not bigger than $1 \cdot 5 \mu$ while the biggest measures about 7 to $8 \mu$.

Literature.—For the purpose of my paper, it will suffice if I deal with the papers dealing with the culturable free living amœbœ whose specific characters have been clearly defined. Of those the most important is by Nägler who described the characters of 7 varieties of amœbœ which he cultivated in Frosch's medium. All these amœbæ are much bigger than the one I have dealt with and all showed cyst formation very readily. Wherry isolated an amœba from water and studied it for 2 years. This showed endogenous budding and binucleated cyst formation. Besides this showed flagellate condition This is also much bigger than my amœbæ.

Vahlkampff describes characters of Amœba Simon. These all show cyst formation and pulsating vacuole.

From all these it appears that this amœba differs from all previously described ones by the following characters.

(1) The power of rapid growth in ordinary agar and potato,

(2) By its small size,

(3) Want of cyst formation.

*Illustrations.*

1. Plate No. I is made from 24 hours bouillon culture stained by Giemsa. The plate shows an actual field showing the large number of amœbæ. It is drawn under $\frac{1}{12}$ apochromatic lens and No. 6 eye-piece.

Plate No. II is a preparation from a plate culture in ordinary agar. Stained by Iron-hæmatoxylin drawn under $\frac{1}{12}$ apochromatic lens and No. 12 eye-piece.

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## *On the Process of Development of Rohita, Catla and Cirrhina Mrigala in confined Waters in Bengal.*

BY BEPIN BEHARY DAS, M.A.

### *Introduction.*

The principal and the most economic of the Bengal fresh water fishes of the carp family are the *Rohita, Catla, Cirrhina mrigala* and *Calbasu.* In Bengal, the मत्स्यदेश of ancient India, where rice being the staple food, fish eating was considered necessary by the Hindu *Sastrakars*, the culture of these fish was thought of as an important element in domestic economy. This culture was, however, entirely in the hands of fishermen who, though not a race of great erudition, was undoubtedly one of keen intellect and from whom originated the famous व्यासदेव and the renowned Pandavas and Kouravas. It is true that those of superior intellect under great paternal influence developed themselves into the greatest geniuses and Rajas but those who were left to their professional calling did not leave to posterity any written account of their work. It is therefore very unfortunate that no literature as to the mode of culture of fish in vogue in ancient times has been discovered. All that is known about their work is that fishes of the carp family were regularly stocked in household tanks from where

they could be easily fished for use. The practical methods in connection with the culture, it appears, have been handed down from father to son as is the case with most of artisan classes in India. From what could be gathered from the present generation of Bengal fishermen who command information of about one century back, it is clear, that they had no knowledge of the spawning habits of the fish like *Rohita*, &c., which form the subject matter of the present paper. Consequently no attempts were made to culture the fish by inducing them to spawn. It is however certain that the practical fisherman of Bengal attempted to find out whether it was possible to raise fry by the spawning of the fish in Tanks. There is also evidence of attempts for finding out the conditions under which the fish wonld spawn. The sum total of their knowledge in this direction is that these fish do not breed in tanks or in confined waters in Bengal, while the inferior members of the family Cyprinidæ do breed in confined waters. With our present knowledge about the spawning habits of these fish it will be wrong to reflect on the spirit of research of our fishermen. It was not for the practical fisherman to undertake such a work and he never felt any great necessity for such a knowledge in his culture of fish. In some cases local knowledge of great scientific value has been noticed and worked up to but there had been no attempt to publication. This is due mostly to the poor condition of the fishermen—they, being pressed very hard for the bare necessities of life, were unable to think of anything beyond what brought them something to go on with.

"In cool sequestered vale of life

They led the noiseless tenor of their way."

In all cases the practical fishermen were able to obtain fry for his economic culture. This ready supply of fry from the rivers made him think of the spawning condition of the fish not as a thing of great importance in his culture. He, however, knows that the fish have roe in some cases and milt in others and that the ripening of the roe and the milt takes place once in one year *viz.*, in the rainy season. After the first showers when there is flood in the rivers tiny larval fish are fonnd in great numbers. These are collected by a very careful method devised by the fishermen themselves and cultured in small nursery tanks before they attain the shape and form of fish in the course of two weeks. There are a class of fishermen whose work is to nurse the larval fish for two weeks after which the small fry are caught by a fine net and sold to others who either stock the fish in larger tanks or who nurse them in a second system of tanks where they grow up to fingerlings in two months. These are then introduced into stocking tanks where the growth is a matter of chance;—in some tanks the rate of growth is small, while in others it is extraordinarily great—depending chiefly on the natural nutrition of the tank.

Thus the system of culture in use in this country starts with the collection of the fry from rivers and ends with the stocking of the fingerlings in stocking tanks. There is no arrangement to examine the fish before it is taken out for consumption—and no rule to preserve it, if it is not fully grown. The fish is generally caught for the first and the last time in its life

and when once caught it is not allowed to retun to water even if it has not attained the adult stage. There is however a tendency to allow it to grow up as it is considered more profitable to kill it when adult than when it is young. This indiscriminate destruction of young fish is due partly to the pressing needs of the poor fishermen and also to the manner of leasing out the fisheries.

As the Tanks are mostly owned by the Zamindars and others who are not fishermen the fish culture in them is involved with some difficulty. The culturist in most cases has to take lease of the tanks and pay rent to the owners or the culture is done by the owners themselves. In the latter case the owners have to incur expenses for the purchase of fry and the cost of netting fry and the fish. In the majority of such cases the owners of the Tanks who might be called amature fish culturists do not sell their fish to the public but reserve them for their own use on extraordinary occasions and feasts. So that the fish produced by the amature culturists is not available for the regular consumer. On the other hand the fish produced by the fishermen is the source of a regular business which forms the means of livelihood of many of them. This fish forms a part of the fish supply in Bengal markets. The quantity however is very small and many of the fishermen engaged in the business complain of the small pofit. It is therefore clear that without an improvement in the method of culture, Pisciculture in tanks will cease to be of any use as regards the supply of fish to the public and also as regards a business by itself. The enquiry into Fishery

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matters was undertaken by Sir K. G. Gupta and on his recommendation one student was deputed to study the culture of the Carps in Europe which are similar to our Rohita &c., and another for Shad in America which are similar to our Hilsa. The writer of the present paper was elected to study the culture of carps in European countries where the results have been very promising. The following account is a brief statement of the work undertaken by him since his return to India.

*Experiments and Observations.*

Soon after my return from Europe I arranged in March 1910 four tanks spawning experiments in Bhagalpur. These were not more than 10 *cottas* each in extent and were constructed by erecting small embankments in a sloping land. The site was in the Hathia-nullah in Barari, Bhagalpur. *Broodfish (Rohita, Catla* and *mrigal)* were collected from the river Ganges and also from the several tanks in and about Bhagalpur. These broodfish were with the exception of a few only not properly mature for reproduction and the mates were not exactly matched on account of the great scarcity of broodfish. But before any experiment could be tried the spawning tanks were filled up by an accident. This was on account of the slip of a huge bank of sand from the water-works. Thus no results were obtained in 1910-11.

My next attempt was at Berhampur in the District of Murshidabad. Arrangements were made for six spawning experiments with *Rohita, Mrigal* and *Calbasu.* Four of these were tried in four newly excavated tanks, 50 feet by 50 feet and 8 to 10 feet deep.

The other two experiments were tried in two old tanks which were cleared of the silt from the bottom and re-excavated for the purpose. Broodfish were collected from the Bhagirathee and Bhandardaha Beel as well as from some of the tanks in Kasimbazar and Polinda. In the selection of the broodfish great attention was paid to sexual maturity of the fish—only fully mature ones were taken for the experiment. There were no mature *Catla* found although more than 100 fish were examined. In this selection of the broodfish it was noticed that while in some tanks the *Rohita* and *mrigal* were found mature enough for reproduction, in others there was no development of the genitals although the fish were in an adult stage weighing 5 to 6 seers each. The same was the case with fish from the Bhagirathee and the Bhandardaha Beel. The development of the genitals of the fish under varied circumstances is a subject by itself and has not been properly studied. This is left alone for the present. I am gathering materials to work up this subject and hope to complete it in future. To return to our point, only fully mature fish were used for the experiments at Berhampore. The *Rohita* and *mrigal* weighed from 4 to 6 seers each and the *Calbasu* were from 1½ to 2 seers. In selecting the match (Spawning party) great attention was paid to make the males strangers to the females. This was done by putting together males of one tank or river with females from a distant place, e.g., the males from the Bhagirathee were put together with females from a tank in Polinda. This kind of mating is believed in Austrian and Bohemian culture stations to increase

the sexual attraction and enhance the breeding. This is also said to improve the breed.

During the collection of the broodfish the males were kept separate from the females and every care was taken to see that the fish did not get any way injured—not a single scale was allowed to come off. They were transferred from one tank to the other by means of large wooden barrels full of water and carried on bulluck carts. Only two fish were taken at a time in a barrel containing 70 gallons of water.

The experiments proper ware started in the second week of June. The ratio of the females to the males in a spawning party was not allowed to be less than 1 : 2 nor was it more than 3 : 4. In the course of five weeks from the starting of the experiments a large number of fry was discovered in five of the experimental tanks. These were found to be *mrigal* in three cases and *Rohita* in other two; while in the tank in which the *calbasu* were put no fry were observed.

The tanks were situated at a distance of one mile from where I stayed, I visited the tanks three or four times during the day while a watchman was on duty all the time near the Tanks. No spawning movement was noticed by me or the watchman. But after five weeks swarms of tiny fish were observed in the four new tanks and in one of the old tanks as stated above.

The fry were fed with powdered lumps of dried blood collected from the slaughter house. The fry got accustomed to this feeding like the trout fingerlings to minced liver in Europe, They also increased

fairly well. In two weeks they increased to one inch. Everything was going on in order and there was every hope of rearing the young fish to maturity. But on account of an accident nothing further could be done. On the 17th September, 1911, there was a very heavy rainfall continuing for three days at Berhampur. All the tanks and the adjoining drain which was in connection with the Bhagirahee formed one sheet of of water. The result was that all the fry were lost and none could not be reared to maturity.

The conclusion arrived at by these experiments was that the *rohita* and the *mrigal* under favourable circumstances do breed in still water of tanks. The spawning habits of the fish as also their spawning movements were however not studied, nor was it possible to study the development of the embryo in the egg.

I am very much thankful to the Hon'ble Maharaja Manindra Chandra Nandi of Cossimbazar and to Babu S. V. Sen, Zamindar, Khagra for rendering me valuable assistance in respect of collecting broodfish for these experiments and in a variety of ways.

During the following four years experiments were conducted by the Fishery Department in Berhampur, Cuttack and Bankipur under the direction of the Deputy Director of Fisheries. Fish from the rivers were used as broodfish. The results of these experiments led to no defenite conclusion regarding the breeding habits of the fish. In some cases the *catla* and the *mrigal* were supposed to have spawned from the presence of the fry in the tanks discovered afterwards but neither any observation about the

nature of spawning, nor any study about the embryonic development of the fry within the egg was done. It was decided that in still water the spawning of the fish in tanks is a matter of chance *i.e.*, under peculiar circumstances if the female fish sheds her ova directly when the male sheds his spermatozoon, fecundation takes place. The fish were considered to spawn only when there is a current. The work of the Fishery Department in this direction then consisted in obtaining the fry from the river and supplying them to the public at cost price.

In 1915 I tried to artificially fertilise the eggs of *rohita*, *catla* and *mrigal* and subsequently to incubate them in Macdonald jars. In an experiment of this kind done at Berhampur in 1910 according to the report of Babu S. V. Sen, Zamindar, Khagra, a few fry were obtained. I however was unable to come to any conclusion about it because myself never proceeded with the incubation of the eggs which were considered to have been fertilised. I was able only to mix the ova which were obtained by a caesarian operation on the female with the milt which could be stripped out of the males. I was compelled to leave the products, which on account of a change in appearance was considered to have been fertilized, in charge of Babu S. K. Sen who after some time reported to have obtained a few fry. In 1915 I was able to make a series of experiments of a similar nature at Cuttack. More than 25 experiments were done by taking out the ova by cutting open the female fish and mixing them with the milt which was obtained by stripping the male. There was a change in the appearance of

the ova when it was mixed with the milt. As there was no development of the embryo on incubation in Macdoland jars and in hatching trays under different conditions, the change in the appearance of the ova was certainly due to the absorption of water which might contain a trace of some salt and not to any real fertilisation. In these experiments I was able to try only the *rohita* and *calbasu* from Tanks. It was our intention to try the river fish—but no mature males could be obtained for our experiments—all those caught in the Mahanuddy were fully spent.

The conclusion arrived at by these experiments was that the artificial fertilization of the ova of *rohita*, &c., was very uncertain on account of their gelatinous nature. For this reason stripping is not possible even when the first is ripe. The extraction of the eggs by opening the fish evidently does not give the ova which are really fit for fertilisation. These experiments were started from the 1st of July L915 and all the male *rohita* and *mrigal* from the rivers were found to be fully spent. From this and from my previous observations on the development of the sexual organs of the fish it appears that in an adult fish (*rohita* &c.,) the ovaries and the testes start to develop from the middle of March and become fully ripe by the end of May. So that the breeding season commences early in June.

With this state of our knowledge about the breeding habits of the *rohita*, &c., I was lucky to find, that the exact nature of their spawning. I was present in the village Talbandi in June 1916 when the fish were actually spawning in a "Bandh." "Talbandi" is 13

miles from Garbetta in the District of Midnapur. The "Bandh" or the Embankment in this village was con-constructed about 25 years ago. The construction is very much like a large spawning tank in Bohemia. During the rains the water area extends over more than 50 *bighas* but in dry weather it reduces itself to 10 *bighas* or less. The embankment is at the foot of a sloping country mostly covered with a juungle of Sal trees. The slope is very gentle being less than 1 : 50. So that the banks were nearly flat except on the side in which the embankment was constructed in order to form the reservoir. The condition was very similar to that in the spawning tanks in Europe and also to that in my experimental tanks in Bhagalpur. Here therefore I got everything what I was looking for in my experimental spawning tanks; there was the embankment, very gentle slope of the side, the bed which had been exposed to the sun and air during the major part of the year, &c. And the most important thing in them was that there were the broodfish. The people also reported that these fish regularly spawn almost every year. One thing in this place was not similar to the European spawning tanks for carp. It was this that the sides were not overgrown with grass &c., to which the firstilised eggs stick as in the case of the eggs of the common gold fish. The original intention for the construction of the Reservoir was irrigation but they introduced in quantity of fry in order to get some fish as a bye-product. These ultimately grew up and started spawning. Near about this reservoir I erected temporary hatchery. There was an arrangement for working four Macdoland

hatching jars, two wooden hatching trays and the final waste water was introduced into an excavation in the earth 3 ft. × 2 ft. × $1\frac{1}{2}$ ft. deep. The water from the last was allowed to flow out through a brass mesh.

Thus equipped I was watching the movement of the fish in the reservoir. In the afternoon of the 13th June 1916, I found that some of the fish were moving about but this movement could not be considered to be due to spawning. This movement ceased under cover of night. It was drizzling but there was no breeze. Towards morning rather a strong breeze started and the fish were seen in groups coming in very shallow water where it was only one foot deep. Some of the fish were lying quite flat on one side for a time then they ran about round and round not very far from a central spot where they again returned. The water was very much coloured on account of the washing of the laterite of the soil and for this reason the exact outline of the fish could not be seen when in motion. But when some of them were lying on one side as stated above the fish could be distinguished. I saw many times large *catla* fish which would be more than 25 seers each. The *mrigals* were seen with the *mrigals* at time but during the whole period, all the *rohita*, *mrigals* and *catlas* were moving together. The fish were not at all shy and could be taken without any trouble. In this heterogenous mixture the fish were touching one another and at times gathered together in heap. There were *boalis* in the reservoir and two of the breeding fish were attacked by the *boali*. In one case the *boali* was killed by the bystanders. The spawning started

at 4 A.M., in the morning and in one hour's time large numbers of fertilised ova were drifted to the banks which were at about 20 yards from the spawning centre. I collected some of these and started their incubation in my hatchery. The spawning continued till 8 A.M. There was no current of any kind either flowing into the Reservoir or out of it. The drizzling rain was so small that the water was getting soaked in the sand. The breeze however continued and became stronger. Many of the eggs on that account were thrown over the dry sand which formed the banks of the "Bundh." A good quantity of the eggs was thus lost. Those which remained in the water were collected by those who do business in fry or those who culture fish in tanks. I arranged to introduce water by earthenware pitchers in order to put back the eggs into water and thus a portion was saved. The eggs remained in water in any position as their density was nearly the same as that of water. As the spawning continued the eggs were drifted more and more and in larger quantities towards the shore. When the spawning ceased the water was full of eggs from a depth of three feet to the shore. For ten hours the eggs were not allowed to be disturbed. After that time the development of the embryo within the egg shell was perceptible. The spinal column was seen nearly surrounding the yolk sac. It was then that they considered the eggs to be ripe (ডিম পাকিয়াছে). The eggs were collected by men who came from the adjoining villages and also from distant places. The collection of the eggs started from 2 P.M., and continued till 6 P.M. The owners of the Bundh did not charge any price for the eggs

but they distributed them among the assembled mass in order to ensure even distribution as much as possible. The management in this distribution was difficult and riots were not very uncommon that day. The eggs were collected by dipping with an earthen or metallic vessel about one seer in capacity. When the eggs became scarce on account of constant dipping they used a fine cloth to gather them together. When they collected as much of the eggs as they could, the work in the Bundh was finished. They took away the eggs with water in earthen "Handis" and put them in small excavations in the earth. These were about 3 feet by 4 feet and 2 feet deep. Some of them were larger while others were smaller. They are called "Hapor" হাপর and are filled with rain-water collected from the low lying lands. The eggs remained there till they were 24 hours old in which time they hatched out. After the eggs had all hatched out the larval fry were collected by means of a fine cloth and put into another similar excavation filled with fresh water. The fry are then ready for sale to those, who culture fish in tanks. These people come from over 20 miles. The sale is very brisk for the first two or three days. It sometime continues for 15 days but after seven or eight days the fry become weak and begin to die for want of food. During this period the only work of the fry seller is to add fresh water to the pits or "Hapors" and to protect them from overheating by the direct rays of the sun.

In my extempore hatching I incubated the eggs (1) in Macdonald jars (2) in a glass aquarium (3) in Zinc and (4) wooden troughs. The incubating appara-

tus were put in series and a gentle current of water about 1·5 quarts per minute was made to pass through them. Fine brass mesh was put seperating the diferent incubating vessels. For want of proper equipment microscopic examination of the embryo could not be done during the successive stages on the spot. From an examination of a series of specimens preserved by my wife, I am able to work out the growth stages of the embryo within the egg and also when it hatched out. For want of time I not was able to study more than a few of these specimens in the laboratory of our Science Association which has been my home, my alma mater, my guide, philosopher and friend ever since the dawn of my intelligence I am studing there specimens which show the development of the fry up to one month and hope to present you a thorough account in the next meeting. The following is the description of the different stages as far as I was able to complete.

(1) First hour after spawning

(2) Twelve hours ,, ,,

(3) Sixteen hours ,, ,,

(4) Twenty four hours after spawning just before hatching.

The egg shell is not shown in figs. 2,3 & 4.

(5) Seventy six hours after spawning or two days after hatching, side view.

(6) Ditto, dorsal view.

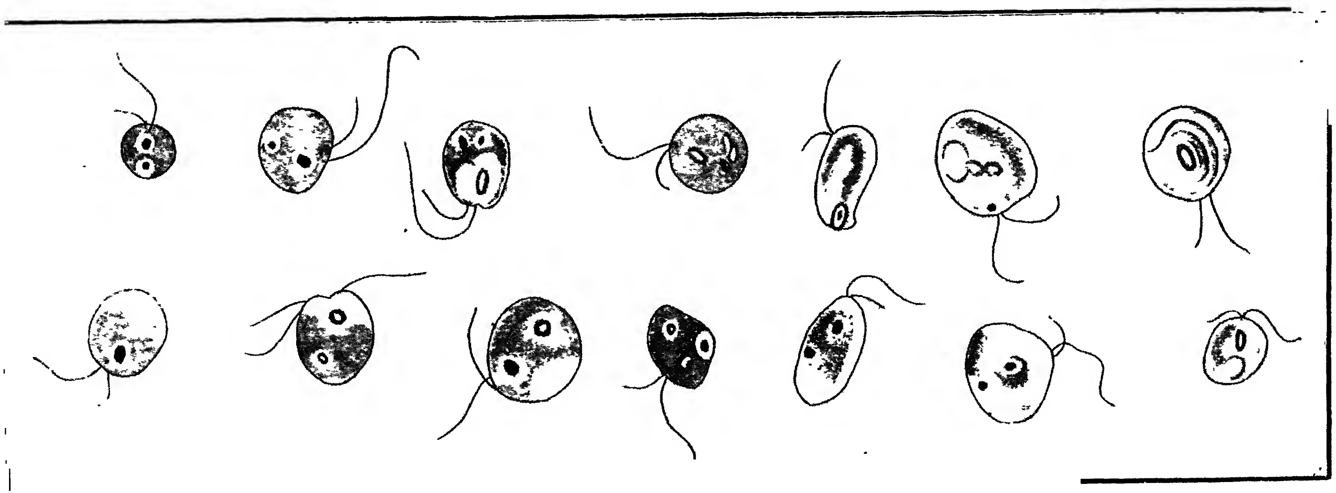
## *Further Experiments on Electrically maintained Vibrations.*

By C. V. Raman, M.A. and Ashutosh Dey.

(See Proceedings, Vol. II, Part I, 1916).

Chatterjee. PLATE I

Chatterjee. PLATE II.



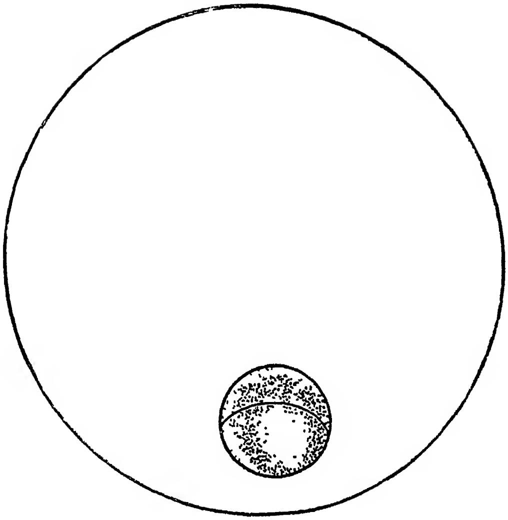


Fig. 1.

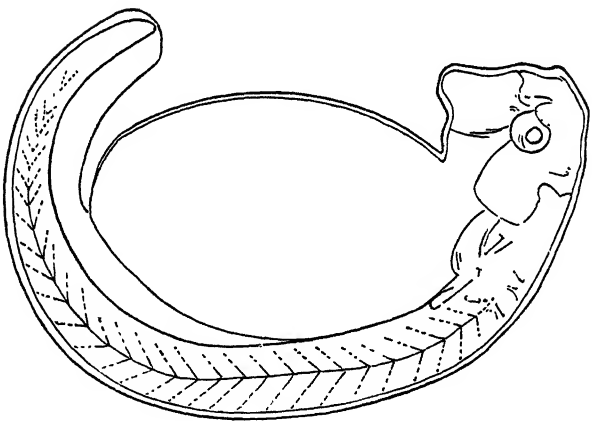


Fig. 2.

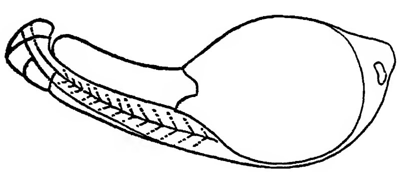


Fig. 3.

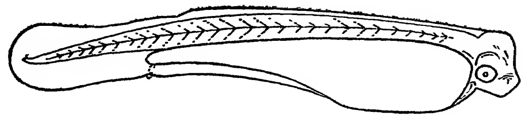


Fig. 4.

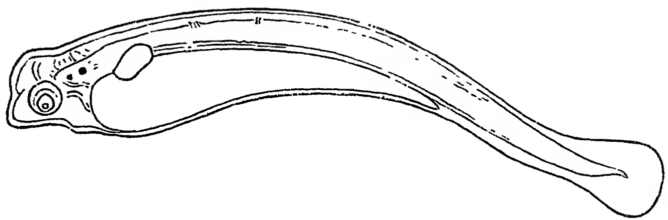


Fig. 5.

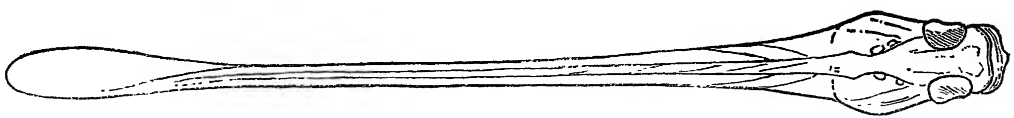


Fig. 6.

PROCEEDINGS

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Vol. III. Part II.

# On Aerial Waves Generated by Impact*.

## *Part II.*

By Sudhansukumar Banerjee, M.Sc.

### 1. *Introduction.*

The origin and characteristics of the sound produced by the collision of two solid spheres were discussed by me at some length in the first paper under the same title that was published in the Philosophical Magazine for July, 1916. It was shown in that paper that the sound is not due to the vibrations set up in the spheres, which, in any ordinary material, are both too high in pitch to be audible, and too faint in intensity, but to aerial waves set up by the reversal of the motion of the spheres as a whole. The intensity of the sound in different directions for the case, in which the two spheres were of the same material and diameter, was investigated by the aid of a new instrument which will be referred to as "the Ballistic Phonometer".† The intensity was found to be a maximum

* This is the complete paper of which an abstract was read at the Annual Meeting held on the 23rd of November, 1916. *Vide*, Report for the year 1915.

† This name was suggested by Prof. E. H. Barton, D.Sc., F.R.S., writing in the *Science Astracts*, page 399, Sept., 1916.

along the line of collision, falling off gradually in other directions to a value which is practically zero on the surface of a cone of semi-vertical angle 67° and rising again to a second but feebler maximum in a plane at right angles to the line of collision.

In view of the interesting results obtained for the case of two equal spheres, it was arranged to continue the investigation and to measure the distribution of intensity when the colliding spheres were not both of the same radius or material. A mathematical investigation of the nature of the results to be expected in these cases was also undertaken. In order to exhibit the results of the experiment and of the theoretical calculation, a plan has now been adopted which is much more suitable than the one used in the first paper. This will be best understood by reference to fig. 1 (Plate I), which refers to the case of two spheres of the same material and diameter.

This figure has been drawn by taking the point at which the spheres impinge as origin and the line of collision as the axis of $x$, and setting off the indications of the Ballistic Phonometer as radii vectores at the respective angles which the direction in which the sound is measured makes with the line of collision. The curve thus represents the distribution of intensity round the colliding spheres in polar co-ordinates, the points at which the intensity of the sound is measured being assumed to be all at the same distance from the spheres. The results are brought much more vividly before the eye by a diagram of this kind than by plotting the results on squared paper.

2. *Case of two spheres of the same material but of different diameters.*

Figure 2 (Plate II), which shows the observed distribution of intensity when two spheres of wood of diameters 3 inches and $2\frac{1}{4}$ inches collide with each other, is typical of the results obtained when the impinging spheres are nearly of the same density and are of different diameters. There is a distinct asymmetry about a plane perpendicular to the line of impact. In addition to the maxima of intensity in the two directions of the line of collision, we have the maxima in lateral directions, which are not at right angles to this line. The directions in which the intensity is a minimum are also asymmetrically situated.

For the explanation of these and other results, we have naturally to turn to the mathematical theory, which rests upon the fact that the sound is due to the wave-motion set up in the fluid by the sudden reversal of the motion of the spheres. Let $a$ and $b$ be the radii of the two spheres and $\rho_a$ and $\rho_b$ be their densities. Then the masses of the spheres are $\frac{4}{3}\pi\rho_a a^3$ and $\frac{4}{3}\pi\rho_b b^3$ respectively. Denoting the changes in velocity which the spheres undergo as a result of the impact by $U_a$ and $U_b$ respectively, by the principle of constant momentum we have $U_a : U_b = \rho_b b^3 : \rho_a a^3$. The ratio $U_a : U_b$ thus depends only on the diameters and the densities of the spheres, while, of course, the actual values of $U_a$ and $U_b$ would depend on the relative velocity before impact and the co-efficient of restitution. It is obvious that if we leave out of account the duration of impact, that is, regard the

changes in velocity of the spheres as taking place practically instantaneously, the character and the ratio of the intensities of the sound produced in different directions would be completely determined by the sizes of the spheres and the ratio of their changes of velocity, that is, by *their diameters* and *their masses*; when the spheres are of the same material, the nature of the motion in the fluid, set up by the impact, depends only on the radii of the spheres.

The complete mathematical problem of finding the nature of the fluid motion set up by the reversal of the motion of the spheres, taking the finite duration of impact into account, would appear to be of great difficulty. In my first paper, I have shown that when a single sphere of radius $a$ undergoes an instantaneous change of velocity U, the wave motion produced is given by the expression

$$\psi = -\frac{\sqrt{2}\,Ua^3}{4}\frac{\partial}{\partial r}\left[\frac{e^{-\left(\frac{ct+a-r}{a}\right)}}{r}\cos\left(\frac{ct+a-r}{a}-\frac{1}{4}\pi\right)\right]\cos\theta, \qquad (1)$$

which indicates that it is of the damped harmonic type confined to a small region near the front of the advancing wave. The wave motion, set up in the case of two spheres in contact assumed to undergo instantaneous changes of velocity, would be of a more complicated type. In order to obtain a general idea of the nature of the results to be expected, particularly as to the intensity and character of the sound in different directions, we may consider the analogous acoustical problem of two rigid spheres nearly in contact, which execute small oscillations to and fro

on the line of their centres. This problem may be mathematically formulated and approximately solved in the following manner :—

Given prescribed vibrations

$$U_a \cos \theta_1 e^{ikct} \quad \text{and} \quad U_b \cos \theta_2 e^{ikct}$$

on the surfaces of two spheres of radii $a$ and $b$, nearly in contact, it is required to determine the velocity potential of the wave motion started and the distribution of intensities round the spheres, where $\theta_1$ and $\theta_2$ are the angles measured at the centres A and B of the two spheres in opposite senses from the line joining the centres of the spheres.

Supposing now that an imaginary sphere is constructed, which is of just sufficient radius to envelop the two actual spheres (touching them externally), it is possible from a consideration of the nature of the motion that takes place in the immediate neighbourhood of the two spheres, to determine the vibrations on the surface of this imaginary sphere, which would produce on the external atmosphere the same effect as the vibrations on the surfaces of the real spheres A and B. When the equivalent vibration on the surface of the enveloping sphere has been obtained, we can, by the use of the well-known solution for a single sphere, at once determine the wave motion at any external point.

The radius of the enveloping sphere is evidently $a+b$, and its centre is at a point C, such that $BC=a$ and $CA=b$.

If the point C be taken as origin, and if the equi-

valent vibration on the surface of the enveloping sphere be expressed by the series

$$\Sigma A_n P_n (\cos \theta) e^{ikct}, \tag{2}$$

where $A_n$'s are known constants, the velocity potential of the wave motion produced at any external point is given by

$$\psi = -\frac{(a+b)^2}{r} e^{ik(ct-r+a+b)} \Sigma \frac{A_n P_n (\cos \theta)}{F_n (ik.\overline{a+b})} f_n (ikr), \tag{3}$$

where

$$f_n (ikr) = 1 + \frac{n(n+1)}{2.ikr} + \frac{(n-1)n(n+1)(n+2)}{2.4.(ikr)^2} + \dots + \frac{1.2.3\dots 2n}{2.4.6\dots 2n.(ikr)^n},$$

and

$$F_n (ikr) = (1+ikr) f_n (ikr) - ikr f_n' (ikr). \tag{4}$$

To obtain the equivalent vibrations on the surface of the enveloping sphere, we shall regard the small quantity of fluid enclosed by this sphere as practically *incompressible*, and use the well-known solution by the method of successive images for two spheres in an incompressible fluid.

We know that the velocity potential due to such a system of two spheres in an incompressible fluid can be expressed in the form

$$U_a \phi + U_b \phi', \tag{5}$$

where $\phi$ and $\phi'$ are to be determined by the conditions

$$\nabla^2 \phi = 0, \nabla^2 \phi' = 0,$$

$$\frac{\partial \phi}{\partial r_1} = -\cos \theta_1, \text{ and } \frac{\partial \phi'}{\partial r_1} = 0, \text{ when } r_1 = a,$$

$$\frac{\partial \phi'}{\partial r_2} = -\cos \theta_2, \text{ and } \frac{\partial \phi}{\partial r_2} = 0, \text{ when } r_2 = b,$$

$r_1$ and $r_2$ being radii vectores measured from A and B.

When $\phi$ and $\phi'$ have been determined so as to

satisfy these conditions the equivalent vibrations on the surface of the imaginary enveloping sphere can be taken to be very approximately given by

$$-\left[U_a \frac{\partial\phi}{\partial r} + U_b \frac{\partial\phi'}{\partial r}\right]_{r=a+b} e^{ikct} \tag{6}$$

The functions $\phi$ and $\phi'$ as is well-known can be determined by the method of successive images and if the expressions for the velocity potential due to these images be all transferred to the co-ordinates $(r, \theta)$ referred to the centre C of the imaginary enveloping sphere, we easily obtain

$$2\phi = a^3\left[1 - \frac{b^3}{(a+b)^3} + \frac{b^3}{(a+2b)^3} - \frac{b^3}{(2a+2b)^3} + \frac{b^3}{(2a+3b)^3} - \cdots\right] \frac{P_1(\cos\theta)}{r^2}$$

$$+ 2a^3\left[b - \frac{b^3(a^2+ab-b^2)}{(a+b)^4} + \frac{b^3(2b^2-a^2)}{(a+2b)^4} - \frac{b^3(2a^2+ab-2b^2)}{(2a+2b)^4} + \frac{b^3(3b^2-2a^2)}{(2a+3b)^4} - \cdots\right] \frac{P_2(\cos\theta)}{r^3}$$

$$+ 3a^3\left[b^2 - \frac{b^3(a^2+ab-b^2)^2}{(a+b)^5} + \frac{b^3(2b^2-a^2)^2}{(a+2b)^5} - \frac{b^3(2a^2+ab-2b^2)^2}{(2a+2b)^5} + \frac{b^3(3b^2-2a^2)^2}{(2a+3b)^5} - \cdots\right] \frac{P_3(\cos\theta)}{r^4}$$

$$+ 4a^3\left[b^3 - \frac{b^3(a^2+ab-b^2)^3}{(a+b)^6} + \frac{b^3(2b^2-a^2)^3}{(a+2b)^6} - \frac{b^3(2a^2+ab-2b^2)^3}{(2a+2b)^6} + \frac{b^3(3b^2-2a^2)^3}{(2a+3b)^6} - \cdots\right] \frac{P_4(\cos\theta)}{r^5}$$

$$+ \text{etc.,} \tag{7}$$

$$2\phi' = -b^3\left[1 - \frac{a^3}{(b+a)^3} + \frac{a^3}{(b+2a)^3} - \frac{a^3}{(2b+2a)^3} + \frac{a^3}{(2b+3a)^3} - \cdots\right] \frac{P_1(\cos\theta)}{r^2}$$

$$+ 2b^3\left[a - \frac{a^3(b^2+ab-a^2)}{(b+a)^4} + \frac{a^3(2a^2-b^2)}{(b+2a)^4} - \frac{a^3(2b^2+ab-2a^2)}{(2b+2a)^4} + \frac{a^3(3a^2-2b^2)}{(2b+3a)^4} - \cdots\right] \frac{P_2(\cos\theta)}{r^3}$$

$$-3b^3\left[a^2-\frac{a^3(b^2+ab-a^2)^2}{(b+a)^5}+\frac{a^3(2a^2-b^2)^2}{(b+2a)^5}-\frac{a^3(2b^2+ab-2a^2)^2}{(2b+2a)^5}+\frac{a^3(3a^2-2b^2)^2}{(2b+3a)^5}-\ldots\right]\frac{P_3'(\cos\theta)}{r^4}$$

$$+4b^3\left[a^3-\frac{a^3(b^2+ab-a^2)^3}{(b+a)^6}+\frac{a^3(2a^2-b^2)^3}{(b+2a)^6}-\frac{a^3(2b^2+ab-2a^2)^3}{(2b+2a)^6}+\frac{a^3(3a^2-2b^2)^3}{(2b+3a)^6}-\ldots\right]\frac{P_4(\cos\theta)}{r^5}$$

$$-\text{etc.,} \tag{8}$$

the law of formation of the series within the brackets being obvious.

Coming now to the present problem of two unequal spheres of the same material, let us take

$$a=2 \text{ inches and } b=1 \text{ inch.}$$

Since the change of velocities of the two balls is inversely proportional to their masses, we must have

$$U_b=8U_a$$

Substituting the values for $a$ and $b$ we easily find that

$$2\phi=2^3\left[\left(1+\frac{1}{4^3}+\frac{1}{7^3}+\frac{1}{10^3}+\ldots\right)-\left(\frac{1}{3^3}+\frac{1}{6^3}+\frac{1}{9^3}+\ldots\right)\right]\frac{P_1(\cos\theta)}{r^2}$$

$$+2.2^3\left[1-\left(\frac{5}{3^4}+\frac{8}{6^4}+\frac{11}{9^4}+\frac{14}{12^4}+\ldots\right)-\left(\frac{2}{4^4}+\frac{5}{7^4}+\frac{8}{10^4}+\ldots\right)\right]\frac{P_2(\cos\theta)}{r^3}$$

$$+3.2^3\left[1+\frac{2^2}{4^5}+\frac{5^2}{7^5}+\frac{8^2}{10^5}+\ldots\right)-\left(\frac{5^2}{3^5}+\frac{8^2}{6^5}+\frac{11^2}{9^5}+\ldots\right)\right]\frac{P_3(\cos\theta)}{r^4}$$

$$+4.2^3\left[1-\left(\frac{5^3}{3^6}+\frac{8^3}{6^6}+\frac{11^3}{9^6}+\frac{14^3}{12^6}+\ldots\right)-\left(\frac{2^3}{4^6}+\frac{5^3}{7^6}+\frac{8^3}{10^6}+\ldots\right)\right]\frac{P_4(\cos\theta)}{r^5}$$

$$+\text{etc.,} \tag{9}$$

$$2\phi'=-2^3\left[\left(\frac{1}{2^3}+\frac{1}{5^3}+\frac{1}{8^3}+\frac{1}{11^3}+\ldots\right)-\left(\frac{1}{3^3}+\frac{1}{6^3}+\frac{1}{9^3}+\ldots\right)\right]\frac{P_1(\cos\theta)}{r^2}$$

$$+2.2^3\left[\left(\frac{1}{3^4}+\frac{4}{6^4}+\frac{7}{9^4}+\cdots\right)+\left(\frac{4}{2^4}+\frac{7}{5^4}+\frac{10}{8^4}+\frac{13}{11^4}+\cdots\right)\right]\frac{P_2(\cos\theta)}{r^3}$$

$$-3.2^3\left[\left(\frac{4^2}{2^5}+\frac{7^2}{5^5}+\frac{10^2}{8^5}+\frac{13^2}{11^5}+\cdots\right)-\left(\frac{1^2}{3^5}+\frac{4^2}{6^5}+\frac{7^2}{9^5}+\cdots\right)\right]\frac{P_3(\cos\theta)}{r^4}$$

$$+4.2^3\left[\left(\frac{1^3}{3^6}+\frac{4^3}{6^6}+\frac{7^3}{9^6}+\cdots\right)+\left(\frac{4^3}{2^6}+\frac{7^3}{5^6}+\frac{10^3}{8^6}+\frac{13^3}{11^6}+\cdots\right)\right]\frac{P_4(\cos\theta)}{r^5}$$

$$-\text{etc.} \qquad (10)$$

Summing the series we easily find that the prescribed vibration on the surface of the imaginary enveloping sphere

$$U_s\left[\frac{\partial\phi}{\partial r}+8\ \frac{\partial\phi'}{\partial r}\right]_{r=3\text{ ins.}} e^{ikct}$$

is proportional to

$$\left[\cdot 246\,P_1(\cos\theta)+3\cdot 180\,P_2(\cos\theta)-2\cdot 108\,P_3(\cos\theta)+ 3\cdot 5\,P_4(\cos\theta)+\text{etc.}\right]e^{ikct}.$$

We have seen that when the prescribed vibration on the surface of the imaginary sphere is

$$\Sigma\, A_n\, P_n(\cos\theta).\, e^{ikct},$$

the velocity potential of the wave-motion is

$$\psi=-\frac{(a+b)^2}{r}.\, e^{ik(ct-r+a+b)}\,\Sigma\,\frac{A_n\,P_n(\cos\theta)}{F_n(ik.\overline{a+b})}\,f_n(ikr).$$

Now when $r$ is large,

$$f_n(ikr)=1,$$

so that the factor on which the relative intensities in various directions depend is

$$\Sigma\, A_n\,\frac{P_n(\cos\theta)}{F_n(ik.\overline{a+b})}.$$

Thus if we put this quantity $=F+iG$, the intensity of the vibrations in various directions is measured by $F^2+G^2$.

The distribution of intensities in different directions round the spheres will be influenced to a considerable extent by the value of the wave-length chosen If we take $k\,(a+b) = 2$, the wave-length is $3\pi$ inches and if we take $k\,(a+b) = 3$, the wave-length is $2\pi$ inches. From the expression (1) for the wave-motion produced by a single sphere undergoing an instantaneous change of velocity, it is seen that the wave-length to be chosen is of the same order as the circumference of the sphere. From this, it appears that for a system of two spheres whose radii are 1 inch and 2 inches respectively, the wave-length to be chosen should be some value intermediate between $2\pi$ and $4\pi$, probably nearer $2\pi$ than $4\pi$; for, in the actual case of impact, the smaller ball which would undergo by far the greater change in velocity would probably influence the character of the motion to a greater extent than the larger sphere. At the same time it must not be forgotten that the analogy between the case of impact and the case of periodic motion cannot be pushed very far, in as much as the fluid motion due to impact is undoubtedly of different character in different directions, and not throughout the same as in the periodic case.

Now taking $k\,(a+b) = 2$, we find (neglecting a constant factor) that

$$F = \cdot 0992\ P_1\ (\cos\ \theta) + \cdot 2840\ P_2 (\cos\ \theta) - \cdot 03535\ P_3\ (\cos\ \theta) - \cdot 01461\ P_4\ (\cos\ \theta) + \text{etc.}$$

$$G = \cdot 0496\ P_1\ (\cos\ \theta) - \cdot 4040\ P_2\ (\cos\ \theta) - 01767\ P_3\ (\cos\ \theta) + \cdot 0315\ P_4\ (\cos\ \theta) + \text{etc.} \quad (11)$$

The values of F, G and $F^2+G^2$ for different directions have been calculated and are shown in the following table :—

TABLE I.

| Angles (in degrees.) | F × const. | G × const. | $(F^2+G^2)$ × (const.) |
|---|---|---|---|
| 0 | + 329,000 | − 338,000 | 223,144 |
| 10 | + 325,908 | − 326,525 | 212,552 |
| 20 | + 297,435 | − 273,545 | 162,738 |
| 30 | + 251,537 | − 214,337 | 109,300 |
| 40 | + 189,095 | − 114,659 | 48,946 |
| 50 | + 114,215 | − 24,167 | 13,572 |
| 60 | + 33,341 | + 74,407 | 6,565 |
| 70 | − 43,647 | + 155,306 | 25,961 |
| 80 | − 107,073 | + 205,212 | 53,574 |
| 90 | − 147,250 | + 213,625 | 67,405 |
| 100 | − 158,815 | + 178,920 | 57,322 |
| 110 | − 140,329 | + 106,238 | 30,836 |
| 120 | − 95,149 | + 8,675 | 9,106 |
| 130 | − 34,099 | − 99,267 | 10,957 |
| 140 | + 35,677 | − 202,159 | 42,100 |
| 150 | + 1,02,819 | − 289,237 | 94,130 |
| 160 | + 157,865 | − 353,605 | 150,280 |
| 170 | + 192,648 | − 392,229 | 190,913 |
| 180 | + 201,000 | − 404,000 | 203,617 |

Now taking $k(a+b)=3$, we find (neglecting a constant factor) that

$$F = \cdot105P_1(\cos\theta) + 1\cdot06P_2(\cos\theta) + \cdot016P_3(\cos\theta) - \cdot281P_4(\cos\theta) - \text{etc.}$$

$$G = -\cdot1225P_1(\cos\theta) + \cdot186P_3(\cos\theta) - \cdot024P_4(\cos\theta) - \text{etc.} \quad (12)$$

The values in different directions have been calculated for these expressions and are shown in the following table :—

TABLE II.

| Angles (in degrees.) | F × const. | G × const. | $(F^2+G^2)$ × (const.) |
|---|---|---|---|
| 0 | + 900,000 | + 38,000 | 811,444 |
| 10 | + 890,608 | + 28,804 | 794,265 |
| 20 | + 849,305 | − 2,390 | 720,805 |
| 30 | + 752,167 | − 45,754 | 567,620 |
| 40 | + 572,469 | − 90,446 | 335,284 |
| 50 | + 309,902 | − 123,998 | 111,476 |
| 60 | − 5,783 | − 135,346 | 18,261 |
| 70 | − 313,014 | − 118,446 | 111,893 |
| 80 | − 542,728 | − 73,554 | 300,178 |
| 90 | − 635,375 | − 9,000 | 403,306 |
| 100 | − 571,364 | + 60,786 | 329,762 |
| 110 | − 371,618 | + 118,638 | 154,545 |
| 120 | − 96,799 | + 149,218 | 31,610 |

| Angles (in degrees.) | F × const. | G × const. | $(F^2+G^2)\times$ (const.) |
|---|---|---|---|
| 130 | + 184,472 | + 144,494 | 54592, |
| 140 | + 412,409 | + 105,758 | 180,980 |
| 150 | + 559,907 | + 44,650 | 315,625 |
| 160 | + 630,625 | − 21,410 | 398,607 |
| 170 | + 654,606 | − 69,748 | 433,925 |
| 180 | + 658,000 | − 88,000 | 440,708 |

The values of $F^2+G^2$ shown in Tables I and II have been plotted in polar co-ordinates in figs. 3 and 4. It is seen that in both cases, the intensity in the direction of the larger ball is greater than in the direction of the smaller ball. The asymmetry is most marked when $k\,(a+b)$ has the larger value.

The intensity of the sound in different directions due to the impact of two spheres of wood, diameters 3 inches and $1\frac{1}{2}$ inches respectively, has been measured with the ballistic phonometer and is shown in Fig. 5.

It is seen that the curve is intermediate in form between those shown in fig. 3 and fig. 4, exactly as anticipated. The agreement between theory and experiment is thus very striking in this case.

### 3. *Two spheres of the same diameter but of different materials.*

We have seen in the preceding section that in the expressions F and G for two spheres of the same material but of unequal diameters, the terms contain-

ing the second order zonal harmonic $P_2$ (cos $\theta$) usually preponderate, and that the intensity diagram is accordingly a curve which consists of four loops. A different result is obtained in the case of two spheres of the same diameter but of markedly unequal densities. The zonal harmonic of the first order preponderates in this case and the intensity diagram is a curve consisting of only two loops. To obtain this result theoretically we have to proceed on exactly the same lines as in the preceding pages.

Taking $a=1$ inch and $b=1$ inch, we easily find from the expressions (7) and (8) that

$$
\begin{aligned}
2\phi = & \left[1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+\frac{1}{5^3}-\text{ etc.}\right]\frac{P_1(\cos\theta)}{r^2} \\
& +2\left[1-\frac{1}{2^4}+\frac{1}{3^4}-\frac{1}{4^4}+\frac{1}{5^4}-\text{ etc.}\right]\frac{P_2(\cos\theta)}{r^3} \\
& +3\left[1-\frac{1}{2^5}+\frac{1}{3^5}-\frac{1}{4^5}+\frac{1}{5^5}-\text{ etc.}\right]\frac{P_3(\cos\theta)}{r^4} \\
& +4\left[1-\frac{1}{2^6}+\frac{1}{3^6}-\frac{1}{4^6}+\frac{1}{5^6}-\text{ etc.}\right]\frac{P_4(\cos\theta)}{r^5} \\
& +\text{ etc.} \\
2\phi' = & -\left[1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+\frac{1}{5^3}-\text{ etc.}\right]\frac{P_1(\cos\theta)}{r^2} \\
& +2\left[1-\frac{1}{2^4}+\frac{1}{3^4}-\frac{1}{4^4}+\frac{1}{5^4}-\text{ etc.}\right]\frac{P_2(\cos\theta)}{r^3} \\
& -3\left[1-\frac{1}{2^5}+\frac{1}{3^5}-\frac{1}{4^5}+\frac{1}{5^5}-\text{ etc.}\right]\frac{P_3(\cos\theta)}{r^4} \\
& +4\left[1-\frac{1}{2^6}+\frac{1}{3^6}-\frac{1}{4^6}+\frac{1}{5^6}-\text{ etc.}\right]\frac{P_4(\cos\theta)}{r^5} \\
& -\text{ etc.}
\end{aligned}
\tag{13}
$$

Summing the series we easily find that the vibrations

$$\left[U_a\frac{\partial\phi}{\partial r}+U_b\frac{\partial\phi'}{\partial r}\right]_{r=2} e^{ikct}$$

on the surface of the enveloping sphere can be expressed in the form

$$\tfrac{1}{2}\Big[(U_a-U_b)\times{\cdot}2254\ P_1(\cos\theta)+(U_a+U_b)\times{\cdot}3550\ P_2(\cos\theta)$$
$$+(U_a-U_b)\times{\cdot}3645\ P_3(\cos\theta)+(U_a+U_b)\times{\cdot}3080\ P_4(\cos\theta)$$
$$+(U_a-U_b)\times{\cdot}2325\ P_5(\cos\theta)+\text{etc.}\Big]\,e^{ikct} \qquad (14)$$

If the ball of radius $b$ is four times heavier than the one of radius $a$, we have

$$U_a=4\ U_b.$$

So that the vibrations on the surface of the enveloping sphere is proportional to the expression

$$\Big[{\cdot}6762\ P_1(\cos\theta)+1{\cdot}7750\ P_2(\cos\theta)+1{\cdot}0935\ P_3(\cos\theta)$$
$$+1{\cdot}5400\ P_4(\cos\theta)+{\cdot}6975\ P_5(\cos\theta)+\text{etc.}\Big]\,e^{ikct}$$

Now taking $k(a+b)=1$, which will give a wave-length equal to the circumference of the enveloping sphere, we get

$$F=\ {\cdot}13524\ P_1(\cos\theta)-{\cdot}04987\ P_2(\cos\theta)-{\cdot}0074\ P_3(\cos\theta)$$
$$+{\cdot}0007\ P_4(\cos\theta)+\text{etc.}$$

$$G=-{\cdot}0676\ P_1(\cos\theta)-{\cdot}0798\ P_2(\cos\theta)+{\cdot}0047\ P_3(\cos\theta)$$
$$+{\cdot}0012\ P_4(\cos\theta)-\text{etc.}$$

The values of F and G, and of $F^2+G^2$ in different directions obtained from the preceding expressions are shown in Table III.

TABLE III.

| Angles (in degrees.) | F × const. | G × const. | $(F^2+G^2)\times$ (const.) |
|---|---|---|---|
| 0 | + 786,000 | − 1,415,000 | 2,620,021 |
| 10 | + 793,732 | − 1,374,897 | 2,521,061 |
| 20 | + 813,819 | − 1,256,037 | 2,240,132 |
| 30 | + 835,068 | − 1,068,615 | 1,839,986 |
| 40 | + 845,629 | − 826,059 | 1,397,992 |
| 50 | + 828,667 | − 549,652 | 989,741 |
| 60 | + 768,690 | − 262,257 | 660,005 |
| 70 | + 654,594 | + 7,901 | 427,780 |
| 80 | + 482,433 | + 237,049 | 288,493 |
| 90 | + 252,125 | + 403,500 | 225,913 |
| 100 | + 228,907 | + 495,515 | 297,466 |
| 110 | − 331,298 | + 509,107 | 368,642 |
| 120 | − 647,986 | + 454,821 | 626,929 |
| 130 | − 954,405 | + 347,884 | 1,031,220 |
| 140 | − 1,229,335 | + 211,923 | 1,555,385 |
| 150 | − 1,458,496 | + 71,667 | 2,130,948 |
| 160 | − 1,629,521 | − 47,667 | 2,655,850 |
| 170 | − 1,734,880 | − 128,811 | 3,026,866 |
| 180 | − 1,770,000 | − 157,000 | 3,157,549 |

The values of $(F^2 + G^2)$ shown in Table III have been plotted in polar co-ordinates and are shown in fig. 6. It is seen that the maximum intensity in the direction of the heavier ball is greater than that in the direction of the lighter one.

The experimental curve of intensity of sound due to impact of a sphere of wood, diameter 2¼ inch, with a billiard ball of nearly the same size is shown in fig. 7. It is found that the directions of minimum intensity are not quite in the plane perpendicular to the line of impact, they being nearer the side of the lighter ball.

A result of some importance indicated by theory is that when one of the spheres is much heavier than the other, replacing the former by a still heavier sphere of the same diameter should not result in any important alteration in the distribution of the intensity of sound in different directions due to impact. This is clear from expression (14). For when $U_a$ is much larger than $U_b$, any diminution in the value of $U_b$ should not appreciably affect the value of the expression. This indication of theory is in agreement with experiment. Several series of measurements have been made with various pairs of balls of the same size but of different densities, namely, wood and marble, wood and iron, billiard ball and iron ball and so forth. Generally similar results are obtained in all cases. It was noticed also that the form of the intensity distribution as shown by the ballistic phonometer was not altogether independent of the thickness of the mica-disk used in the instrument. This is not surprising, as the behaviour of the mica-disk, before the pointer attached

to it ceases to touch the mirror of the indicator, would no doubt depend to some extent on the relation between its natural frequency and the frequency of the sound waves set up by impact. The best results have been obtained with a disk neither so thick as to be relatively insensitive, nor so thin as to remain with its pointer in contact with the indicator longer than absolutely necessary.

### *4. The general case of spheres of any diameter and density.*

When the impinging spheres are both of different diameters and of different densities, the results generally obtained are that the sound is a maximum on the line of impact in either direction and a minimum which approaches zero in directions asymmetrically situated with reference thereto. Generally speaking no maxima in lateral derections are noticed, that is, the curve consists of two nearly closed loops. The difference of the intensity of the sound in the two directions of the line of impact may sometimes be considerable. As a typical case, the results obtained by the impact of a sphere of wood 3 inches in diameter with a brass sphere 1⅛ inches in diameter are shown in fig. 8. It is observed that the sound due to impact is actually of greater intensity on the side of the small brass ball. As a matter of fact the result generally obtained is that the intensity is greater on the side of the ball of the denser material, even if its diameter be smaller.

The mathematical treatment of the general case is precisely on the same lines as in the two preceding sections. It is found in agreement with the experimental result that in practically all cases in which both

the densities and the diameters are different, that the zonal harmonic of the first order is of importance and that the intensity curve consists of two nearly closed loops, as in the case of two spheres of the same diameter but of different densities.

### 5. *Summary and Conclusion.*

The investigation of the origin and character of the sound due to the direct impact of two similar solid spheres which was described in the Phil. Mag. for July, 1916, has been extended in the present paper to the cases in which the impinging spheres are not both of the same diameter or material. The relative intensities of the sound in different directions have been measured by the aid of the ballistic phonometer, and in order to exhibit the results in an effective manner, they have been plotted in polar co-ordinates, the point at which the spheres impinge being taken as the origin, and the line of collision as the axis of $x$. As might be expected, the curves thus drawn show marked asymmetry in respect of the plane perpendicular to the line of impact.

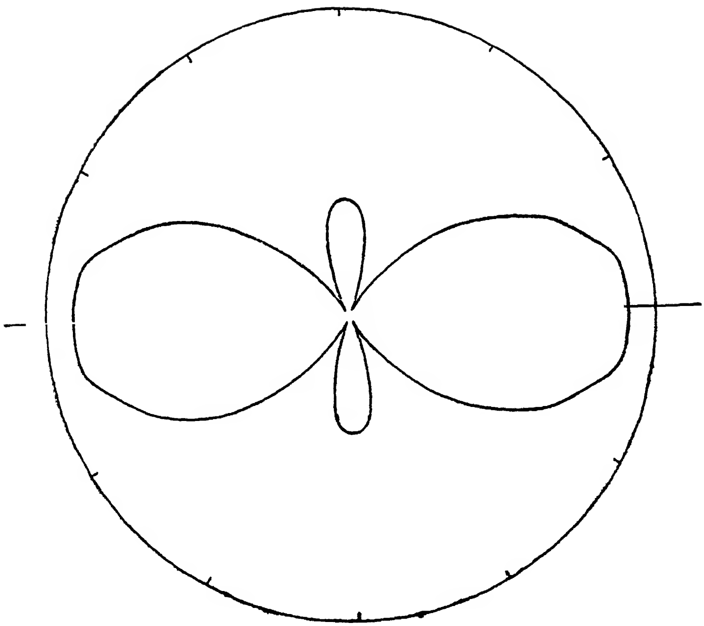
A detailed mathematical discussion of the nature of the results to be expected is possible by considering the analogous case of two rigid spheres nearly in contact which vibrate bodily along the line of centres. By choosing an appropriate wave-length for the resulting motion, intensity curves similar to those found experimentally for the case of impact are arrived at. A further confirmation is thus obtained of the hypothesis regarding the origin of the sound suggested by the work of Hertz and of Lord Rayleigh on the theory of elastic impact.

When the impinging spheres, though not equal in size, are of the same or nearly the same density, the intensity curve drawn for the plane of observation shows the sound to be a maximum along the line of impact in either direction, and also along two directions making equal acute angles with this line. The sound is a minimum along four directions in the plane.

In practically all other cases, that is when the spheres differ considerably either in density alone, or both in diameter and density, the intensity is found to be a maximum along the line of impact in either direction, and to be a minimum along directions which are nearly but not quite perpendicular to the line of impact. The form of the intensity curve is practically determined by the diameters and the masses of the spheres.

The investigation was carried out in the Physical Laboratory of the Indian Association for the Cultivation of science. It is hoped when a suitable opportunity arrives to study also the case of oblique impact. The writer has much pleasure in acknowledging the helpful interest taken by Prof. C. V. Raman in the progress of the work described in the present paper.

CALCUTTA,
*The 15th of June, 1917.*



**Fig, 1.**

Observed distribution of sound intensity around two equal colliding spheres of the same material.

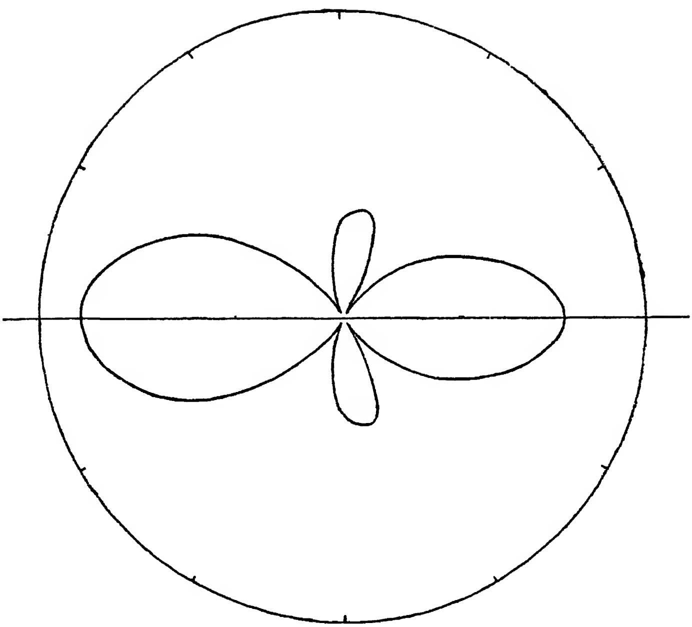


**Fig. 2.**

Observed distribution of intensity due to impact of two unequal spheres of wood.

Sphere on left; 3 inches diameter.

Sphere on right; $2\frac{1}{4}$ inches diameter.

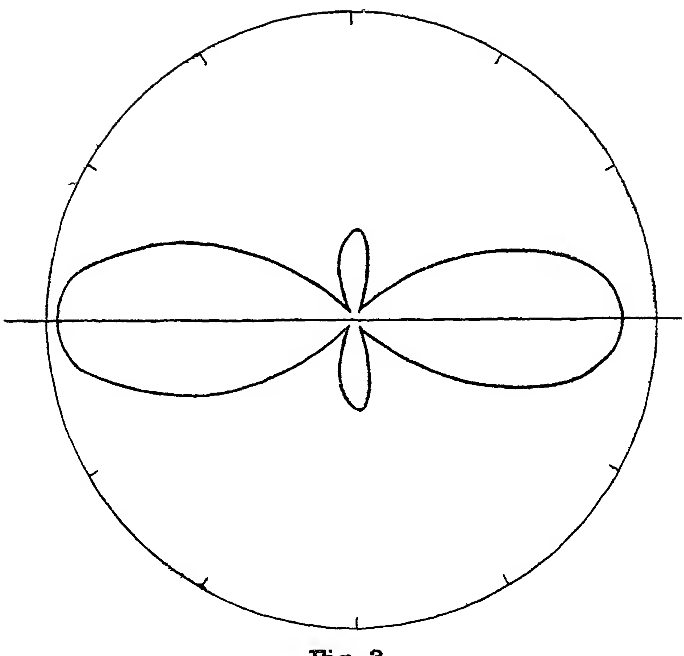


**Fig. 3.**

Calculated Form of Intensity Curve due to two Spheres of diameters 2 : 1. [$k\,(a+b)=2.$]

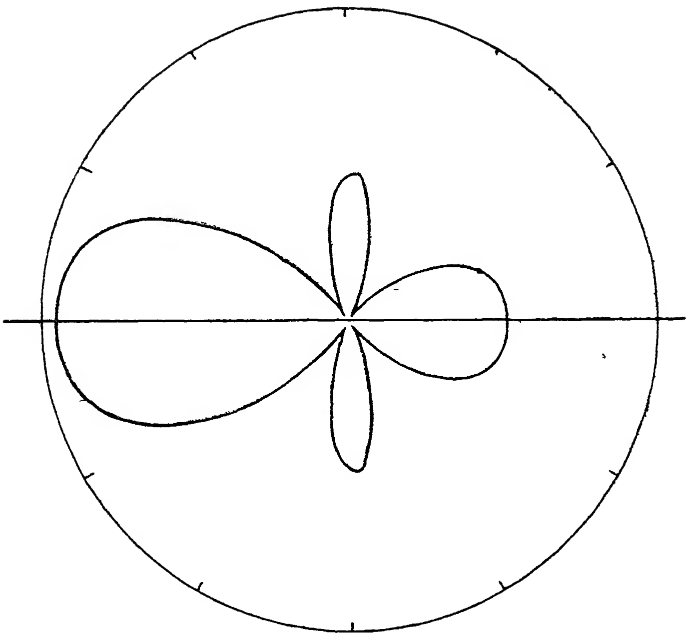


**Fig. 4.**

Same as fig. 3, but with $k\ (a+b)=3$.

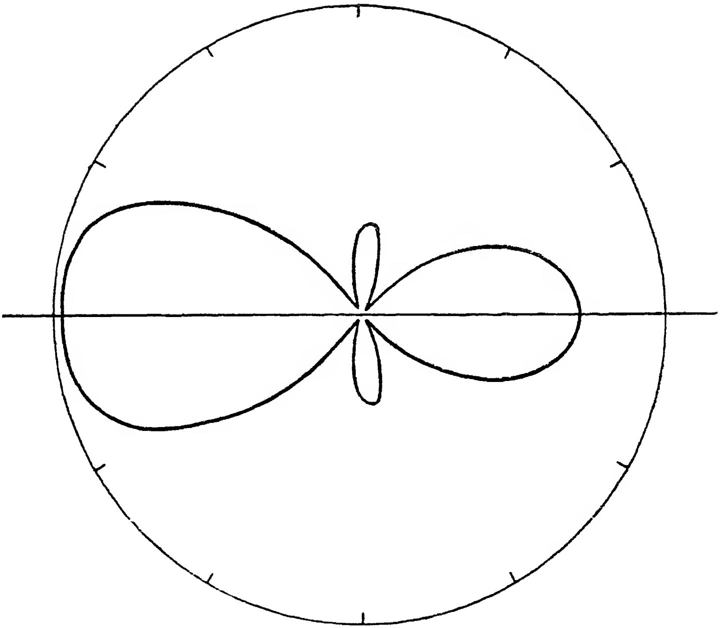


**Fig. 5.**

Observed Form of Intensity Curve due to impact of Spheres of diameters 2 : 1.

(Material, wood ; diameters 3 inches and $1\frac{1}{2}$ inches respectively).

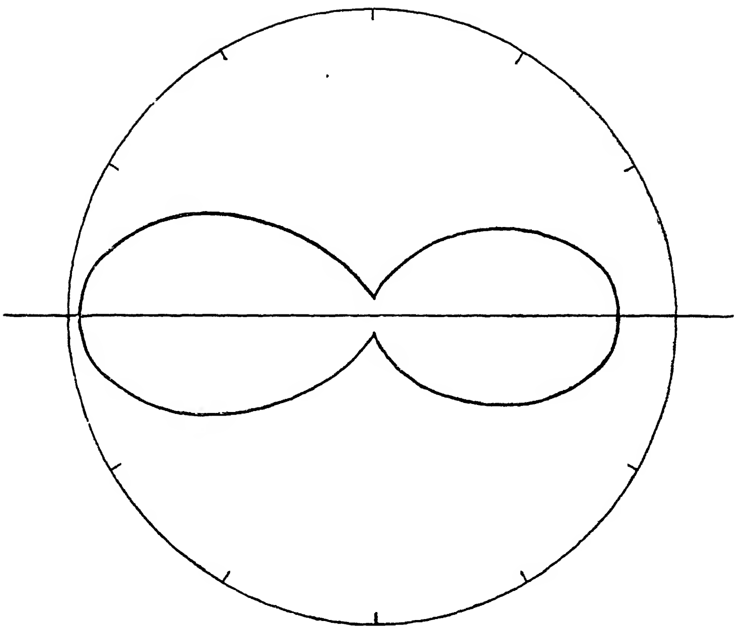


**Fig. 6.**

Calculated Form of Intensity Curve due to two equal Spheres of densities 4 : 1.

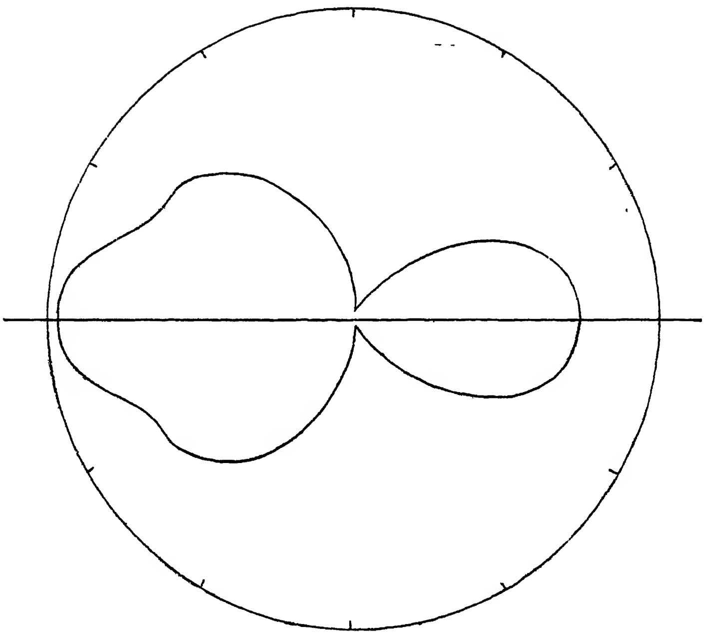


**Fig. 7.**

Observed Form of Intensity Curve due to impact of Spheres of nearly equal diameters but different materials

Sphere on left:
Material, billiard ball;
Diameter, $2\frac{1}{8}$ inches;
Mass, 150 gms.

Sphere on right:
Material, wood;
Diameter, $2\frac{1}{4}$ inches;
Mass, 66 gms.

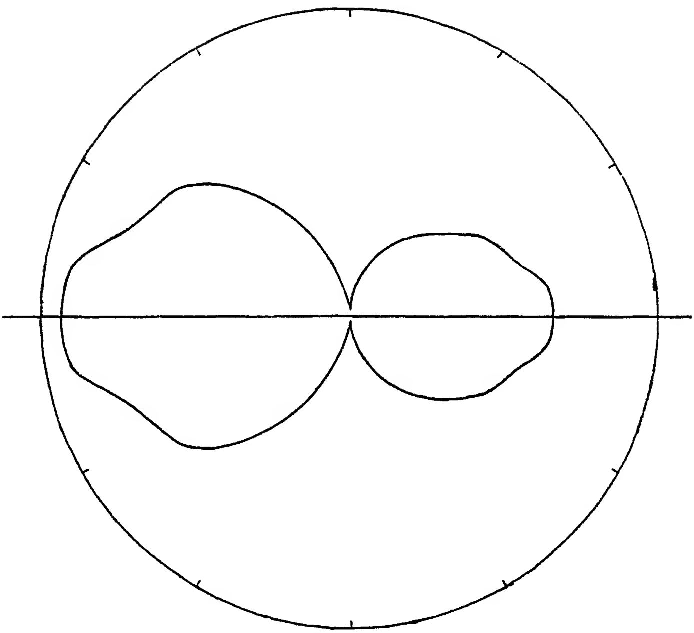


**Fig. 8.**

Observed Form of Intensity Curve due to impact of two Spheres of different diameters and densities.

| | |
|---|---|
| Sphere on left : | Sphere on right : |
| Material, brass ; | Material, wood ; |
| Diameter, $1\frac{1}{8}$ inches ; | Diameter, 3 inches ; |
| Mass, 118 gms. | Mass, 158 gms. |

PROCEEDINGS

OF THE

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. III. PART III.

# On the Diffraction of Light by Cylinders of large Radius.

BY NALINIMOHAN BASU, M.Sc.

## *Introduction.*

1. C. F. Brush has recently published a paper containing some interesting observations on the diffraction of light by the edge of a cylindrical obstacle*. Brush worked with cylinders of various radii, (the finer ones being screened on one side so as to confine diffraction to the other side only) and observing the fringes formed within a few millimetres of the diffracting edge through a microscope, found that they appeared brighter and sharper with every increase in the radius of the cylinder. The fringes obtained with a smooth rod of one or two centimetres radius differed very markedly from those formed by a sharp edge or by a cylinder of small radius. They were brighter, more numerous, showed greater contrast between the maxima and minima of illumination, and their spacing was different from that given

* "*Some Diffraction Phenomena: Superposed Fringes*" by C.F. Brush, *Proceedings of the American Philosophical Society*, 1913, pages 276-282. See also *Science Abstracts*, No. 1810, 1913.

by the usual Fresnel formulæ for diffraction by a straight edge. Brush also observed that when the radius of the cylinder was a millimetre or more, the fringes did not vanish when the focal plane of the microscope was put forward so as to coincide with the edge of the cylinder. Sharp narrow fringes were observed with the focal plane in this position, becoming broader and more numerous as the radius of the cylinder was increased.

2. To account for these phenomena Brush has suggested an explanation, the nature of which is indicated by the title of his paper. The diffraction-pattern formed by the cylinder is, according to Brush, the result of the superposition of a number of diffraction-patterns which are almost, but not quite, in register. He regards the cylindrical diffracting surface as consisting of a great many parallel elements, each of which acts as a diffracting edge and produces its own fringe-pattern which is superposed on those of the other elements. Brush has made no attempt to arrive mathematically or empirically, at any quantitative laws of the phenomena described in his paper. A careful examination of the subject shows that the view put forward by him presents serious difficulties and is open to objection. One of the defects of the treatment suggested by Brush is that it entirely ignores the part played by the light regularly reflected from the surface of the obstacle at oblique or nearly grazing incidences. I propose in the present paper (*a*) to describe the observed effects in some detail, drawing attention to some interesting features overlooked by Brush, (*b*) to show that they can be interpreted in a manner entirely

different from that suggested by him, and (*c*) to give a mathematical theory together with the results of a quantitative experimental test.

3. Reference should be made here to the problem of the diffraction of plane electro-magnetic waves by a cylinder with its axis parallel to the incident waves. The solution of this problem for a perfectly conducting cylinder has been given by J. J. Thomson*, and for a dielectric cylinder by Lord Rayleigh†. These solutions are however suitable for numerical computation only when the radius of the cylinder is comparable with the wave-length. A treatment of the problem in the case of a cylinder of any radius has recently been given by Debye‡. He considers the electromagnetic field round a perfectly reflecting cylinder, whose axis is taken for the axis of $z$, with polar co-odinates $r$, $\varphi$, and waves in the plane of $xy$ polarised in the direction of $z$, the electric component in $z$ being $e^{ikx}$. Expressing the disturbance-field in the form

$$Z = -\Sigma e^{\frac{in\pi}{2}} \cdot \frac{J_n(ka)}{H_n(ka)} \cdot H_n(kr) \cos n\varphi$$

(in which $J_n$ is the usual Bessel function, $H_n$ is Hankel's second cylindrical function and $k = \frac{2\pi}{\lambda}$), Debye

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* "*Recent Researches in Electricity and Magnetism*", p. 428.

† *Phil Mag.*, 1881. Scientific works, Vol. 1, p. 534.

‡ P. Debye, "*On the Electromagnetic field surrounding a Cylinder and the theory of the Rainbow*", *Phys. Zeitschr.* 9. pp. 775-778, Nov., 1908. Also, *Deutsch. Phys Gesell. Verh.* 10.20. pp. 741-749, Oct., 1908, and *Science Abstracts*, No. 258, 1909.

transforms the solution into the simple form

$$Z = -\sqrt{\frac{a\cos\frac{\varphi}{2}}{2r}} \cdot e^{-ik\left(r - 2a\cos\frac{\varphi}{2}\right)}.$$

Debye's work is of considerable significance but his final result is valid only for points at a great distance from the surface of the cylinder, whereas the phenomena considered in the present paper are those observed in its immediate neighbourhood. No complete mathematical treatment of the subject now dealt with appears to have been given so far.

*General Description of the Phenomena.*

4. The experimental arrangements adopted are those shewn in the diagram (fig. 1.).

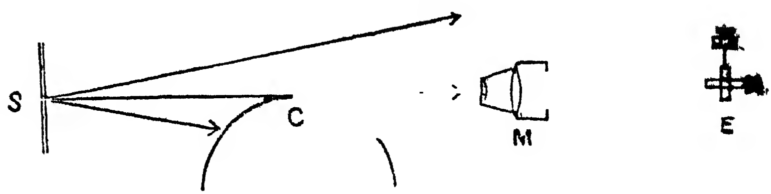



**Fig. I.**

Light from a slit S falls on a polished cylinder of metal or glass and passes it tangentially at C*. The axis of the cylinder is parallel to the slit. A collimating lens may, if necessary, be interposed between the slit and the cylinder. The fringes bordering the shadow of the edge C are observed through the miscroscope-objective M and the micrometer eyepiece E. The latter may be placed at any convenient distance from

* A glass cylinder may be used without inconvenience as the light transmitted through the cylinder is refracted out to one side and does not enter into the field under observation. Very little light is, in fact, transmitted through the cylinder at oblique incidences.

the objective so as to give the necessary magnification. The effects are best seen with monochromatic light which may be obtained by focussing the spectrum of the electric arc on the slit with a small direct-vision prism. For photographic work, the eyepiece E is replaced by a long light-tight box in front of which the objective is fixed, and at the other end of which the photographic plate is exposed. Sufficient illumination for photographing the fringes may be secured by using the arc and illuminating the slit by the greenish-yellow light transmitted by a mixture of solutions of copper sulphate and potassium bichromate.

5. The phenomena observed depend on the position of the focal plane of the objective with reference to the diffracting edge of the cylinder, and an interesting sequence of changes is observed as the focal plane of the objective is gradually moved towards the light, up to and beyond the edge C (fig. 1) at which the incident light grazes the cylinder. Some idea of these changes will be obtained on a reference to Plate I, figs. I to VIII, in which the fringes photographed with a cylinder of radius 1·54 cm. are reproduced. (A Zeiss objective of focal length 1·7 cm. was used and the magnification on the original negative was 135 diameters.)

6. To interpret the phenomena it is convenient to compare them with those obtained when the cylinder is replaced by a sharp diffracting edge in the same position. Using the cylinder it is found that, when the focal plane is beween the objective and the cylinder but several centimetres distant from the latter, the fringes are practically of the same type as those due

to a sharp edge. They are diffuse, few in number (not more than seven or eight being visible even in monochromatic light) and the first band is considerably broader and more luminous than the rest. The fringes become narrower and more numerous (retaining their characteristics) as the focal plane is brought nearer the cylinder till the distance between the two is about two centimetres. At this stage some new features appear; the contrast between the minima and maxima of illumination becomes greater than in the fringes of the usual Fresnel type, and the number that can be seen and counted in monochromatic light increases considerably. These features become more and more marked as the focal plane approaches the cylinder, and the dark bands become almost perfectly black. The difference between the intensity of the first maximum and of those following it also becomes less conspicuous. Figs. I to III represent these stages. A considerable brightening-up of the whole field is also noticed as the focal plane approaches the cylinder, but this is not shewn in the photographs as the exposures obtained with the light of the arc were very variable. When the focal plane is within a millimetre or two of the edge at which the incident light grazes the cylinder, a change in the law of spacing of the fringes also becomes evident, the widths of the successive bright bands decreasing less rapidly than in the fringes of the Fresnel type. Fig. IV illustrates this feature which is most marked when the focal plane coincides with the edge of the cylinder. At this stage, of course, the fringes due to a sharp diffracting edge would vanish altogether.

7. When the focal plane is gradually moved further in, so that it lies between the cylinder and the source of light, some very interesting effects are observed. The fringes contract a little and the first band instead of remaining in the fixed position defined by the geometrical edge, moves into the region of the shadow, and is followed by a new system of fringes, characterised by intensely dark minima, that appears to emerge from the field occupied by the fringes seen in the previous stages. (See Plate I, figs. V and VI.). The first band of this new system is considerably more brilliant than those that follow it. It is evident on careful inspection that the fringes that move into the shadow form an independent system. For it is found that the part of the field from which the new system has separated out appears greatly reduced in intensity in comparison with the part on which it is still superimposed. When the separation of the field into two parts is complete, a few diffraction-fringes of the usual Fresnel type are observed at the geometrical edge of the shadow of the cylinder. (See Plate I, figs. VII and VIII in which this position is indicated by an arrow).

8. A comparison of the effects described in the preceding paragraph and of those obtained with a sharp diffracting edge in the same position furnishes the clue to the correct explanation of all the phenomena observed and dealt with in the present paper. With a sharp edge the fringes of the Fresnel type disappear when the focal plane coincides with it, and reappear without alteration of type when the focal plane is between the edge and the source of light. As already

remarked and shown in Plate I, figs. VII and VIII, fringes of this type may also be observed with the cylinder when the focal plane is in this position, and in addition we have inside the shadow an entirely separate system of fringes characterised by perfectly black minima and a series of maxima with intensities converging to zero. This latter system has nothing in common with the diffraction phenomena of the Fresnel class and has obviously an entirely different origin. That it is formed exclusively by the light reflected from the surface of the cylinder is proved by the fact that it may be cut off without affecting the rest of the field by screening the surface of the cylinder. It is accordingly clear that the light reflected from the surface of the cylinder plays a most important part in the explanation of the phenomena, and that the edge of the cylinder grazed by the incident rays alone acts as a diffracting edge in the usual way, and not all the elements of the surface as supposed by Brush. We shall accordingly proceed on this basis to consider the theory of the fringes observed in various positions of the focal plane.

*Theory of the Fringes at the edge of the Cylinder.*

9. When the focal plane coincides with the edge at which the incident light grazes the cylinder, it is permissible to regard the fringes seen as formed by simple interference between the light that passes the cylinder unobstructed and the light that suffers reflection at the surface of the cylinder at various incidences. For if a sharp diffracting edge were put in the focal plane, no diffraction fringes would be

visible. The positions of the minima in the field may be readily calculated.

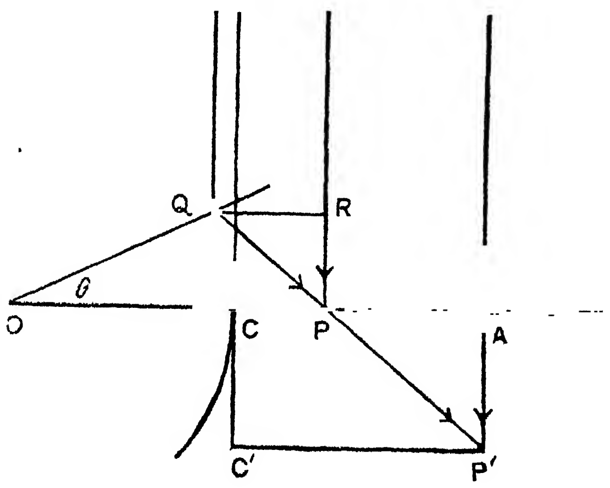



Fig. 2.

In fig. 2, let O be the centre of the cross-section of the cylinder in the plane of incidence, and let C be the point at which the incident light grazes the cylinder. It is sufficient for practical purposes to consider the incident beam as a parallel pencil of rays. The ray meeting the cylinder at the point Q is reflected in the direction QP. Let the angle $QOA=\theta$, so that the angle $OQP=\frac{\pi}{2}+\theta$, and the angle $OPQ=\frac{\pi}{2}-2\theta$. Let $a$ be the radius of the cylinder and $CP=x$. The difference of path, $\delta$, between the direct ray and the reflected ray reaching the point P is evidently equal to $QP-RP$ which can be easily shewn to be given by

$$\delta=a\sin\theta\,(\sec 2\theta-1).$$

Similarly we shall have

$$x=a\sec 2\theta\,(\cos\theta-\cos 2\theta).$$

Therefore, neglecting 4th and higher powers of $\theta$, we have

$$\delta=2a\theta^3, \text{ and } x=3a\theta^2/2,$$

so that $\delta=2a\left(\frac{2x}{3a}\right)^{\frac{3}{2}}$.

Since the rays suffer a phase-change of half a wave-length by reflexion, the edge C will form the centre of a dark band and the successive minima are therefore given by

$$x=\frac{3a}{2}\cdot\left(\frac{n\lambda}{2a}\right)^{\frac{2}{3}}=\tfrac{3}{4}\cdot(2a)^{\frac{1}{3}}\cdot(n\lambda)^{\frac{2}{3}},$$

where $n=1, 2, 3$, etc. The results calculated according to the above theory and those found in experiment are given in Table I.

TABLE I.

Widths of bright bands in cm.

$a=1\cdot54$ cm. $\lambda=6562\times10^{-8}$ cm.

| $n$ | Observed widths. | Calculated widths. |
|---|---|---|
| 1 | 0·00174 | 0·001775 |
| 2 | 0·00102 | 0·001019 |
| 3 | 0·00086 | 0·000875 |
| 4 | 0·00076 | 0·000781 |
| 5 | 0·00069 | 0·000717 |
| 6 | 0·00068 | 0·000671 |

The discrepancies are within the limits of experimental error. When making these measurements, the focal plane was in the first instance set in ap-

proximate coincidence with the edge of the cylinder by noting the stage at which a further movement of the focal plane towards the light results in a movement of the fringes into the region of the shadow. There was however a slight uncertainty in regard to this adjustment and the best position of the focal plane was finally ascertained by actual trial.

10. The ratio between the maxima and minima of illumination in the fringes at the edge of the cylinder may readily be calculated. Dividing up the pencil of rays incident on the cylinder into elements of width $a \sin \theta . d\theta$, or $a\theta . d\theta$ approximately, the width of the corresponding elements of the reflected pencil in the plane of the edge is, $dx$, that is, $3a\theta . d\theta$. The amplitude of the disturbance at any point in this plane due to the reflected rays is thus only $1/\sqrt{3}$ of that due to the direct rays, multiplied by the reflecting power of the surface. If the reflecting power be unity (as is practically the case at such oblique incidences), the ratio of the intensities of the maxima and minima is $(1 + 1/\sqrt{3})^2 : (1 - 1/\sqrt{3})^2$, that is approximately 14:1. The dark bands are thus nearly, but not quite, perfectly black.

### *Theory of the Fringes at the Edge of the Shadow.*

11. If the fringes be observed in a plane (such as C′P′ in fig. 2) which is farther from the source of light than the edge of the cylinder, the diffraction and mutual interference of the direct and the reflected rays have both to be taken into account. Since the reflected rays form a divergent pencil while the incident rays are parallel, the effect of the former at any

point sufficiently remote from the cylinder would be negligible in comparison with the effect of the latter. If $d$, the distance of the plane of observation from the edge of the cylinder, be sufficiently large, the problem thus reduces to one of simple diffraction of the incident waves by the straight edge C. The position of the minima of illumination with reference to the geometrical edge of the shadow would then be given approximately by the simple formula

$$x' = \sqrt{2nd\lambda} = \sqrt{d\lambda/2}\sqrt{4n}$$

where $x' = C'P'$, and $d = C\,C'$

or with great accuracy by Schuster's formula

$$x' = \sqrt{(8n-1)d\lambda/4} = \sqrt{d\lambda/2}\ \sqrt{(8n-1)/2}.$$

These two formulæ give results which do not differ materially except in regard to the first two or three bands as can be seen from Table II.

TABLE II.

| (1) $n$ | (2) $\sqrt{4n}$ | (3) $\sqrt{(8n-1)/2}$ | (4) Proportionate widths of bands as per column (2) | (5) Proportionate widths of bands as per column (3) |
|---|---|---|---|---|
| 1 | 2·000 | 1·871 | 2·000 | 1·871 |
| 2 | 2·828 | 2·739 | 0·828 | 0·868 |
| 3 | 3·464 | 3·391 | 0·636 | 0·652 |
| 4 | 4·000 | 3·937 | 0·536 | 0·546 |
| 5 | 4·472 | 4·416 | 0·472 | 0·479 |
| 6 | 4·899 | 4·848 | 0·427 | 0·432 |
| 7 | 5·292 | 5·244 | 0·393 | 0·396 |

12. If $d$ be not large, the intensity of the reflected rays is not negligible. The following considerations enable us to find a simple formula for the position of the minima of illumination which takes both diffraction and interference into account. We may, to begin with, find the positions of the minima assuming the case to be one of simple interference between the direct and the reflected rays. The expression for the path difference, $\delta'$, of the rays arriving at the point P′ is readily seen from fig. 2 to be given by the formulæ

$$\delta' = (d + a \sin\theta)(\sec 2\theta - 1),$$

$$\text{and} \quad x' = d \tan 2\theta + a (\cos\theta \sec 2\theta - 1).$$

These two relations may, to a close approximation, be written in the form

$$\delta' = 2d\theta^2 + 2a\theta^3,$$

$$\text{and} \quad x' = 2d\theta + 3a\theta^2/2.$$

Putting $d = o$, we get the formula already deduced (see paragraph 9 above) for the fringes at the edge C of the cylinder. On the other hand if $d$ be greater than $a$, we may, to a sufficient approximation, write

$$\delta' = 2d\theta^2 \quad \text{and} \quad x' = 2d\theta,$$

and the positions of the points at which the direct and the reflected rays are in opposite phases are given by the formula

$$x' = \sqrt{2nd\lambda}$$

13. But, as remarked above, the simple formula $x' = \sqrt{2nd\lambda}$ also gives the approximate positions of the minima in the diffraction-fringes at a considerable distance from the cylinder where the effect of the reflected rays is negligible. It is thus seen that the formulæ

$$\left.\begin{aligned} n\lambda &= 2d\theta^2 + 2a\theta^3, \\ \text{and} \quad x' &= 2d\theta + 3a\theta^2/2, \end{aligned}\right\} \quad \cdots \quad \cdots \quad \text{(A)}$$

suffice to give the approximate positions of the minima of illumination at the edge of the cylinder (at which point the fringes are due only to simple interference of the direct and the reflected rays) and also at a considerable distance from it (in which case they are due only to the diffraction of the incident light). *A priori*, therefore, it would seem probable that the formula would hold good also at intermediate points, that is for all values of $d$. That this is the result actually to be expected may be shewn by considering the effect due to the reflected rays at various points in the plane of observation. The reflected wave-front is the involute of the virtual caustic (see fig. 3). At the edge C, the radius of curvature of the wave-front is zero and increases rapidly as we move outwards from the edge of the cylinder. The reflected rays accordingly suffer the most rapid attenuation due to divergence in the direction of the incident rays and less rapid attenuation in other directions. In any plane C′P′, therefore, the effect of the reflected light is negligible in the immediate neighbourhood of the point C′, and would be most perceptible at points farthest removed from C′.* On the other hand, the fluctuations of intensity due to the diffraction of the direct rays are most marked in the neighbourhood of C′, that is, for the smallest values of $\theta$. We should accordingly expect to find that, when $d$ is not zero, the first few bands are practically identical in position with those due to simple diffraction, and those following are due to simple interference between the direct and

* Debye's formula *(loc. cit.)* shows that the intensity of the reflected light becomes very small as $\varphi$ approaches $\pi$.

the reflected rays. The formulæ given above satisfy both of these requirements. For it is obvious from the manner in which they have been deduced that they satisfy the second requirement. The first requirement is also satisfied as, by putting $\theta$ small, the formulæ reduce to $n\lambda=2d\theta^2$ and $x'=2d\theta$; in other words $x'=\sqrt{2nd\lambda}$ for the minima of illumination, which is also the usual approximate diffraction-formula. Accordingly the complete formulæ $n\lambda=2d\theta^2+2a\theta^3$ and $x'=2d\theta+3a\theta^2/2$ would (on eliminating $\theta$) give the positions of the minima over the entire field with considerable accuracy.

14. The statements made in the preceding paragraph are however subject to an important qualification. The validity of the formula obtained rests on the basis that, for large values of $d$, the positions of the minima of illumination are given by the simple formula $x'=\sqrt{2nd\lambda}$. This however is only an approximation, as the accurate values are to be found from Schuster's formula (see Table II) when the reflected light is negligible. When $d$ is so large that the formulæ $n\lambda=2d\theta^2+2a\theta^3$, and $x'=2d\theta+3a\theta^2/2$ give nearly the same positions for the minima as the simple relation $x'=\sqrt{2nd\lambda}$, they should therefore cease to be strictly valid. The actual positions of the minima for such values of $d$ should agree more closely with those given by Schuster's formulá and should, when $d$ is very large, agree absolutely with the same. This qualification is however of importance only with reference to the first two or three bands obtained for fairly large values of $d$. The differences in respect of the other bands would be negligibly small.

15. To test the foregoing results measurements were made of the widths of the bright bands for a series of values of $d$ up to 2 cm. Table III shows the observed values, the values calculated from my formula, and the values according to Schuster's formula (which would be valid for a sharp diffracting edge in the same position). To calculate the positions of the minima given by the relations $n\lambda = 2d\theta^2 + 2a\theta^3$, and $x' = 2d\theta + 3a\theta^2/2$, the first equation was solved for $\theta$ by Horner's method and the resulting values substituted in the second equation. The measurements of the width of the first band ware rather rough on account of the indefiniteness of its outer edge. The agreement between the observed widths and the widths calculated from my formulæ is seen to be fairly satisfactory for values of $d$ up to 3 mm. For larger values of $d$, the observed widths agree more closely with those calculated from Schuster's formula as explained in paragraphs 11 and 14 above.

## TABLE III.

Widths of bright bands in cm. $\times 10^{-5}$.

$a = 1{\cdot}54$ cm. $\lambda = 6562 \times 10^{-8}$ cm.

| $n$ | Observed values. | Calculated [Formula (A)] | Calculated [Schuster's formula.] | Observed values. | Calculated [Formula (A)] | Calculated [Schuster's formula.] | Observed values. | Calculated [Formula (A)] | Calculated [Schuster's formula.] |
|---|---|---|---|---|---|---|---|---|---|
| | $d = {\cdot}5$ mm. | | | $d = 1$ mm. | | | $d = 1{\cdot}5$ mm. | | |
| 1 | 277 | 294 | 239 | 397 | 385 | 339 | 471 | 458 | 415 |
| 2 | 133 | 140 | 111 | 173 | 170 | 157 | 198 | 199 | 193 |
| 3 | 107 | 111 | 83 | 134 | 134 | 118 | 154 | 155 | 145 |
| 4 | 95 | 98 | 70 | 119 | 116 | 99 | 133 | 133 | 121 |
| 5 | 86 | 89 | 61 | 100 | 103 | 87 | 114 | 119 | 106 |
| 6 | 80 | 81 | 55 | 93 | 97 | 78 | 106 | 110 | 96 |
| | $d = 2$ mm. | | | $d = 2{\cdot}5$ mm. | | | $d = 3$ mm. | | |
| 1 | 527 | 522 | 479 | 594 | 578 | 536 | 606 | 636 | 587 |
| 2 | 224 | 225 | 222 | 256 | 247 | 249 | 274 | 273 | 272 |
| 3 | 173 | 175 | 167 | 194 | 197 | 187 | 211 | 201 | 202 |
| 4 | 141 | 146 | 140 | 161 | 160 | 156 | 179 | 178 | 174 |
| 5 | 131 | 133 | 123 | 138 | 145 | 137 | 153 | 154 | 150 |
| 6 | 108 | 121 | 115 | 129 | 129 | 124 | 141 | 141 | 136 |
| | $d = 5$ mm. | | | $d = 10$ mm. | | | $d = 20$ mm. | | |
| 2 | 349 | 340 | 351 | 486 | 477 | 497 | 699 | 672 | 703 |
| 3 | 267 | 263 | 264 | 375 | 366 | 373 | 514 | 516 | 528 |
| 4 | 223 | 223 | 221 | 315 | 308 | 313 | 442 | 435 | 442 |

### *Theory of the Fringes between the Edge and the Source of Light.*

16. As already remarked in paragraph 7, the direct and the reflected pencils tend to separate into distinct parts of the field when the focal plane of the observing microscope is put forward so as to lie between the edge of the cylinder and the source of light. Why this is so will be readily understood on a reference to fig. 3. The rays reflected from the surface when

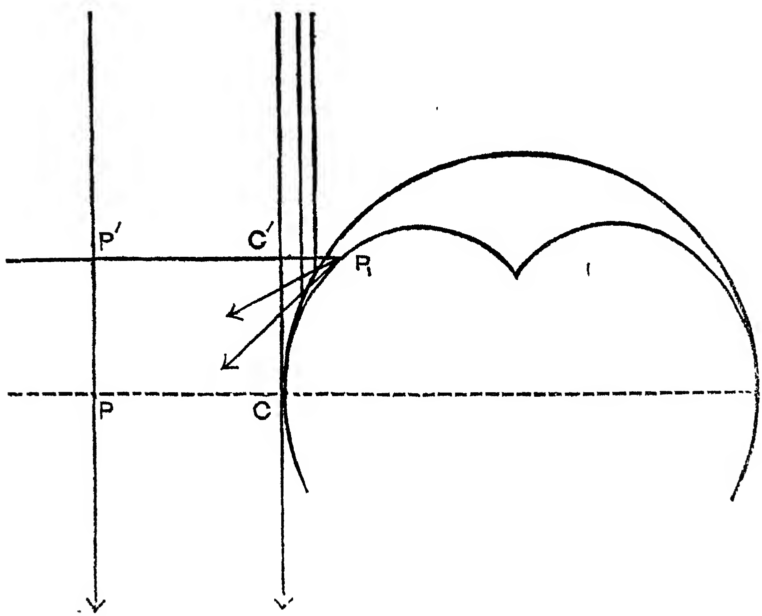



**Fig. 3.**

produced backwards would touch the enveloping surface which lies within the cylinder. This surface which is virtually the caustic of the reflected rays terminates at the edge C of the cylinder, and when the focal plane of the observing microscope is moved forward from CP to a position $P'C'P_1$ in front of the

edge, the boundary of the field on the right-hand side would shift into the region of the shadow and would, in fact, lie on the surface of the caustic at the point $P_1$. If the plane $P'C'P_1$ is considerably forward of PC, the field is seen divided into two parts. The first part $P'C'$ consists of the direct rays alone (the reflected rays meeting $P'C'$ being too oblique to enter into the field of the observing microscope) and should obviously be bounded at $C'$ by a few diffraction-fringes of the ordinary Fresnel type. The second part of the field $P_1C'$ is due to the reflected rays alone and requires separate consideration.

17. In the case considered above, that is, when the focal plane is considerably in advance of the edge, the fringe-system within the shadow due to the reflected light is of the same type as that found by Airy in his well-known investigation on the intensity of light in the neighbourhood of a caustic. For the elementary pencils into which the reflected rays may be divided up diverge from points lying along the caustic, and if the point $P_1$ at which the focal plane intersects the caustic is sufficiently removed from the edge C at which the latter terminates, Airy's investigation becomes fully applicable, but not otherwise. The rays emerging from the point $P_1$ after passage through the objective of the microscope becomes a parallel pencil, while pencils emerging from points on either side of $P_1$ become convergent and divergent respectively. The reflected wave-front after passage through the objective has thus a point of inflexion, on either side of which it may be taken to extend indefinitely provided the arc $CP_1$ be long

enough. Assuming the focal length to be $f$, and the equation of the wave front to be $\xi = A\eta^3$, the value of $A$ may be readily found. The equation of the caustic is

$$(4x^2+4y^2-a^2)^3-27a^4x^2=0.$$

From this, or directly by an approximate treatment, it may easily be shewn that the radius of curvature of the caustic at the point C is 3/4 of the radius of the cylinder. For our present purpose it is thus sufficient to treat the caustic as equivalent to a cylinder of radius $3a/4$ touching the given cylinder at C. We have

$$A=\tfrac{1}{6}\,\frac{d^3\xi}{d\eta^3}=\tfrac{1}{6}\,\frac{d}{d\eta}\left(\frac{d^2\xi}{d\eta^2}\right),$$

where $\frac{d^2\xi}{d\eta^2}$ is the measure of the convergence or divergence of the normals to the wave-front in the neighbourhood of the point of inflexion. Substituting the values obtained from the formulæ of geometrical optics, it is found that

$$A=\tfrac{1}{6}.\,\frac{3a}{4}.\,\frac{1}{f^3}.=\frac{a}{8f^3}.$$

The equation to the wave-front accordingly is

$$\xi=\frac{a}{8f^3}.\,\eta^3.$$

The illumination in the fringe-system is then given by Airy's formula

$$\mathrm{I}=4\left[\int_0^\infty \cos\tfrac{\pi}{2}\,(\omega^3+m\omega)\,d\omega\right]^2,$$

where $m=4.\,2^{\frac{1}{3}}.\,a^{-\frac{1}{3}}.\,\lambda^{-\frac{2}{3}}.\,x_1$,

$x_1$ being the distance of any point in the focal plane measured from the point of intersection with the caustic. The integral gives a series of maxima of which the

first is the most intense, and the rest gradually converge to zero. The minima of illumination are zeroes.* As the focal plane is moved further and further towards the source of light, the fringe-system moves inwards along the caustic but remains otherwise unaltered.

18. The foregoing treatment of the reflected fringe-system in terms of Airy's theory ceases to be valid when the focal plane is not sufficiently in advance of the edge, and the arc $CP_1$ of the caustic is therefore not large enough. For the reflected wave-front on one side of the point of inflexion then becomes limited in extent and its equation cannot with sufficient accuracy be assumed to be of the simple from $\xi = A\eta^3$. In fact when the focal plane is at the edge of the cylinder, and $CP_1$ is zero, the point of inflexion on the reflected wave-front coincides with the edge, and is its extreme limit. At this stage, of course, the fringes seen in the field are due only to the interference of the direct and the reflected wave-trains. The phenomena noticed as the focal plane is advanced towards the source of light, represent a gradual transition from this stage to one in which Airy's theory becomes fully applicable. In the transition-stages the field of illumination is a continuous whole of which however the different parts present distinct characteristics. First, within the geometrical shadow of the edge we have a finite number of fringes (one, two or more according to the position of the plane of observation, but not an

*Graphs of Airy's integral and references to the literature will be found in an interesting paper by Aichi and Tanakadate (*Journal of the College of Science*, Tokyo, Vol. XXI, Art. 3.)

indefinitely large number as contemplated by Airy's theory); these may be regarded as the interference-fringes in the neighbourhood of the caustic due to the reflected light alone. Following these we have a long train of fringes due to the interference of the direct and the reflected pencils. The first few of these should evidently be modified by the diffraction which the direct rays suffer at the edge C before they reach the observing microscope. Finally, we may also have a part of the field in which the illumination is due only to the direct pencil, the reflected rays not entering the objective of the microscope owing to their obliquity. This part of the field should appear less brightly illuminated than the rest.

19. A complete theoretical treatment of the transition-stages described in the preceding paragraph is somewhat difficult, and has to be deferred to some future occasion. There is no difficulty, however, in calculating the positions of the fringes due to the interference of the direct and the reflected pencils when the focal plane is in advance of the edge, provided the diffraction-effect due to the latter is neglected. It is easily shown that the path difference between the direct and reflected rays at a point P′ is given by

$$\left.\begin{aligned} \delta &= -2d\theta^2+2a\theta^3, \\ x' &= -2d\theta+3a\theta^2/2, \end{aligned}\right\} \qquad \text{(B)}$$

$x'$ being equal to C′P′, and $d =$ CC′. By putting $\delta = n\lambda$ and eliminating $\theta$, the positions of the minima of illumination may be calculated. A complete agreement of the results thus obtained with those found in experiment cannot however be expected as the fringes are narrow and the modifications due to

diffraction are not negligible. As regards the fringes alongside the caustic due to the reflected rays, we should expect to find an appreciable disagreement between these widths and those found from Airy's theory, so long as the latter is not fully applicable. The divergence should be most marked when the region of the caustic under observation is nearest the edge of the cylinder and for the fringes which are farthest from the caustic.

20. The foregoing conclusions have been tested by a series of measurements made with the focal plane in various positions in advance of the edge. To prove that the boundary of the field within the shadow is the caustic and not the surface of the cylinder, measurements were made of the length $C'P_1$, the rays incident on the cylinder being a parallel pencil. The results are given below.

| $d$ in cm. | Observed value of $C'P_1$ in cm. | Calculated value of $C'P_1$ in cm. |
|---|---|---|
| 0.1 | 0.00454 | 0.00433 |
| 0.13 | 0.00750 | 0.00733 |

The following shows the widths of the fringes in the neighbourhood of the caustic when the focal plane was 1·6 mm. in advance of the edge, and those calculated from Airy's theory.

| | | | | | | |
|---|---|---|---|---|---|---|
| Observed widths in cm. $\times 10^{-5}$ | 159, | 69, | 56, | 51, | 45, | 43 |
| Calculated from Airy's formula | 155, | 70, | 57, | 50, | 46, | 43 |

The agreement in both cases is satisfactory.

21. Table IV shows the results of measurements made of the fringes in the transition-stages when the focal plane was only a little in advance of the edge and

Airy's theory is inapplicable. The observed results are in general agreement with the indications of theory set out in paragraph 19. It will be seen that the fringes farthest in the region of the shadow show a fair agreement with Airy's theory, and the others are more nearly in agreement with the widths calculated from formula (B.)

TABLE IV.

Widths of bright bands in cm. $\times 10^{-6}$

$a = 1 \cdot 54$ cm. $\lambda = 6562 \times 10^{-8}$ cm.

| Observed widths. | Calculated [Airy's formula.] | Calculated [Formula (B.)] | Observed widths. | Calculated [Airy's formula.] | Calculated [Formula (B.)] |
|---|---|---|---|---|---|
| | $d = \cdot 2$ mm. | | | $d = \cdot 4$ mm. | |
| 1546 | 1550 | — | 1555 | 1550 | — |
| 852 | 696 | 894 | 777 | 696 | 754 |
| 722 | 570 | 777 | 628 | 570 | 674 |
| 672 | 504 | 705 | 590 | 504 | 636 |
| 622 | 461 | 637 | 566 | 461 | 594 |
| 590 | 429 | 602 | 500 | 429 | 543 |
| | $d = \cdot 6$ mm. | | | $d = \cdot 8$ mm. | |
| 1513 | 1550 | — | 1466 | 1550 | — |
| 700 | 696 | — | 706 | 696 | — |
| 626 | 570 | 586 | 563 | 570 | — |
| 536 | 504 | 549 | 503 | 504 | 508 |
| 493 | 461 | 520 | 475 | 461 | 483 |
| 446 | 429 | 497 | 448 | 429 | 463 |

*Summary and Conclusion.*

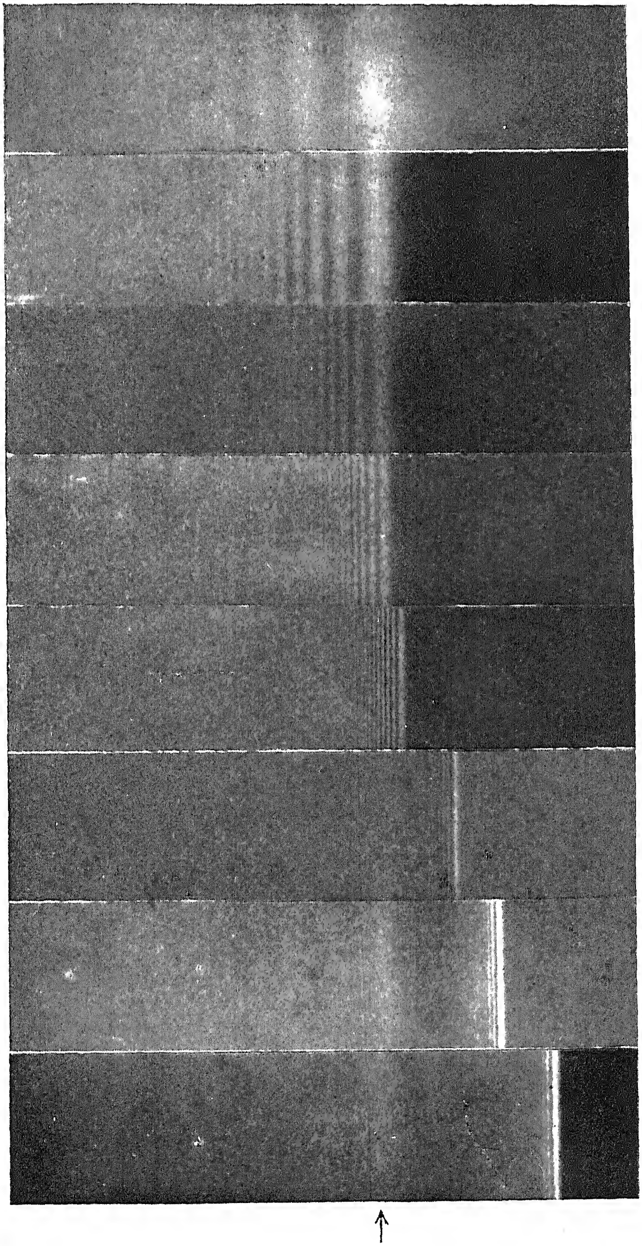
22. C. F. Brush has published some observations of considerable interest on the diffraction of light by cylindrical edges. The views put forward by him to explain the phenomena, however, present serious difficulties and are open to objection. My attention was drawn to this subject by Prof. C. V. Raman at whose suggestion the present work was undertaken by me in order to find the true explanation of the effects and to develop a mathematical theory which would stand quantitative test in experiment. This has now been done, and in the course of the investigation various features of importance overlooked by Brush have come to light. The following are the principal conclusions arrived at: *(a)* The fringes seen in the plane at which the incident light grazes the cylinder are due to the simple interference of the direct and reflected rays, the position of the dark bands being given by the formula $x=\frac{3}{4}\cdot(2a)^{\frac{1}{3}}\cdot(n\lambda)^{\frac{2}{3}}$; *(b)* the fringes in a plane further removed from the source of light than the cylinder are of the Fresnel class due to the edge grazed by the incident rays, but modified by interference with the light reflected from the surface behind the edge. The positions of the dark bands in these fringes are given by the formulæ* $x=2d\theta+3a\theta^2/2$, $n\lambda=2d\theta^2+2a\theta^3$, from which $\theta$ is to be eliminated; *(c)* when the focal plane of the observing microscope is on the side of the cylinder towards the light, the direct and the reflected rays do not both cover exactly the same part of the field, and by putting the focal

* This formula is subject to a small correction which is of importance only when $d$ is large.

plane sufficiently forward towards the light, they may be entirely separated. When this is the case, the fringes of the ordinary Fresnel type due to the edge of the cylinder may be observed, and inside the shadow we have also an entirely separate system of fringes due to the reflected rays, the first and principal maximum of which lies alongside the virtual caustic formed by oblique reflection; the distribution of intensity in this system can be found from the well-known integral due to Airy; *(d)* but when the focal plane is not sufficiently in front of the edge, the caustic and the reflecting surface are nearly in contact, and Airy's investigation of the intensity in the neighbourhood of a caustic requires modification. It is then found that only a finite number of bands (one, two, three or more according to the position of the plane of observation) are formed within the limits of the shadow and not an indefinitely large number as contemplated by Airy's theory. The rest of the fringes seen in the field are due to the interference of the direct and the reflected but modified to a certain extent by diffraction at the rays edge of the cylinder.

Indian Association for the Cultivation of Science.

CALCUTTA, 1917.




The Diffraction of Light by a Cylinder of radius 1·54 cm.

# PROCEEDINGS

OF THE

## INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. III. PART IV.


Friday, the 10th August, 1917 at 5 P.M. Rai Bahadur Dr. Chuni Lal Bose, I.S.O., Rasayanacharya, M.B., F.C.S.

## Disappearance of volumes by dissolution of substances in water.


BY JITENDRA NATH RAKSHIT.


The fact that the curves for specific gravities and percentages of substances in water are neither straight nor simple, suggested that there must be some regular systematic cause which creates this complication. To discover the exact truth it became useful to consider the disappearance of volumes by the dissolution of substances in water. In several cases the number of disappeared volumes in 100 parts of *mixture* have been calculated but these data are, however, not strictly applicable for this purpose. Consequently the disappearances of volumes when a fixed quantity of substance is dissolved in increasing quantities of water are calculated by the following method.

Let 100 gms. of pure substance of specific gravity S be mixed with water of Wg gms. weight and specific

gravity Ws to produce (100 + Wg) gms of mixture of specific gravity S′. Then,

$$\text{Vol. of 100 gms. of pure substance} = \frac{100}{S} \text{ c.c.}$$

$$\text{,, ,, Wg ,, ,, water} = \frac{Wg}{Ws} \text{ c.c.}$$

$$\text{and ,, ,, (100+Wg) ,, mixture} = \frac{100+Wg}{S'} \text{ c.c.}$$

$$\text{Therefore, disappearance of volume} = \frac{100}{S} + \frac{Wg}{Ws} - \frac{100+Wg}{S'} \text{ c.c.}$$

$$= K \text{ c.c. (say)}$$

When the gravities are measured at 4°C. compared with water at 4°C., $W_s = 1$. And when they are measured at t°C. compared with water at t°C., all the three above items are similarly influenced by the assumption of $W_s$ at t°C. = 1; in this case this disappearance of volume will be at t°C. But when the gravities are taken at t°C. and compared with water at 4°C. then $W_s$ = specific gravity of water at t°C. compared with water at 4°C., and the disappearance of volume found will be at t°C. Truly S in "$\frac{100}{S}$" is the specific gravity of the substance when it is in pure state before passing into solution, which is kept as such in cases of the gases or solids, "$\frac{100}{S}$" being a constant figure for particular substances.

For the construction of the following tables either specific gravity tables from "Landolt Bornstein—Physikalisch-chemische Tabellen" 3rd Edn. 1905, abbreviation for which is written as L, were used, or otherwise, as mentioned.

SULPHURIC ACID DOMKE. L, PAGE 326.

| P. C. W/W. | Wg. | 0/4 S′ | 0/4 K | 5/4 S′ | 5/4 K | 15/4 S′ | 15/4 K | 20/4 S′ | 20/4 K |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 9900 | 1·0075 | 29·44 | 1·0073 | 26·64 | 1·0061 | 23·86 | 1·0051 | 22·90 |
| 5 | 1900 | 1·0364 | 24·44 | 1·0355 | 22·74 | 1·0332 | 20·42 | 1·0317 | 19·45 |
| 10 | 900 | 1·0735 | 22·56 | 1·0718 | 21·16 | 1·0681 | 19·04 | 1·0661 | 18·23 |
| 20 | 400 | 1·1510 | 19·64 | 1·1481 | 18·67 | 1·1424 | 17·15 | 1·1394 | 16·52 |
| 30 | 233·33 | 1·2326 | 16·93 | 1·2291 | 16·31 | 1·2220 | 15·23 | 1·2185 | 14·82 |
| 40 | 150 | 1·3179 | 14·33 | 1·3141 | 13·93 | 1·3065 | 13·41 | 1·3028 | 13·01 |
| 50 | 100 | 1·4110 | 12·26 | 1·4070 | 12·03 | 1·3990 | 11·61 | 1.3951 | 11·45 |
| 60 | 66·67 | 1·5154 | 10·69 | 1·5111 | 10·55 | 1·5024 | 10·27 | 1·4982 | 10·18 |
| 70 | 40·28 | 1·6293 | 8·19 | 1·6245 | 8·10 | 1·6151 | 7·94 | 1·6105 | 7·88 |
| 80 | 25 | 1·7482 | 7·50 | 1·7429 | 7·45 | 1·7324 | 7·34 | 1·7272 | 7·30 |
| 90 | 11·11 | 1·8361 | 5·11 | 1·8306 | 4·59 | 1·8198 | 4·54 | 1·8144 | 4·52 |
| 100 | 0 | 1·8517 | 0 | 1·8463 | 0 | 1·8357 | 0 | 1·8305 | 0 |

## * NITRIC ACID. VELEY AND MANLEY. L. PAGE 325.

| 4/4 | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 0·625 | 15900·00 | 1·0035 | 20·66 |
| 1·323 | 7458·57 | 1·0076 | 21·86 |
| 2·630 | 3702·28 | 1·0154 | 22·60 |
| 3·42 | 2821·67 | 1·0199 | 21·86 |
| 6·11 | 1535·18 | 1·0356 | 21·07 |
| 10·75 | 830·23 | 1·0644 | 21·14 |
| 21·95 | 355·58 | 1·1365 | 19·57 |
| 38·10 | 162·46 | 1·2500 | 17·34 |
| 51·24 | 95·16 | 1·3376 | 14·11 |
| 79·59 | 25·64 | 1·4750 | 5·31 |
| 95·62 | 4·58 | 1·5219 | 0·71 |
| 99·97 | 0 | 1·5420 | 0 |

## * NITRIC ACID. LUNGE AND REY. L. PAGE 325.

| 15/4 | | | |
|---|---|---|---|
| 1·06 | 9333·96 | 1·0051 | 22·00 |
| 5·35 | 1769·16 | 1·0290 | 20·09 |
| 9·85 | 915·22 | 1·0554 | 19·88 |
| 18·16 | 450·66 | 1·1065 | 19·17 |
| 31·68 | 215·65 | 1·1953 | 17·54 |
| 60·37 | 65·64 | 1·3754 | 11·08 |
| 99·97 | 0 | 1·5204 | 0 |

* These two sp. gr. tables of nitric acid do not agree, apparently the first one seems to be wrong being itself irregular.

STANNIC CHLORIDE. GERLACH. L. PAGE 337.

| 15/15 | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 10 | 900 | 1·082 | 20·55 |
| 20 | 400 | 1·174 | 18·87 |
| 30 | 233·33 | 1·279 | 17·48 |
| 40 | 150 | 1·404 | 16·70 |
| 50 | 100 | 1·556 | 16·23 |
| 70 | 40·28 | 1·973 | 13·95 |
| 100 | 0 | 2·234 | 0 |

FORMIC ACID. RICHARDSON. L. PAGE 355.

| 20/4 | | | |
|---|---|---|---|
| 1 | 9900·00 | 1·0020 | 19·36 |
| 4 | 2350·00 | 1·0094 | 7·87 |
| 10 | 900·00 | 1·0247 | 7·58 |
| 30 | 233·33 | 1·0730 | 4·97 |
| 50 | 100·00 | 1·1208 | 3·62 |
| 70 | 40·28 | 1·1656 | 1·88 |
| 80 | 25·00 | 1·1861 | 1·53 |
| 90 | 11·11 | 1·2045 | 0·77 |
| 100 | 0 | 1·2213 | 0 |

ACETIC ACID. OUDEMANS. L. PAGE 354.

| P. C. W/W. | Wg. | *5/4 | | 15/4 | | 20/4 | |
|---|---|---|---|---|---|---|---|
| | | S' | K | S' | K | S' | K |
| 1 | 9900·00 | 1·0017 | 11·74 | 1·0007 | 10.52 | 0·9997 | 9·78 |
| 5 | 1900·00 | 1·0083 | 11·23 | 1·0067 | 9·76 | 1·0055 | 9·56 |
| 10 | 900·00 | 1·0167 | 11·19 | 1·0142 | 9·58 | 1·0126 | 9·30 |
| 20 | 400 00 | 1·0325 | 10·50 | 1·0284 | 8·93 | 1·0261 | 8·69 |
| 30 | 233·33 | 1·0467 | 9·65 | 1·0412 | 8·15 | 1·0383 | 7·97 |
| 40 | 150·00 | 1·0590 | 8·69 | 1·0523 | 7·32 | 1·0488 | 7·15 |
| 50 | 100·00 | 1·0692 | 7·71 | 1 0615 | 6·44 | 1 0575 | 6·32 |
| 60 | 66·67 | 1·0771 | 6·69 | 1·0685 | 5 50 | 1·0642 | 5·43 |
| 70 | 40·28 | 1·0825 | 5·45 | 1·0733 | 4·39 | 1·0686 | 4 34 |
| 80 | 25·00 | 1·0847 | 4·52 | 1·0748 | 3·48 | 1·0699 | 3·47 |
| 90 | 11·11 | 1·0819 | 3·18 | 1·0713 | 2·17 | 1 0660 | 2·16 |
| 100 | 0 | *Solid | 0 | 1·0553 | 0 | 1·0497 | 0 |

* S in this case is assumed to be 1.0553 which is only true at 15/4.

METHYL ALCOHOL. DITTMAR AND FAWSITT. L. PAGE 357.

| 0/4 | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S′ | K |
| 1 | 9900·00 | 0·9981 | 5·38 |
| 5 | 1900 00 | 0·9914 | 6·27 |
| 10 | 900·00 | 0·9843 | 7 56 |
| 20 | 400·00 | 0·9723 | 9·22 |
| 30 | 233·33 | 0 9606 | 9·75 |
| 40 | 150·00 | 0·9457 | 9·08 |
| 50 | 100 00 | 0·9287 | 8·08 |
| 60 | 66·67 | 0·9092 | 6·77 |
| 70 | 40·28 | 0·8869 | 5·53 |
| 80 | 25·00 | 0·8631 | 3·60 |
| 90 | 11·11 | 0·8375 | 1·87 |
| 95 | 5·62 | 0·8240 | 0·89 |
| 100 | 0 | 0·8102 | 0 |

ETHYL ALCOHOL. HEHNER. ANALYST. V.

| 15.5/15.5 | | | |
|---|---|---|---|
| 1 | 9900 00 | 0·9981 | 6·94 |
| 5 | 1900·00 | 0·9914 | 8·63 |
| 10 | 900·00 | 0·9841 | 9·82 |
| 20 | 400·00 | 0·9716 | 11·36 |
| 30 | 233·33 | 0·9578 | 11·29 |
| 40 | 150·00 | 0·9396 | 9·90 |
| 50 | 100·00 | 0·9182 | 8·16 |

ETHYL ALCOHOL. HEHNER. ANALYST. V. *ctd.*

| 15.5/15.5 | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 60 | 66·67 | 0·8956 | 6·55 |
| 70 | 40·28 | 0·8721 | 5·40 |
| 80 | 25·00 | 0·8483 | 3·62 |
| 90 | 11·11 | 0·8228 | 2·04 |
| 95 | 5·26 | 0·8089 | 1·11 |
| 100 | 0 | 0·7938 | 0 |

PROPYL ALCOHOL. PAGLIANI. L. PAGE 360.

| 0/4 | | | |
|---|---|---|---|
| 10·0 | 900·00 | 0·9878 | 9·84 |
| 18·2 | 443·95 | 0·9805 | 11·22 |
| 25·0 | 300·00 | 0·9707 | 10·06 |
| 35·7 | 180·11 | 0·9511 | 7·72 |
| 40·0 | 150·00 | 0·9425 | 6·85 |
| 62·5 | 60·00 | 0·8974 | 3·71 |
| 86·9 | 15·07 | 0·8502 | 1·83 |
| 100 | 0 | 0·8190 | 0 |

ISO-BUTYL ALCOHOL. TRAUBE. L. PAGE 360.

| 20/4 | | | |
|---|---|---|---|
| 2 | 4900·00 | 0·9949 | 7·64 |
| 4 | 2400·00 | 0·9922 | 9·15 |
| 6 | 1566·67 | 0·9895 | 9·63 |
| 8 | 1150·00 | 0·9869 | 10·65 |
| 100 | 0 | 0·8029 | 0 |

ISO-AMYL ALCOHOL. TRAUBE. L. PAGE 360.

| 20/4 | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 1 | 9900·00 | 0·9967 | 7·35 |
| 2 | 4900·00 | 0·9951 | 7·33 |
| 2·5 | 3900·00 | 0·9946 | 8·44 |
| 100 | 0 | 0 8121 | 0 |

GLYCERINE. NICOL. L. PAGE 366.

| 20/20 | | | |
|---|---|---|---|
| 5 | 1900·00 | 1·0118 | 2·47 |
| 10 | 900·00 | 8·0239 | 2·48 |
| 20 | 400·00 | 1·0488 | 2·41 |
| 25 | 300·00 | 1·0617 | 2·39 |
| 30 | 237·33 | 1·0747 | 2·31 |
| 40 | 150·00 | 1·1012 | 2·12 |
| 50 | 100·00 | 1·1283 | 1·89 |
| 60 | 66·67 | 1·1556 | 1·58 |
| 70 | 40·28 | 1·1829 | 0·83 |
| 80 | 25·00 | 1·2101 | 0·85 |
| 90 | 11·11 | 1·2372 | 0·44 |
| 100 | 0 | 1·2635 | 0 |

* GLYCERINE. LENZ.

| 12–14/12–14. | | | |
|---|---|---|---|
| P. C. W/W | Wg. | S'. | K. |
| 1 | 9900·00 | 1·0025 | 3·73 |
| 2 | 4900·00 | 1·0049 | 3·18 |
| 5 | 1900·00 | 1·0123 | 3·10 |
| 10 | 900·00 | 1·0245 | 2·71 |
| 20 | 400·00 | 1·0498 | 2·51 |
| 30 | 233·33 | 1·0771 | 2·65 |
| 40 | 150·00 | 1·1045 | 2·45 |
| 50 | 100·00 | 1·1320 | 2·12 |
| 60 | 66·67 | 1,1582 | 1·57 |
| 70 | 40·28 | 1·1889 | 1·17 |
| 80 | 25·00 | 1·2159 | 0·99 |
| 90 | 11·11 | 1·2425 | 0·48 |
| 100 | 0 | 1·2691 | 0 |

ACETONITRILE. TRAUBE. L. PAGE 363.

| 15/4. | | | |
|---|---|---|---|
| 2·73 | 3563·00 | 0·99570 | 14·13 |
| 5·25 | 1803·80 | 0·99289 | 14·60 |
| 8·22 | 1116·54 | 0·98865 | 13·78 |
| 12·51 | 699·36 | 0·98064 | 11·59 |
| 16·56 | 503·86 | 0·97427 | 11·25 |
| 100 | 0 | 0·7890 | 0 |

*Lewkowtisch Chemical Technology of Fats and Waxes, 3rd Edn., 1904, p. 1102.

ACETONE. SQUIBB. L. PAGE 366.

| 4/4. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S'. | K. |
| 20 | 400·00 | 0·9794 | 13·34 |
| 30 | 233·33 | 0·9660 | 12·15 |
| 40 | 150·00 | 0·9527 | 11·44 |
| 50 | 100·00 | 0·9334 | 9·58 |
| 60 | 66·67 | 0·9106 | 7·49 |
| 70 | 40·28 | 0·8877 | 6·11 |
| 80 | 25·00 | 0·8626 | 3·94 |
| 90 | 111·11 | 0·8371 | 2·23 |
| 100 | 0 | 0·8082 | 0 |

| 15/15. | | | |
|---|---|---|---|
| 20 | 400·00 | 0·9755 | 12·98 |
| 30 | 233·33 | 0·9604 | 11·79 |
| 40 | 150·00 | 0·9554 | 11·09 |
| 50 | 100·00 | 0·9247 | 9·25 |
| 60 | 66·67 | 0·9019 | 7·41 |
| 70 | 40·28 | 0·8790 | 6·22 |
| 80 | 25·00 | 0·8536 | 4·10 |
| 90 | 11·11 | 0·8260 | 2·13 |
| 95 | 5·26 | 0·8113 | 1·05 |
| 100 | 0 | 0·7966 | 0 |

M. ELROY.

| 15/4. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S | K |
| 10 | 900·00 | 0·9868 | 12·86 |
| 20 | 400 00 | 0·9744 | 12·65 |
| 30 | 233·33 | 0·9604 | 11·88 |
| 40 | 150·00 | 0·9449 | 10·98 |
| 100 | 0 | 0·7973 | 0 |

NICOTINE. PRIBRAM. L. PAGE 363.

| 20/4. | | | |
|---|---|---|---|
| 4·73 | 2014·16 | 1·0015 | 5·77 |
| 10·19 | 881·35 | 1·0054 | 5·90 |
| 20·75 | 381·93 | 1·0132 | 6·01 |
| 49·30 | 102·84 | 1·0329 | 5·67 |
| 62·81 | 59·21 | 1·0391 | 5·16 |
| 75·13 | 33·12 | 1·0394 | 4·15 |
| 86·89 | 15·09 | 1·0299 | 2·42 |
| 100 | 0 | 1·0095 | 0 |

AMMONIA. GRUNEBERG. L. PAGE 329.

| 15/15. | | | |
|---|---|---|---|
| 1·05 | 9423·81 | 0·995 | $\frac{100}{S} - 147{\cdot}85$ |
| 2·15 | 4551·11 | 0·990 | 146·92 |
| 3·30 | 2930·30 | 0·985 | 146·14 |

AMMONIA. GRUNEBERG. L. PAGE 329.

| 15/15. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S′ | K |
| 4·50 | 2122·22 | 9·980 | $\frac{100}{S}$ − 145·33 |
| 5·75 | 1639·13 | 0·975 | 144·46 |
| 12·60 | 693·65 | 0·950 | 141·78 |
| 19·80 | 405·05 | 0·925 | 140·95 |
| 27·70 | 261·01 | 0·900 | 140·11 |
| 33·40 | 199·40 | 0·885 | 138·90 |

AMMONIA. WACHSMUTH. L. PAGE 329.

| 15/15. | | | |
|---|---|---|---|
| 1·17 | 8447·00 | 0·995 | 142·95 |
| 2·26 | 4324·78 | 0·990 | 144·61 |
| 3·48 | 2772·19 | 0·98 | 143·76 |
| 4·71 | 2023·14 | 0·980 | 143·32 |
| 5·97 | 1575·04 | 0·975 | 142·90 |
| 12·80 | 681·25 | 0·950 | 141·11 |
| 20·26 | 393·58 | 0·925 | 140·01 |
| 28·34 | 258·86 | 0.900 | 139·20 |
| 33·64 | 197·26 | 0.885 | 138·62 |

AMONIA. LUNGE. L. PAGE 329.

| 15/14. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S'. | K |
| 1·14 | 8671·93 | 0·995 | $\frac{100}{S}$ – 136·34 |
| 2·31 | 4247·82 | 0·990 | 129·16 |
| 3·55 | 2716·90 | 0·985 | 140·46 |
| 4·80 | 1983·33 | 0·980 | 150·94 |
| 6·05 | 1552·06 | 0·975 | 140·99 |
| 20·18 | 395·54 | 0·925 | 139·86 |
| 30·03 | 233·00 | 0·895 | 138·87 |
| 35·6 | 180·90 | 0·880 | 138·14 |

AMMONIA. CARIUS. L. PAGE 329.

| 16/16. | | | |
|---|---|---|---|
| 1·08 | 9159·28 | 0·995 | 146·52 |
| 2·35 | 4155·32 | 0·990 | 142·98 |
| 3·55 | 2716·90 | 0·985 | 142·89 |
| 4·75 | 2005·26 | 0·980 | 142·96 |
| 6·00 | 1566·66 | 0·975 | 142·74 |
| 12·65 | 690·51 | 0·950 | 141·59 |
| 20·00 | 400·00 | 0·925 | 140·54 |
| 29·00 | 244·82 | 0·900 | 138·31 |
| 35·65 | 180·55 | 0·885 | 136·45 |

## HYDROCHLORIC ACID. LUNGE AND MARCHLEWSKI. L. PAGE 324.

| 15/4. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 1·15 | 8595·65 | 1·0050 | $\frac{100}{S}$ −48·99 |
| 1·52 | 6478·94 | 1·0069 | 49·07 |
| 2·93 | 3312·96 | 1·0140 | 49·92 |
| 5·18 | 1830·50 | 1·0251 | 51·11 |
| 12·38 | 707·75 | 1·0609 | 53·00 |
| 20·29 | 392·85 | 1·1014 | 54·27 |
| 31·28 | 219·69 | 1·1589 | 55·98 |
| 39·15 | 155·42 | 1·2002 | 57·25 |

## CAUSTIC SODA. PICKERING. L. PAGE 329.

| 15/4. | | | |
|---|---|---|---|
| 1 | 9900·00 | 1·0106 | +13·65 |
| 2 | 4900·00 | 1·0219 | +11·57 |
| 3 | 3233·33 | 1·0331 | + 9·49 |
| 4 | 2400·00 | 1·0443 | + 8·21 |
| 5 | 1900·00 | 1·0555 | + 6·83 |
| 10 | 900·00 | 1·1111 | + 0·72 |
| 20 | 400·00 | 1·2219 | − 8·83 |
| 30 | 233·33 | 1·3312 | −16·87 |
| 40 | 150·00 | 1·4343 | 20·30 |
| 50 | 100·00 | 1·5303 | 30·60 |

CAUSTIC POTASH. PICKERING. L. PAGE 329.

| 15/4. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 1 | 9900·00 | 1·0083 | $\frac{100}{S}$ −8·92 |
| 2 | 4900·00 | 1·0175 | 9·59 |
| 3 | 3233·33 | 1·0267 | 10·43 |
| 5 | 1900·00 | 1·0452 | 11·83 |
| 10 | 900·00 | 1·0918 | 15·10 |
| 20 | 400·00 | 1·1884 | 20·38 |
| 30 | 233·33 | 1·2905 | 24·76 |
| 40 | 150·00 | 1·3991 | 28·55 |
| 50 | 100·00 | 1·5143 | 31·98 |

SODIUM CHLORIDE. KARSTEN. L. PAGE 322.

| P. C. W/W. | Wg. | 0/4 $\dot{S}'$ | 0/4 $\dot{K}$ | 15/4 $\dot{S}'$ | 15/4 $\dot{K}$ | 20/4 $\dot{S}'$ | 20/4 $\dot{K}$ |
|---|---|---|---|---|---|---|---|
| 1 | 9900 | 1·0076 | $\frac{100}{S} - 23{\cdot}58$ | 1·0064 | $\frac{100}{S} - 27{\cdot}64$ | 1·0054 | $\frac{100}{S} - 28{\cdot}77$ |
| 5 | 1900 | 1·0384 | 25·85 | 1·0355 | 29·75 | 1·0341 | 30·69 |
| 10 | 900 | 1·0771 | 28·32 | 1·0726 | 31·50 | 1·0707 | 32·38 |
| 20 | 400 | 1·1562 | 32·41 | 1·1497 | 34·52 | 1·1473 | 35·39 |
| 25 | 300 | 1·1972 | 34·09 | ... | ... | 1·1879 | 36·19 |

d. TARTARIC ACID. PRIBRAM. L. PAGE 356.

| 20/4. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S'. | K. |
| 1·01 | 9800·99 | 1·0028 | $\frac{100}{S}$ – 54·83 |
| 5·09 | 1864·64 | 1·0215 | 55·45 |
| 10·89 | 818·27 | 1·0491 | 55·57 |
| 20 70 | 383·09 | 1,0978 | 56·18 |
| 30·16 | 231 56 | 1·1486 | 56·71 |
| 44·33 | 125·58 | 1·2312 | 57·41 |
| 49·95 | 100·20 | 1·2655 | 57 81 |

CHLORAL HYDRATE. RUDOLPHI. L. PAGE 363.

| 20·2/4. | | | |
|---|---|---|---|
| 0·5 | 19900·00 | 1 0012 | – 40·80 |
| 2 0 | 4900·00 | 1 0065 | 58·89 |
| 5 0 | 1900·00 | 1·0198 | 57·80 |
| 10·0 | 900 00 | 1 0440 | 56·26 |
| 20·0 | 400 00 | 1·0956 | 55·66 |
| 33⅓ | 200 00 | 1·1711 | 55·82 |
| 50 | 100 00 | 1·2713 | 57·13 |
| 66⅔ | 50 00 | 1 3998 | 57·06 |
| 80 | 25·00 | 1 5134 | 57·55 |

PHENOL. TRAUBE. L. PAGE 363

| 15/4. | | | |
|---|---|---|---|
| 1·14 | 8671·93 | 1 00037 | 89·72 |
| 2·20 | 4445·45 | 1 00133 | 89·66 |
| 5 18 | 1830 50 | 1·00418 | 90·34 |

CANE SUGAR. PLATO. L. PAGE 364.

| 0/4. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S' | K |
| 1 | 9900·00 | 1·0039 | $\frac{100}{S}$ – 60·16 |
| 5 | 1900·00 | 1·02033 | 59·96 |
| 6 | 1566·67 | 1 02449 | 60·01 |
| 10 | 900·00 | 1·04135 | 60·20 |
| 20 | 400·00 | 1·08546 | 60·59 |
| 30 | 233·33 | 1·13274 | 60·93 |
| 40 | 150·00 | 1·18349 | 61·23 |
| 50 | 100·00 | 1·23775 | 61·57 |
| 60 | 66·67 | 1·29560 | 61·96 |
| 70 | 40·28 | 1·35719 | 63·04 |

* CANE SUGAR.

| 17·5/17·5. | | | |
|---|---|---|---|
| 1 | 9900·00 | 1·00388 | 61·36 |
| 5 | 1900·00 | 1·0197 | 61·36 |
| 10 | 900·00 | 1·0401 | 61·44 |
| 20 | 400·00 | 1·0833 | 61·55 |
| 30 | 233·33 | 1·1297 | 61·73 |
| 40 | 150·00 | 1·1794 | 61·97 |
| 50 | 100·00 | 1·2328 | 62·23 |
| 60 | 66·67 | 1·2899 | 62·54 |
| 70 | 40·28 | 1·3509 | 62·94 |
| 80 | 25·00 | 1·4159 | 63·28 |
| 90 | 11·11 | 1·4849 | 63·71 |

* Allen's Commercial Organic Analysis. Part I, 4th Edn. 1909, pages 292-295.

## * INVERT SUGAR. HERZFELD

| 17·5/17·5. | | | |
|---|---|---|---|
| P. C. W/W. | Wg. | S | K. |
| 10 | 900 00 | 1 04034 | $\frac{100}{S}$ – 61·22 |
| 15 | 566 67 | 1·06154 | 61·34 |
| 20 | 400 00 | 1 08357 | 61·43 |
| 25 | 300 00 | 1·10616 | 61·61 |

## * LAVULOSE LIPPMANN

| 20/4. | | | |
|---|---|---|---|
| 1·01 | 9800 99 | 1 0021 | 61·91 |
| 4 971 | 1911 48 | 1·0178 | 61·46 |
| 10 5199 | 850 48 | 1 0405 | 61·50 |
| 20 2638 | 393 49 | 1 0821 | 61·85 |
| 30 1157 | 232 83 | 1·1279 | 61·84 |

## * ANHYDROUS DEXTROSE. SALOMON.

| 17·5/17·5. | | | |
|---|---|---|---|
| 1 988 | 4930 18 | 1·0075 | 62 55 |
| 5·873 | 1602 77 | 1 023 | 61·72 |
| 10 570 | 846 07 | 1 042 | 61·85 |
| 15 984 | 525 62 | 1 0649 | 61 86 |

## ANHYDROUS MALTOSE. SALOMON.

| 17·5/17·5. | | | |
|---|---|---|---|
| 1 987 | 4932 71 | 1 00785 | 60 80 |
| 5·87 | 1603 57 | 1 02340 | 61·04 |
| 9·637 | 937 66 | 1 03900 | 61 05 |
| 18·587 | 432 63 | 1 07740 | 61·73 |
| 26 928 | 271 36 | 1 1155 | 61·55 |

* Allen's Commercial Organic Analysis Part I, 4th Edn 1909, pages 292-295

## CONCLUSIONS.

1. In some cases the increase of dilution cases the increase of disappearance of volumes ; but there are several others which are very remarkable, the disappearance of volumes increases at first with the dilution then after reaching maximum the disappeared volumes begin to reappear more and more as the dilution increases. Maximum gravities of solutions of some chemicals, and percentage contractions are offshoots of *this* cardinal principle.

2. The maximum contractions are constants and different for different substances. There are some similarities noticeable according to the similarity and gradation of chemical properties of them which are cleanly marked in the cases of alcohols.

3. It is evident in the cases of sulphuric acid, acetic acid and sodium chloride, that the disappearance of volumes at all dilutions diminishes as the temperature rises. It is, however, not yet ascertained whether this apparent re-appearance of volume is due to the differences of co-efficients of expansion of water and the substance or to the decrease of the intensity of cause effecting the contraction. It remains as a subject for further investigation.

OPIUM FACTORY,
*Ghazipur, U. P.*

# On the Application of Cochineal Stain on Calcite and Aragonite.

By Sureschandra Datta, M.Sc.,

*Professor of Chemistry Ripon College, Calcutta.*

In a previous communication (1) I have described the application of aniline black as a stain to distinguish between calcite and aragonite and it has been found that with the help of aniline black these two natural carbonates can be very easily distinguished. So far as I am aware cochineal too has never been used to differentiate between calcite and aragonite and this note is meant to put on record the results I have obtained by staining calcite and aragonite with cochineal in presence of some acids and salts.

The way in which the stain has been fixed on calcite and aragonite is very simple. These minerals are separately powdered. Each is boiled in sufficiently dilute solutions of some salts and acids as mentioned below and in the latter case care must be taken so that the carbonate powders are not completely lost in the acids. Cochineal solution of sufficient strength is added next and the whole thing is boiled again when the powders of the minerals under examination are observed to have stains fixed on them. There is always some difference between the stains assumed by calcite and aragonite. The stains fixed are observed under the boiling solutions which are poured off and the stained minerals are washed with hot water and the stains whether fixed or not and their intensities whether decreased or not are taken notice of. It is not out of place to add here that when the minerals under examination are heated separately with sufficiently strong cochineal solution only, there are observed no stains on the said minerals.

(1) *On the staining of Calcite and Aragonite by means of Aniline black*—read in the Third Quarterly Meeting of the Indian Association for the Cultivation of Science, 23rd September, 1916.

The following is the table of the results of my experiments—the experiments being conducted on the method outlined above :—

| The acid or salt in which calcite and aragonite are separately boiled before the addition of strong cochineal solution in which the minerals are boiled again. | Stains under the hot solutions on calcite and aragonite. | Stains after washing with hot water on calcite and aragonite. | Colour of precipitate, if any. | Difference between the stains on calcite and aragonite. |
|---|---|---|---|---|
| Ammonium-Fluoride | Cal.–Deep blue-black.<br>Ara.–Light blue-black. | Same.<br>Intensities slightly decrease. | Cal.–Deep blue-black<br>Ara.–Light blue-black. | A. |
| Ammonium-Molybdate | Cal.–Deep rose or lilac.<br>Ara.–Pink. | Same.<br>Intensities slightly decrease | No precipitate. | A. |
| Ammonium-acetate | Cal.–Blue-black<br>Ara.–Bluish with reddish. | Same.<br>Intensities slightly decrease. | Cal.–Blue-black.<br>Ara.–dirty red. | A. |
| Ammonium-bromide | Cal.–Deep rose<br>Ara.–Light rose or pink. | Same.<br>Intensities slightly decrease. | No precipitate. | C.<br>But there is difference enough to differentiate the two minerals by means of stains. |

| The acid or salt in which calcite and aragonite are separately boiled before the addition of strong cochineal solution in which the minerals are boiled again. | Stains under the hot solutions on calcite nnd aragonite. | Stains after washing with hot water on calcite and aragonite. | Colour of precipitate, if any. | Difference between the stains on calcite and aragonite. |
|---|---|---|---|---|
| Barium chloride | Cal.–Blue.<br>Ara.–Bluish with reddish. | Same. | No precipitate. | B. |
| Sulphuric acid | Cal.–Bluish.<br>Ara.–Reddish. | Same.<br>Intensities decrease. On heating with cochineal solution again:—<br>Cal.–Deep red.<br>Ara.–Light red. | Cal.–Blue black.<br>Ara.–Dirty reddish. | A. |
| Hydrochloric acid | Cal.–Bluish.<br>Ara.–Violet. | Same.<br>The colour on aragonite becomes intense on washing with hot water. | Cal.–Blackish green.<br>Ara.–Brownish green. | A. |

| The acid or salt in which calcite and aragonite are separately boiled before the addition of strong cochineal solution in which the minerals are boiled again. | Stains under the hot solutions on calcite and aragonite. | Stains after washing with hot water on calcite and aragonite. | Colour of precipitate, if any. | Difference between the stains on calcite and aragoinite. |
|---|---|---|---|---|
| Nitric acid | Cal.–Bluish.<br>Ara.–Reddish. | Same.<br>On treating with cochineal solution :—<br>Cal.–Slightly red.<br>Ara.–Very light red | Cal.–Grayish.<br>Ara.–Dirty Red. | A. |
| Acetic acid | Cal.–Blue.<br>Ara.–Reddish. | Same. | Cal.–Blue-black<br>Ara.–Dirty red | A. |
| Formic acid | Cal.–Blue.<br>Ara.–Bluish with reddish. | Intensities decrease to a great extent. On treating with cochineal :—<br>Cal.–Red.<br>Ara.–Bluish. | Cal.–Grayish blue-black.<br>Ara.–Dirty red | A. |

| The acid or salt in which calcite and aragonite are separately boiled before the addition of strong cochineal solution in which the minerals are boiled again. | Stains under the hot solutions on calcite and aragonite. | Stains after washing with hot water on calcite and aragonite. | Colour of precipitate, if any. | Difference between the stains on calcite and aragonite. |
|---|---|---|---|---|
| Lactic acid | Cal.–Blue.<br>Ara.–Bluish with reddish. | Same.<br>On treating with cochineal :—<br>Cal.–Red<br>Ara.–Reddish | Cal.–Grayish blue black.<br>Ara.–Dirty reddish. | A. |
| Oxalicacid | Cal.–Blue.<br>Ara.–Reddish. | Same. | No precipitate. | C. |
| Silver nitrate<br>By silver nitrate solution calcite and aragonite are distinguished as mentioned in current literature (1) But cochineal solution with silver nitrate has never been used before. | Cal.–Black.<br>Ara.–Brownish black. | Same.<br>Intensities decrease | No precipitate. | A. |
| Ammonium phosphate | Cal.–Bluish<br>Ara.–Very light bluish. | Same. | No precipitate. | C. |

(1) Doelter—*Petrogenesis*, Bd. 1, p 112.

**N. B** A—Very pronounced. B—Pronounced. C—Not very pronounced. The stains on the powders of calcite and aragonite are better distinguished under water than when they are dry.

# A new Process for the Carbonisation of sea weeds.

*(Abstract.)*

By Dr. Rasik Lal Datta, D.Sc.

This process relates to the carbonisation of weeds in a closed chamber in a regulated current of air and passing the products of combustion through a spiral condenser for the condensation of tar which is formed in good quantity by the heat of the carbonisation and finally leading the gas through a scrubber in which a solution of alkali is allowed to trikle down. The iodine which is volatilised during the carbonisation is kept back in the condenser and the last traces in the scrubber. The iodine from tar and the alkaline solution may be recovered in any known manner.

PROCEEDINGS

OF THE

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. III. PART V.

# On the Flow of Energy in the Electromagnetic Field surrounding a Perfectly Reflecting Cylinder.

BY T. K. CHINMAYANANDAM, B.A. (HONS.),
*Research Scholar in the Indian Association for the Cultivation of Science, Calcutta.*

## § 1. *Introduction.*

In a paper recently contributed to this journal,* Mr. N. Basu has discussed the general features of the phenomena observed in the immediate neighbourhood of a perfectly reflecting cylinder, on which plane light waves are incident in a direction at right angles to its axis. Further investigation was, however, necessary in order to establish the formulæ for the distribution of light intensity in the various parts of the field. These formulæ have now been obtained, and subjected to a detailed experimental test. Besides describing the results of a photometric study that has been carried out, the present paper also deals with the form of the

* N. Basu.—*On the Diffraction of Light by Cylinders of large Radius,* Proc. of the Ind. Association, Vol. III, part 3, 1917.

lines of flow of energy through the field, which, it is thought, may prove of interest with reference to the work of Profs. R. W. Wood* and Max Mason† on the simpler case of the interference field due to two point sources of light.

It may be remarked here that the phenomena, which form the subject of this paper may be strikingly shown on a large scale, without the aid of a microscope, by using a cylindrical surface of very large radius as the diffracting "edge." A strip of thick plate glass, two inches wide and about a yard long, may be bent into a circle of some yards radius by resting it on supports near the two ends, and loading the latter sufficiently. A slit illuminated by a Cooper-Hewitt lamp and placed at some distance from the surface in a line with it, may be used as the source of light. A very large number of fringes may then be seen with a low-power eye-piece, if the plane of observation be within a few feet of the cylindrical "edge." At greater distances, the fringes widen out; their visibility and number decrease, and their spacing alters with increasing distances from the cylinder in such manner as to approximate more and more closely to that of the diffraction fringes due to a straight edge. Some photographs which have been taken with the arrangement described above are shown in Plate VI, where figures (a), (b), and (c) correspond to the phenomena in planes at increasing distances from the edge.

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* R. W. Wood.—*On the Flow of Energy in a System of Interference Fringes*, Phil. Mag., 18, p. 250.

† Max Mason.—*The Flow of Energy in an Interference Field*, Phil. Mag., 20, p. 290.

## § 2. *The Form of the Illumination Curves.*

Debye* has shown from the electro-magnetic theory, that at a great distance from a cylinder (assumed to be of large radius) on which plane waves are incident, the disturbance due to it is practically the same as that to be expected from the principles of geometrical optics, this statement however not being taken as correct in respect of points lying in a direction very nearly the same as that of the incident rays. Debye's results suggest a simple, method of finding the distribution of intensity at points lying within the region of light, in the immediate neighbourhood of the cylinder. Let AOB (Fig. 1) represent the section of the

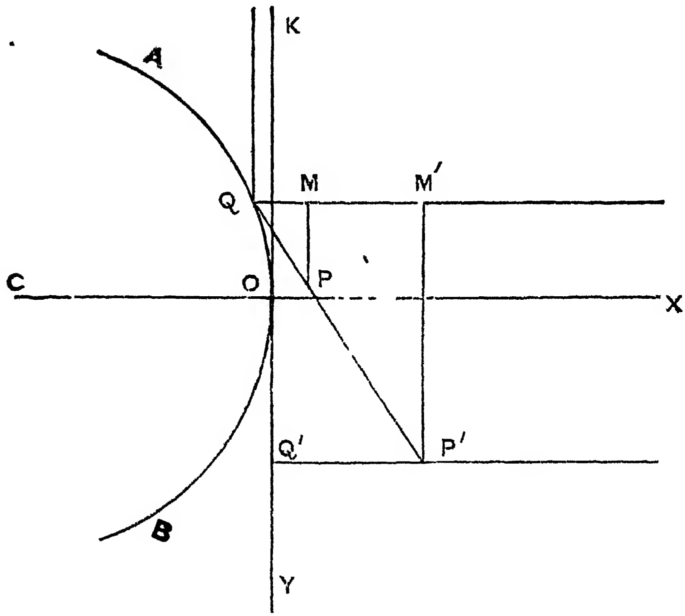



Fig. 1

* Debye.—Phys. Zeitschr., 9. pp. 775-778, Nov, 1908, also Science Abstracts, 1909, p 88

cylinder, and KOY the direction of the incident rays. We may assume that the fringes observed in the plane OX containing the edge of the cylinder grazed by the rays, are due solely to the interference of these rays with those reflected from the surface of the cylinder at varying angles. This would also be the case as regards any plane such as QR in advance of the edge. But the phenomena in a plane such as Q′P′ which lies on the remote side of the edge, would not admit of such simple treatment, especially when we consider the effect at points lying not far from the boundary OY of the direct and reflected rays. In such a plane, the intensity at any point on the right of the boundary may be regarded as due to the superposition of three factors, *(a)* the effect due to the direct rays, *(b)* that due to the reflected rays, and *(c)* a diffraction effect mainly perceptible in the neighbourhood of the boundary. If the cylinder were replaced by a perfectly reflecting semi-infinite screen lying in the plane CO with its edge at O, the diffraction effect would be found as in Sommerfeld's well-known investigation,* by superposing upon the direct rays, a radiation emitted by the edge of the screen. It will be observed that in the present case, the intensity of the rays regularly reflected from the cylinder, as given by the formulæ of Geometrical Optics, is zero along the boundary OY, increasing slowly as we move away from the boundary into the region of light, and thus presents no discontinuity. It thus seems justifiable to assume that so far as regards the phenomena in the region on the

* Sommerfeld.—*On the Math. Theory of Diffraction*, Math. Annalen, Vol. XLVII, p. 317, 1895.

right-hand side of the boundary, the diffraction effect *(c)* is practically the same as in the case of a semi-infinite screen with its edge at O.

The distribution of intensity in the field may be readily found on the foregoing assumptions. Take the edge O as origin of co-ordinates, and rectangular axes OX, OY, perpendicular and parallel respectively to the direction of the incident rays, and let the angle OCQ, assumed to be small, be denoted by $\theta$. The path difference between the direct and the reflected rays reaching any point P is

$$\delta = 2\pi/\lambda\ (QP - MP) + \pi$$
$$= 2k\theta^2\ (y + a\theta) + \pi \text{ approximately,}$$

if $k$ be written for $2\pi/\lambda$.

Again, if the amplitude of the incident light be taken as unity, that of the reflected light may be written as

$$\left(\frac{\rho}{\rho + QP}\right)^{\frac{1}{2}}$$

where $\rho$ is the radius of curvature of the reflected wave on emergence at Q. Since

$$\rho = \frac{a\theta}{2} \text{ approximately}$$

$$\frac{\rho}{\rho + QP} = \frac{a\theta}{2y + 3a\theta}$$

In any plane in advance of that passing through the edge of the cylinder ($y = -d$, say), the expression for the intensity of illumination at any point is

$$I = 1 + s - 2\sqrt{s} \cos \phi \quad \ldots \quad \ldots \quad (1)$$

where $\phi = 2k\theta^2\ (y + a\theta)$

and $s = \dfrac{a\theta}{3a\theta + 2y}$.

The positions of the maxima and minima of illumination are given by

$$\frac{dI}{d\theta}=\frac{ds}{d\theta}\left(1-\frac{\cos\phi}{\sqrt{s}}\right)+2\sqrt{s}\sin\phi\,\frac{d\phi}{d\theta}=0. \qquad \dots \qquad (2)$$

Since $\frac{d\phi}{d\theta}=4k\theta(y+a\theta)+2ka\theta^2$, and contains the factor $2\pi/\lambda$, it will be large, so that the first term in equation (2) is negligible. That equation can hence be written

$$\sin\phi=0.$$

$$\text{or} \qquad 2\theta^2\,(y+a\theta)=m\lambda/2 \quad \dots \qquad \dots \quad (3)$$

while the relation between $\theta$ and $x$ is given by

$$x=(y+a\theta)2\theta-a\theta^2/2$$

$$=2y\theta+3a\theta^2/2. \quad \dots \qquad \dots \qquad \dots \quad (4)$$

From (1) and (3) it, is seen that the intensities of the successive maxima and minima are respectively proportional to

$$I_{max.}=\left\{1+\left(\frac{a\theta}{3a\theta-2d}\right)^{\frac{1}{2}}\right\}^2$$

$$I_{min}=\left\{1-\left(\frac{a\theta}{3a\theta-2d}\right)^{\frac{1}{2}}\right\}^2$$

The intensity curve has been plotted out in Plate I (*a*) for the case $a=1{\cdot}5$ cm. and $d=0{\cdot}2$ cm. It will be seen that in this plane $I_{max.}$ begins nearly with a value $(1+1)^2=4$, and drops down gradually in successive fringes to a limiting value of $\left(1+\frac{1}{\sqrt{3}}\right)^2=2\,49.$ $I_{min.}$ increases from a value nearly zero to a limiting value $\left(1-\frac{1}{\sqrt{3}}\right)^2=0{\cdot}18$. The 'visibility' of the successive fringes in the plane of observation, therefore, decreases slowly.

At the plane $y=0$, the illumination is given by

$$I=1+\tfrac{1}{3}-\frac{2}{\sqrt{3}}\cos\phi \quad , \qquad \dots \quad (5)$$

The intensity curve is shown in Plate I (*b*). $I_{max}$

has a constant value of $\left(1+\frac{1}{\sqrt{3}}\right)^2$, and $I_{min.}$ a value $\left(1-\frac{1}{\sqrt{3}}\right)^2$. The 'visibility' of the fringes is thus stationary along this plane for a considerable distance from the edge.

Passing on to consider the distribution of Intensity in any plane Q′P′ below the plane $y=0$, we have, as remarked above, to add to the effect of the direct and the reflected rays a diffraction effect. We shall represent the latter effect by that due to a single source placed at the edge O, the amplitude of the disturbance due to it, at a point in the region bounded by OX and OY being given by Sommerfeld's expression*

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{r}}\cos\left(kr-nt+\frac{\pi}{4}\right)\left\{\pm\frac{1}{\cos\frac{\varphi+\varphi'}{2}}-\frac{1}{\cos\frac{\varphi-\varphi'}{2}}\right\} \tag{6}$$

where $r$ is the distance of the point from the edge O, $\varphi, \varphi'$ are the angles which the diffracted and the incident beams respectively make with the direction XO.

If we denote the angle P′OY by $\alpha$

$$\varphi=3\pi/2-\alpha \quad \varphi'=\pi/2.$$

so that expression (6) becomes

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{r}}\cos\left(kr-nt+\frac{\pi}{4}\right)\left(\pm 1-\frac{1}{\sin\frac{\alpha}{2}}\right)$$

$$=-\frac{1}{4\pi}\,\frac{2}{\alpha}\sqrt{\frac{\lambda}{r}}\cos\left(kr-nt+\frac{\pi}{4}\right)$$

since $\alpha$ is small so far as our investigation is concerned.

Now $r=\text{P}'\text{O}=y+\frac{x^2}{2y}$ ; $\alpha=\frac{x}{y}$, approximately,

* Sommerfeld—*loc. cit.*

so that the above expression reduces to

$$-\frac{\sqrt{y\lambda}}{2\pi x}\cos\left(k.\overline{y+\frac{x^2}{2y}}-nt+\frac{\pi}{4}\right)\quad\text{(approx.)}.$$

The total disturbance at P′ is thus

$$\xi=\cos\,(ky-nt)-\sqrt{s}\cos\,(ky-nt+k\,\overline{2a\theta^3+2y\theta^2})$$

$$-\frac{\sqrt{y\lambda}}{2\pi x}\cos\left(\overline{k.y+\frac{x^2}{2y}}-nt+\frac{\pi}{4}\right).$$

Remembering that, since $x=3a\theta^2/2+2y\theta$, $\frac{x^2}{2y}=2y\theta^2+3a\theta^3$ we get for the intensity of illumination at any point, the expression $I=1+S-2\sqrt{S}\cos\chi$ ... ... (7)

where $S=s+\frac{y\lambda}{4\pi^2x^2}+\frac{\sqrt{sy\lambda}}{\pi x}\cos\left(ka\theta^3+\frac{\pi}{4}\right)$ ... (8)

$$\chi=k\,(2a\theta^3+2y\theta^2)+\epsilon\quad\ldots\quad\ldots\quad\ldots\quad(9)$$

and $\tan\epsilon=\dfrac{\frac{\sqrt{y\lambda}}{\pi x}\sin\left(ka\theta^3+\frac{\pi}{4}\right)}{2\sqrt{s}+\frac{\sqrt{y\lambda}}{\pi x}\cos\left(ka\theta^3+\frac{\pi}{4}\right)}$ ... ... (10)

Equations (4) and (7) give for the positions of the maxima and minima of illumination

$$\left.\begin{aligned}&3a\theta^2/2+2y\theta=x\\&2a\theta^3+2y\theta^2=m\lambda/2-\frac{\epsilon\lambda}{2\pi}\end{aligned}\right\}\quad\ldots\quad\ldots\quad(11)$$

We see that the introduction of the diffraction term has slightly changed the positions of the maxima and minima. The magnitude of the change depends upon $\epsilon$, which is zero when $y=0$ (equation 10), and steadily increases with $y$ to a limiting value of $\left(ka\theta^3+\frac{\pi}{4}\right)$ or $\frac{\pi}{4}$ since $ka\theta^3$ is negligible over the first few bands when $y$ is sufficiently large. By actual calculation it is found that this limit is practically reached, when $y$ is over three times the radius of the cylinder. Under these conditions, equations (11) reduce to

$$x^2=y\lambda(4m-1)/4.\quad\ldots\quad\ldots\quad(12)$$

Formula (12) is identical with Schuster's formula for the case of diffraction of plane waves by a straight edge.

Returning to equation (7), we see that the intensity of illumination of the successive maxima and minima is given by

$$\left.\begin{aligned} I_{\text{max.}} &= (1+\sqrt{S})^2 \\ I_{\text{min.}} &= (1-\sqrt{S})^2 \end{aligned}\right\} \quad \ldots \qquad \ldots \quad (13)$$

where S is given by equation (8).

The intensity curve for a plane 5 mm. behind the "edge" is shown in Plate I (*c*). It will be seen that the ratio of the minima to the maxima is considerably greater than in (*a*) and (*b*). Calculation also shows that the visibility of the successive fringes in this plane of observation should decrease, though somewhat slowly. For still greater distances from the edge of the cylinder, the illumination curves become practically identical with that of the Fresnel type due to a straight edge, the intensity of the reflected rays becoming negligible in comparison with that of the incident and the diffracted rays The ordinates of the curves (*a*), (*b*), and (*c*), (though not the abscissæ) have all been drawn to the same scale, and the curves illustrate the fact that the luminosity of the field as a whole decreases as we recede from the cylinder.

### § 3. *Photometric Study of the field.*

The formulæ obtained above have been tested by two independent methods—(1) by photometric comparison of the maxima and minima of illumination, and (2) by determination of their relative positions.

A small polished cylinder of glass, of about 1·5 cm. radius, was used. It was mounted on one of the

stands of an optical bench, and a microscope objective mounted on another of those stands was brought up close to the cylinder. Light from a narrow slit was passed through a collimating lens and was allowed to fall grazingly on the cylinder. The field was viewed through a micrometer eye-piece, placed at a distance behind the objective. By moving the cylinder towards or away from the objective, the phenomena at different planes $y=d$ could be observed.

The photometric arrangement used to study the relative intensity of the fringes was based on a Polarization method. The beam of light was plane polarized by passage through a Nicol before falling on the cylinder. The eye-piece, (a low-power one) was moved off to a pretty large distance behind the objective, and two narrow slits cut out of aluminium foil and pasted on two glass strips, were mounted, one above the other, between the eye-piece and the objective, so as to allow a small relative motion which could be controlled by a micrometer-screw. A thin mica plate was also fixed up on the upper one (which was movable in the actual experiment), the thickness of the plate and its orientation with respect to the slit being adjusted by trial so that, under the conditions of the experiment, it circularly polarized the light falling on it. The field was viewed through another Nicol fitted with a graduated circle and mounted just behind the slits.

The lower slit was always set on the first bright band, while the upper one was set successively on the different maxima and minima. Equality of illumination of the upper and lower slits was obtained in each case, by rotating the analysing Nicol. The reading for the

crossed position of the analyser being also taken, the ratio of the intensities of illumination could be easily calculated Thus if $\psi$ be the orientation of the analyser in any case ($\psi$ being reckoned from the crossed position) and $I$, $I_0$ the intensities of illumination of the upper and lower slits respectively,

$$I/I_0 = \sin^2\psi.$$

Readings were taken for the first few bands on the planes (1) $y=0$, (2) $y=0{\cdot}5$ cm., and are shown in Table I with the corresponding values calculated from theory.

TABLE I.

| $n$ | $y=0$. | | | | $y=0{\cdot}5$ *cm.* | | | |
|---|---|---|---|---|---|---|---|---|
| | MAXIMUM. | | MINIMUM. | | MAXIMUM. | | MINIMUM. | |
| | $I_n/I_0$. | | $I_n/I_0$. | | $I_n/I_0$. | | $I_n/I_0$. | |
| | Obsd. | Calcd. | Obsd. | Calcd. | Obsd. | Calcd. | Obsd. | Calcd. |
| 1 | 1·00 | 1·00 | 0·09 | 0·07 | 1·00 | 1·00 | 0·46 | 0·42 |
| 2 | 0·96 | 1·00 | 0·08 | 0·07 | 0·91 | 0·93 | 0·45 | 0·44 |
| 3 | 1·00 | 1·00 | 0·12 | 0·07 | 0·91 | 0·91 | 0·52 | 0·46 |
| 4 | 0·94 | 1·00 | 0·15 | 0·07 | 0·83 | 0·90 | 0·52 | 0·46 |

The discrepancies are within the limits of experimental error, so that the theory developed above appears to be substantially correct.

As has been remarked already, the theory was also tested by measurements of the positions of the minima of illumination in different parts of the field. Some explanation is here necessary with regard to the measurements of fringes in a plane in advance of the "edge" ($y$ negative). With the ordinary arrangement

as described above, if the microscope is moved near, so that its focal plane may be in advance of the edge, we are unable to see the exact phenomena in that plane, since the light has to come past the "edge" before it can fall on the objective; and secondly, as has been fully described in Mr. Basu's* paper, the field is complicated by the occurrence of the caustic and its accompanying fringes, formed by reflection from the surface of the cylinder. This difficulty was got over in the present work by turning the cylinder around its axis, till the desired plane of observation coincided with the boundary of the polished surface. What is meant may be better understood by a reference to fig. 1, the process described being equivalent to cutting off the cylinder along CQ and removing the lower half. It is not easy, however, with this arrangement, directly to determine the value of $y$ when it is negative; and in Table II below it has been obtained by calculation from a pair of readings. The source of light was a quartz mercury lamp with a green ray filter.

TABLE II.

Fringes between the Cylidrical Edge and the Source of light.

| | $y=0$ | | $y=-2{\cdot}0$ mm. | | $y=-2{\cdot}9$ mm. | |
|---|---|---|---|---|---|---|
| $n$ | $x_n - x_1$. | | $x_n - x_1$. | | $x_n - x_1$. | |
| | Obsd. | Calcd. | Obsd. | Calcd. | Obsd. | Calcd. |
| 3 | 0·0169 | 0 0169 | 0·0082 | 0·0082 | 0·0062 | 0·0060 |
| 5 | 0·0307 | 0·0302 | 0·0165 | 0·0163 | 0·0123 | 0·0120 |
| 7 | 0·0420 | 0·0417 | 0·0240 | 0·0240 | 0·0183 | 0·0182 |
| 9 | 0·0524 | 0·0522 | 0·0310 | 0·0317 | 0·0235 | 0·0235 |
| 11 | 0·0620 | 0·0619 | 0·0378 | 0·0395 | 0·0290 | 0·0293 |
| | | | mms. | | | |

* Basu—*loc. cit.*

TABLE III.

Fringes behind the Cylindrical Edge.

| $n$ | $d$=0·1 cm. |  | $d$=0·3 cm. |  | $d$=0·5 cm. |  | $d$=0·7 cm. |  |
|---|---|---|---|---|---|---|---|---|
|  | $x_n - x_1$. |  | $x_n - x_1$. |  | $x_n - x_1$. |  | $x_n - x_1$. |  |
|  | Obsd. | Calcd. | Obsd. | Calcd. | Obsd. | Calcd. | Obsd. | Calcd. |
| 2 | 0·0159 | 0·0158 | 0·0250 | 0·0251 | 0·0341 | 0·0326 | 0·0370 | 0·0370 |
| 3 | 0·0284 | 0·0284 | 0·0439 | 0·0450 | 0·0580 | 0·0580 | 0·0660 | 0·0661 |
| 4 | 0·0385 | 0·0390 | 0·0600 | 0·0609 | 0·0776 | 0·0782 | 0·0887 | 0·0897 |
| 5 | 0·0483 | 0·0480 | 0·0740 | 0·0750 | 0·0928 | 0·0938 |  |  |
| 6 | 0·0568 | 0·0570 | 0·0867 | 0·0879 | 0·1116 | 0·1134 |  |  |
| 7 | 0·0651 | 0·0650 | 0·0990 | 0·0997 |  |  |  |  |
| 8 | 0·0715 | 0·0723 | 0·1108 | 0·1110 |  |  |  |  |
| 9 | 0·0794 | 0·0796 |  |  |  |  |  |  |
| 10 | 0·0862 | 0·0863 |  |  |  |  |  |  |

$\lambda$=5461. A. U.; $a$=1 54 cm.

The agreement between the calculated and the observed values is very close and confirms the theory.

§ 4. *The Loci of maxima and minima of Illumination.*

These curves have an interesting property which may be briefly considered here. The equation to the loci is obviously

$$r(1-\cos 2\theta)=n\lambda/2$$

or $$r\theta^2=n\lambda/4 \text{ approx.} \quad . \quad (14)$$

where $r$ is the distance of the point from the surface of the cylinder measured along the reflected ray which passes through the point. The shape of the curves is indicated in Plate II (thick lines), for the case when $a=5$ inches, $\lambda=\cdot016$ inch, $\lambda$ being taken so large for convenience of representation to scale; only the 1st, 3rd, 5th &c., loci are shown.

The equation to the loci can also be got in terms of $\theta$, and $x$ or $y$. Thus on eliminating $y$ from equations (3) and (4) we get

$$x=\frac{m\lambda}{2\theta}-\frac{a\theta^2}{2} \quad \ldots \quad \ldots \quad (15)$$

which gives the abscissæ of the points at which the loci cut the straight lines $\theta=$ const.

The ordinates of these points are given by

$$2y\theta=x-\frac{3}{2}a\theta^2=\frac{m\lambda}{2\theta}-2a\theta^2$$

$$\therefore \quad y=\frac{m\lambda}{4\theta^2}-a\theta \quad \ldots \quad \ldots \quad (16)$$

From (15) and (16) it is seen that

$$dx=-\left(\frac{m\lambda}{2\theta^2}+a\theta\right)\delta\theta,$$

and $$dy=-\left(a+\frac{m\lambda}{2\theta^3}\right)\delta\theta$$

so that $$\frac{dy}{dx}=\theta$$

At any point, therefore, the curves bisect the angle between the directions of the direct and the reflected rays which pass through that point.

As might be expected, the formulæ obtained above for the case of diffraction by a cylinder, reduce to the ordinary formulæ for diffraction by a straight edge, on writing $a=o$, provided, of course, that the light from the other side of the cylinder is cut off by a semi-infinite plane extending to the left of the origin O.

For then, equations (11) become

$$\left.\begin{aligned} x&=2y\theta \\ m\lambda/2-\epsilon\lambda/2\pi&=2y\theta^2 \end{aligned}\right\}$$

and $\epsilon=\frac{\pi}{4}$, so that $x^2=y\lambda\,(4m-1)/4$.

which is Schuster's formula for diffraction of plane waves at straight edge. The results regarding the loci of maxima and minima will also apply for diffraction at a straight edge, under the same conditions.

## § 5. *On the flow of energy in the field.*

We will first take into account only the effects due to the interference of the direct and the reflected rays. The effect due to diffraction at the edge of the cylinder can be brought in later as a correction.

Let us assume, for simplicity, that the light is polarized in the plane of incidence, so that the electric intensity is perpendicular to that plane, and the magnetic intensity lies in it. Then at any point P (fig. 1.) the resultant electric intensity is

$$E=\left[\cos n\left(t-\frac{r_1}{c}\right)+\left(\frac{\rho}{\rho+r_2}\right)^{\frac{1}{2}}\cos\left\{n\left(t-\frac{r_2}{c}\right)+\pi\right\}\right]$$

where $r_1=PM$, $r_2=PQ$ and $\rho$ is the radius of curvature

of the reflected wave at Q. The expression $\left(\frac{\rho}{\rho+r_2}\right)^{\frac{1}{2}}$ will in the rest of the paper be denoted by K. The resultant magnetic intensity at P is

$$\overrightarrow{\mathrm{H}}=\cos n\left(t-\frac{r_1}{c}\right)+\mathrm{K}\cos\left\{n\left(t-\frac{r_2}{c}\right)+\pi\right\}$$

Let now $\mathbf{k}_1$, $\mathbf{k}_2$ be unit vectors at P in the direction of the direct and the reflected rays respectively. The flow of energy is determined by the Poynting vector S where

$$\mathrm{S}=\frac{c}{4\pi}\left[\mathrm{EH}\right]=\frac{c}{4\pi}\left\{\cos n\left(t-\frac{r_1}{c}\right)+\mathrm{K}\cos\left\{n\left(t-\frac{r_2}{c}\right)+\pi\right\}\right\}$$
$$\times\left\{\mathbf{k}_1\left[\cos n\left(t-\frac{r_1}{c}\right)\right]+\mathbf{k}_2\left[\mathrm{K}\cos n\left(t-\frac{r_2}{c}\right)+\pi\right]\right\},$$

or $\frac{4\pi\mathrm{S}}{c}=\mathbf{k}_1\left[\cos^2\chi+\mathrm{K}\cos\chi\cos\chi'\right]+\mathbf{k}_2\left[\mathrm{K}\cos\chi\cos\chi'+\mathrm{K}^2\cos^2\chi'\right]$

where $\chi=n\left(t-\frac{r_1}{c}\right)$ and $\chi'=n\left(t-\frac{r_2}{c}\right)+\pi$.

The time-mean $\overline{\mathrm{S}}$ of the flow of energy is given by

$$\frac{4\pi\overline{\mathrm{S}}}{c}=\tfrac{1}{2}\,\mathbf{k}_1\left[1+\mathrm{K}\cos(\chi'-\chi)\right]+\tfrac{1}{2}\,\mathbf{k}_2\left[\mathrm{K}^2+\mathrm{K}\cos(\chi'-\chi)\right]$$

$$\frac{8\pi\overline{\mathrm{S}}}{c}=\mathbf{k}_1\left[a_1\right]+\mathbf{k}_2\left[a_2\right]$$

if $a_1=1+\mathrm{K}\cos(\chi-\chi')$ and $a_2=\mathrm{K}^2+\mathrm{K}\cos(\chi'-\chi)$

If $\varphi$ be the angle which the direction of $\overline{\mathrm{S}}$ makes with that of the incident rays, it can be easily shown that

$$\tan\varphi=\frac{a_2\sin 2\theta}{a_1+a_2\cos 2\theta}=\frac{2a_2\theta}{a_1+a_2}$$

since $\theta$ is small.

$$\text{Thus}\quad \tan\varphi=\frac{2\theta\left\{\mathrm{K}^2+\mathrm{K}\cos(x'-\chi)\right\}}{1+2\mathrm{K}\cos(\chi'-\chi)+\mathrm{K}^2}\quad\ldots\quad\ldots\quad(17)$$

now $\chi'-\chi=\frac{n(r_2-r_1)}{c}+\pi \quad =\frac{n}{c}2r_2\theta^2+\pi.$

Hence equation (17) becomes

$$\tan\varphi=\frac{2\theta\left\{K^2-K\cos\frac{n}{c}2r_2\theta^2\right\}}{1+K^2-2K\cos\frac{n}{c}2r_2\theta^2} \quad \ldots \quad \ldots \quad (18)$$

The current of energy at the point is

$$\bar{S}=\frac{c}{8\pi}\left\{a_1^2+a_2^2+2a_1a_2\cos 2\theta\right\}^{\frac{1}{2}}$$

$$=\frac{c}{8\pi}\left(1+K^2-2K\cos\frac{n}{c}2r_2\theta^2\right) \quad . \ \ldots \text{(approx.)} \ldots \quad (19)$$

The lines of flow of energy are determined by the condition that at any point $(r_2, \theta)$ the inclination to the

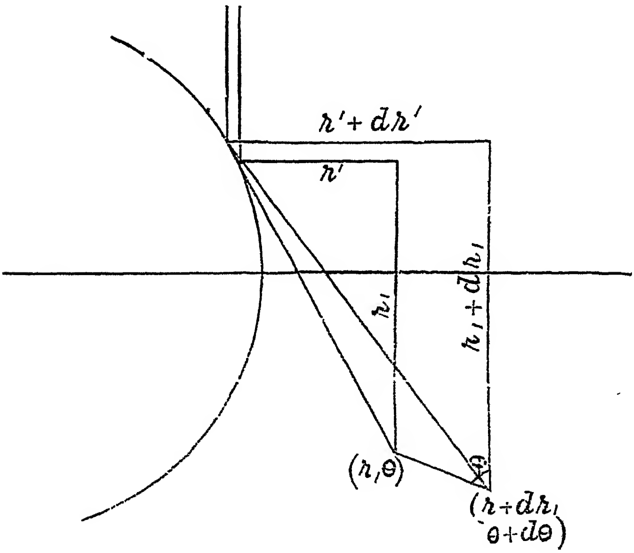



Fig. 2

direct rays is $\varphi$, where $\varphi$ is given by equation (18). From this, we can get the differential equation to the lines of flow in terms of $r_2$ and $\theta$, or dropping the suffix, in terms of $r$ and $\theta$. It is seen from fig. 2, that

$$\tan \varphi = \frac{dr' - a\theta d\theta}{dr_1 - a d\theta}$$

But $r' = r \sin 2\theta = 2r\theta$ ; $r_1 = r \cos 2\theta = r$ (approx.)

$$\therefore \tan \varphi = \frac{2\,(r d\theta + \theta dr) - a\theta d\theta}{dr - a d\theta}$$

$$\text{or } \tan \varphi \left(1 - a\frac{d\theta}{dr}\right) = 2\theta + (2r - a\theta)\,\frac{d\theta}{dr} \quad \ldots \quad \ldots \quad (20)$$

From (18) and (20), we get

$$\frac{d\theta}{dr}\,(2r - a\theta) + 2\theta = \left(1 - a\,\frac{d\theta}{dr}\right)\frac{2K\theta\left(K - \cos\frac{n}{c}\,2r\theta^2\right)}{1 + K^2 - 2K\cos\frac{n}{c}\,2r\theta^2}$$

which on reduction becomes

$$\frac{d\theta}{dr} = -\frac{\theta}{r}\cdot\frac{1 - K\cos\frac{n}{c}2r\theta^2}{1 - 2K\cos\frac{n}{c}\,2r\theta^2} \quad \ldots \quad \ldots \quad (21)$$

Equation (21) may also be written

$$d\,(r\theta) - \frac{K}{2\theta}\cdot\frac{c}{n}\cdot\cos\psi\, d\psi = 0 \quad \ldots \quad \ldots \quad (22)$$

where $\psi = \frac{n}{c}\,2r\theta^2$.

In the immediate neighbourhood of a point, we can regard $\frac{K}{\theta}$ which is equal to $\left\{\frac{a}{\theta\,(2r + a\theta)}\right\}^{\frac{1}{2}}$ as constant since its variation with $\theta$ and $r$ will be small compared with the periodic part. On integrating (22), we get

$$r\theta - \left(\frac{K}{2\theta}\right)\frac{c}{n}\,\sin\psi = \text{const.} \quad \ldots \quad \ldots \quad (23)$$

which determines the shape of the lines of flow in the immediate neighbourhood of a point. The points of intersection of these curves with the loci of maxima and minima of illumination ($\psi = m\pi$), are given by the equation

$$r\theta = \text{const.} \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

This gives the "mean lines of flow" about which energy "crinkles" down. They are shown in Plate II (thin lines). We find that so long as $\theta$ is not very large, these curves are inclined to the direct rays at angles smaller than those corresponding to the maxima and minima loci. If we imagine the latter set of curves as forming successive bright and dark tubes, energy will flow down across the tubes, its direction being periodically shifted such that it tends to flow along the bright tubes and to cut across dark tubes. The shift in its direction goes through one complete cycle, as the energy passes from one dark tube, to the next dark tube, or from one bright tube to the next, so that the 'Wave-length' of a crinkle in the neighbourhood of a point $(r,\theta)$ may be determined by finding the distance, along the curve $r\theta=$const., between two successive points of intersection of that curve and the family of curves $r\theta^2=\frac{m\lambda}{2}$ (the loci of minima of illumination). Thus

$$r\theta^2=\frac{m\lambda}{2}\,;\ r\theta=C$$

giving $\theta=\frac{m\lambda}{2C}$ or $\delta\theta=\frac{\lambda}{2C}$

if $\delta\theta$ be the difference in the $\theta$-co-ordinates between two successive points of intersection. Also if $\sigma$ be the arc measured along the curve $r\theta=C$, we have from equation (20)

$$\left(\frac{d\sigma}{d\theta}\right)^2=\left\{(2r-a\theta)+2\theta\,\frac{dr}{d\theta}\right\}^2+\left\{\frac{dr}{d\theta}-a\right\}^2 \quad \ldots \quad (25)$$

Along the curve $r\theta=C$, $\frac{dr}{d\theta}=-\frac{r}{\theta}$.

Hence from (25) $\left(\frac{d\sigma}{d\theta}\right)^2=a^2+\frac{r^2}{\theta^2}+\frac{2ar}{\theta}$ approximately.

$$\frac{d\sigma}{d\theta}=a+\frac{r}{\theta}=a+\frac{C}{\theta^2}.$$

If $l$ be the "wave-length" of a crinkle,

$$l=\left(a+\frac{C}{\theta^2}\right)\delta\theta=\left(a+\frac{C}{\theta^2}\right)\frac{\lambda}{2C} \qquad \text{approx.}$$

$$=\frac{a\lambda}{2C}+\frac{\lambda}{2\theta^2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (26)$$

C being the same, $l$ increases as $\theta$ decreases, the rate of increase getting larger as the absolute value of $\theta$ diminishes; $l=\infty$ when $\theta=0$. Again as we move away from the cylinder to the right, both C and $\theta$ increase, so that $l$ decreases. These points are brought out in Plate III.

Turning back to equation (23), the lines of flow near the point $r_1\theta_1$ are given by

$$r\theta=C-\frac{K_1}{\theta_1}\frac{\lambda}{4\pi}\sin\psi.$$

The curves will obviously lie between the curves

$$r\theta=C\pm\frac{K_1}{\theta_1}\frac{\lambda}{4\pi}.$$

The deviation from the mean line $r\theta=C$ is proportional to

$$\frac{K_1}{\theta_1}=\left\{\frac{a}{\theta\,(2r+a\theta)}\right\}^{\frac{1}{2}}$$

As either $r$ or $\theta$ increases, this will decrease. The "amplitude" of these crinkles therefore gets smaller and smaller as $r$ and $\theta$ increase, and the crinkles vanish at sufficiently large distances from the cylinder.

The shape of the lines of flow very near the surface of the cylinder is of special interest. The energy which comes crinkling down successive loci of maxima and minima of illumination, when it reaches the first maximum, flows down in a smooth curve which will

meet the mean line of flow only at infinity. The energy does not crinkle along after it has crossed the first maximum of illumination.

It is thus seen that the introduction of a perfectly reflecting cylinder into a field through which plane light waves are passing, has the effect of (1) altering the general direction of flow of energy, and (2) giving a crinkled microscopic structure to the energy current at any point. But we have still to seek an explanation as to how a flow of energy in the manner described above leads to the actual distribution of maxima and minima of illumination in the field. The current of energy at any point is by (19)

$$\overline{S}=\frac{c}{8\pi}\left(1-2K\cos\frac{n}{c}2r\theta^2+K^2\right)$$

and this varies from point to point along each line of flow, being maximum and minimum respectively where it cuts successive curves $2r\theta^2=m\lambda/2$. This variation of the current of energy along its own line of flow can be explained only as due to the change in cross-section of the tube of flow formed by two lines of flow close to each other. The conception of energy as flowing through tubes must therefore give us a better idea of what happens in the field.

The curves which are at every point normal to the lines of flow, *i.e.*, the curves analogous to equipotential curves, can easily be obtained. For these curves,

$$\tan\varphi'=-\frac{1-2K\cos\frac{n}{c}2r\theta^2+K^2}{2\theta\left(K^2-K\cos\frac{n}{c}2r\theta^2\right)} \qquad \text{[cp. } eqn. \text{ (2)]}$$

The differential equation in terms of $\theta$ and $r$ is found to be

$$(dr-ad\theta)+\frac{2K}{1+K^2}\cos\frac{n}{c}2r\theta^2\left[(a-2r\theta)\,d\theta-dr\right]=0.$$

In the immediate neighbourhood of a point, we can as before leave out all variations other than the periodic one, and integrate. Then we get

$$r-a\theta+\frac{2K_1}{1+K_1^2}\sin\frac{n}{c}2r\theta^2\left[\frac{a-2r_1\theta_1}{4r\theta\frac{n}{c}}-\frac{1}{2\theta^2\frac{n}{c}}\right]=\text{const.} \quad (27)$$

which cuts the successive loci of maxima and minima of illumination at points lying on the curve

$$r-a\theta=\text{const.} \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

which is therefore the mean curve about which the actual curves crinkle round. In the rectangular co-ordinates $(x, y)$, equation (28) becomes

$$y=\text{const.}$$

*i.e.*, the mean curves are straight lines parallel to the to the $x$-axis. The curves are shown in Plate IV, in relation to the lines of flow and to the loci of minimum illumination. Suppose one of these curves cuts two successive minima loci at points A and B. Then by equation (27), there is one complete crinkle between A and B. Consider the tube of flow bounded by the lines of flow which pass through A and B. Since the flow of energy should be everywhere normal to the wavy curve AOB, energy is concentrated in the right half of the tube and 'rarefied' in the left half. If we draw the line of flow passing through a point O, midway between A and B, we see that we can conceive of the tube AB as made up of two tubes each of which widens and contracts periodically, and one of which is so shifted relatively to the other, that the broadened part of one falls by

the side of the narrow part of the other. We see also from Plate IV, that the contracted parts of successive tubes lie along the loci of max. illumination.

We may now take into account the diffraction at the edge of the cylinder and obtain the correction to our results necessary for points below the plane $y=0$. Using Sommerfeld's expression as we have done before, the electric intensity at a point P′ (fig. 1.) due to this alone is from (6)

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{r'}}\cos\left\{\frac{n}{c}(r'-ct)+\frac{\pi}{4}\right\}\left\{1-\frac{1}{\sin\frac{\alpha}{2}}\right\}$$

$$=-\frac{1}{2\pi\alpha}\sqrt{\frac{\lambda}{r'}}\cos\left\{\frac{n}{c}(r'-ct)+\frac{\pi}{4}\right\}$$

since $\alpha$ is small; $r'=\mathrm{P'O}$.

Let $\mathrm{P'Q}=r$, and $\mathrm{P'M'}=r_1$.

Then $r'=(r_1-a\theta)\sec\alpha=(r_1-a\theta)\sec 2\theta$

$$=-a\theta+r_1(1+2\theta^2) \text{ approximately.}$$

$$\therefore r'+a\theta=r_1(1+2\theta^2).$$

$$\alpha=\frac{2r\theta}{r-a\theta}.$$

If E and H be the resultant electric and magnetic intensities at P′,

$$\mathrm{E}=\cos n\left(t-\frac{r_1}{c}\right)-\mathrm{K}\cos n\left(t-\frac{r}{c}\right)$$
$$-\frac{1}{2\pi\alpha}\sqrt{\frac{\lambda}{r'}}\cos\left\{n\left(t-\frac{r'+a\theta}{c}\right)-\frac{\pi}{4}\right\}$$

and $\mathrm{H}=\cos n\left(t-\frac{r_1}{c}\right)-\mathrm{K}\cos n\left(t-\frac{r}{c}\right)$

$$-\frac{1}{2\pi\alpha}\sqrt{\frac{\lambda}{r'}}\cos\left\{n\left(t-\frac{r'+a\theta}{c}\right)-\frac{\pi}{4}\right\}$$

If $\mathbf{k}_1, \mathbf{k}_2. \mathbf{k}_3$, be unit vectors measured along $r_1$, $r$ and $r'$ respectively, the time-mean of the Flow of energy is found to be given by

$$\frac{8\pi\bar{S}}{c}=\mathbf{k}_1\left[1-K\cos\frac{n}{c}2r\theta^2-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)\right]$$
$$+\mathbf{k}_2\left[K^2-K\cos\frac{n}{c}2r\theta^2+KK'\cos\frac{\pi}{4}\right]$$
$$+\mathbf{k}_3\left[K'^2-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)+KK'\cos\frac{\pi}{4}\right]$$

K′ being written for $\frac{1}{2\pi\alpha}\sqrt{\frac{\lambda}{r}}$.

Resolving the vectors along and perpendicular to the direction of the direct rays, we get

$$\left.\begin{aligned}\frac{8\pi}{c}\bar{S}_x&=1+K^2+K'^2-2K\cos\frac{n}{c}2r\theta^2-2K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)\\&\qquad+2KK'\cos\frac{\pi}{4}.\\\frac{8\pi}{c}\bar{S}_y&=2\theta\left\{-K\cos\frac{n}{c}2r\theta^2-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)\right.\\&\qquad\left.+2KK'\cos\frac{\pi}{4}+K^2+K'^2\right\}\end{aligned}\right\}29$$

where $\theta^2$ is neglected in comparison with unity and $\alpha$ and $\theta$ are considered to be the same in the small term $\bar{S}_y$. Since K, K′ are each small in the present problem, terms involving their products and squares can be neglected. The expression for $\tan\varphi$ may then be written

$$\tan\varphi=\frac{\bar{S}_y}{\bar{S}_x}=\frac{2\theta\left\{-K\cos\frac{n}{c}2r\theta^2-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)\right\}}{1-2K\cos\frac{n}{c}2r\theta^2-2K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)}\qquad(30)$$

From equation (20)

$$\frac{d\theta}{dr}(2r-a\theta)+2\theta=\left(1-a\frac{d\theta}{dr}\right)\tan\varphi$$

and (30) becomes on reduction

$$\frac{d\theta}{dr}=-\frac{\theta}{r}\cdot\frac{1-K\cos\frac{n}{c}2r\theta^2-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)}{1-2K\cos\frac{n}{c}2r\theta^2-2K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)}\quad\ldots\ (31)$$

Now $K\cos\frac{n}{c}2r\theta^2+K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)$

$$=D\cos\left(\frac{n}{c}2r\theta^2+\epsilon\right)$$

if $D^2=(K+K'/\sqrt{2})^2+\frac{K'^2}{2}=K^2+K'^2+KK'.\sqrt{2}$

and $\tan\epsilon=\frac{K'}{K'+K\sqrt{2}}$.

Then (30) may be written

$$\frac{d\theta}{dr}=-\frac{\theta}{r}\cdot\frac{1-D\cos\left(\frac{n}{c}2r\theta^2+\epsilon\right)}{1-2D\cos\left(\frac{n}{c}2r\theta^2+\epsilon\right)}.\quad\ldots\ (32)$$

In the neighbourhood of any point, D and $\epsilon$ can be regarded as constant and equation (32) integrated.

$$r\theta=C-\frac{D_1}{\theta_1}\sin\left(\frac{n}{c}2r\theta^2+\epsilon\right)$$

The "mean lines of flow" of energy are still given by

$$r\theta=C.$$

but the actual lines of flow which wind about them cut them along the curves

$$\frac{n}{c}2r\theta^2+\epsilon=m\pi\quad\ldots\quad\ldots\ (33)$$

instead of the curves $\frac{n}{c}2r\theta^2=m\pi$. It may be noted that it is equation (33) that determines the position of the maxima and minima of illumination when the effect of diffraction is also taken into account, so that the points of intersection of the lines of flow with the loci of points of max. and min. illumination still lie on the "mean lines of flow."

If the squares and products of K, are not neglected, it can be shown that the mean lines are given by the equation

$$r d\theta\,(1+K'^2+\sqrt{2}\,KK')+\theta dr=0.$$

$$\text{or}\quad r\theta^{1+\alpha}=\text{const.}$$

$$\text{where}\quad \alpha=K'^2+KK.'\sqrt{2}$$

It will be interesting finally to deduce the results for the case of diffraction at a straight edge from the above investigation. Putting $a=o$, K becomes zero also. We have then directly from equations (29)

$$\tan\varphi=\frac{2\theta\left\{K'^2-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)\right\}}{1+K'^2-2K'\cos\left\{\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right\}}$$

$$\text{and}\quad \frac{d\theta}{dr}=-\frac{\theta}{r}\cdot\frac{1-K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)}{1-2K'\cos\left(\frac{n}{c}2r\theta^2+\frac{\pi}{4}\right)}$$

an expression very similar to (21).

The "mean lines of flow" are given by $r\theta=$const. or in the rectangular co-ordinates used, by $x=$const. They are straight lines parallel to the Y-axis, the "mean direction" of energy flow being apparently not altered by the presence of the edge. The actual lines of flow cut the successive loci of points of max. and min. illumination (*i. e.*, the curves $2r\theta^2+\frac{\lambda}{8}=m\frac{\lambda}{2}$) at points which lie along the mean lines of flow. The shape of these lines of flow in the diffraction field due to a straight edge is indicated in the figure in Plate V, which has been drawn for $\lambda=0.5$ inch for convenience of representation.

## § 6. *Summary and Conclusion.*

The present paper deals with the distribution and flow of energy in the immediate neighbourhood of a perfectly reflecting cylinder on which plane light waves are grazingly incident in a direction at right angles to its axis. The following are the principal results which are indicated by theory, and have been verified by photometric study of the field.

(*a*) The positions of the maxima and minima of illumination are determined by eliminating $\theta$ from the pair of equations,

$$2y\theta + 3a\theta^2/2 = x\,;\ 2\theta^2\,(y + a\theta) = m\lambda/2$$

or from the pair of equations

$$2y\theta + 3a\theta^2/2 = x\,;\ 2\theta^2\,(y + a\theta) = m\lambda/2 - \epsilon\lambda/2\pi$$

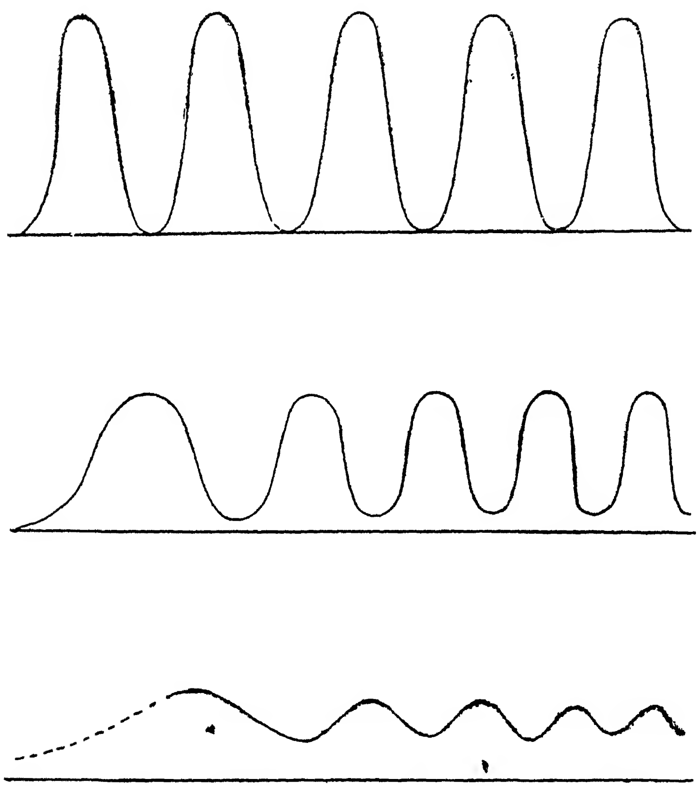
according as the plane of observation is in front of the cylindrical "edge" or behind it. $x$, $y$ are the co-ordinates of any point, the origin being the "edge" of the cylinder. $\epsilon$ is a small angle which for large values of $y$ becomes equal to $\pi/4$.

(*b*) The visibility of the fringes varies in an interesting manner with the position of the part of the field under observation. It is practically constant over the entire plane of observation when this coincides with the plane passing through the "edge," but falls off when it is moved further away from the source of light, the decrease being greatest for the regions farthest from the surface. When the part of the field under observation is between the "edge" and the source of light, the visibility of the fringes reaches the maximum value at the surface of the cylinder, falling off slowly as we recede from it.

(c) The loci of maxima and minima of illumination are given by $r\theta^2 = m\lambda/4$ and the "mean lines of flow" of energy are given by $r\theta$=const. and are for small values of $\theta$, less inclined to the direction of the incident rays than the former set of curves. The actual lines of flow crinkle about these mean lines; the "wave-length" of the crinkles increases as we move along the direction of the incident rays, and decreases as we move in a direction at right angles to it, away from the cylinder. The "amplitude" of the crinkles decreases as $r$ and $\theta$ increase, and vanishes at sufficiently large distances from the cylinder. A good conception of the actual phenomena is obtained, if we imagine energy as flowing through tubes which widen and contract periodically, the widened parts of successive tubes lying on the loci of minimum illumination, and the contracted parts on the loci of maximum illumination (see Plate IV). When the radius of the cylinder is very small, the results obtained are practically identical with those obtained in the case of diffraction by a straight edge.

No reference has so far been made to the phenomena noticed within the region of the geometrical shadow of the cylinder. The writer has made some preliminary observations on this subject, and hopes on a suitable opportunity to continue the work, which might prove of interest in relation to the general problem of the diffraction of electro-magnetic waves by cylindrical or spherical surfaces of large radius.

*October, 1917.*

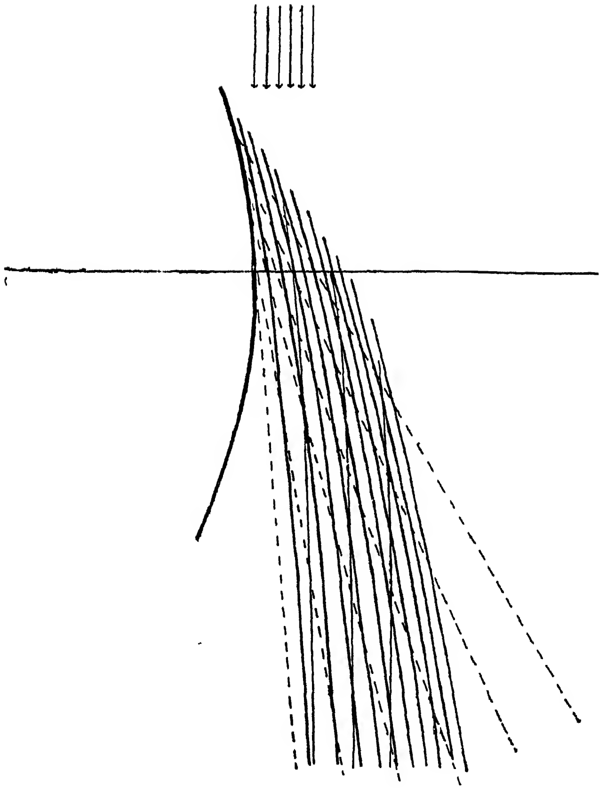


(a)

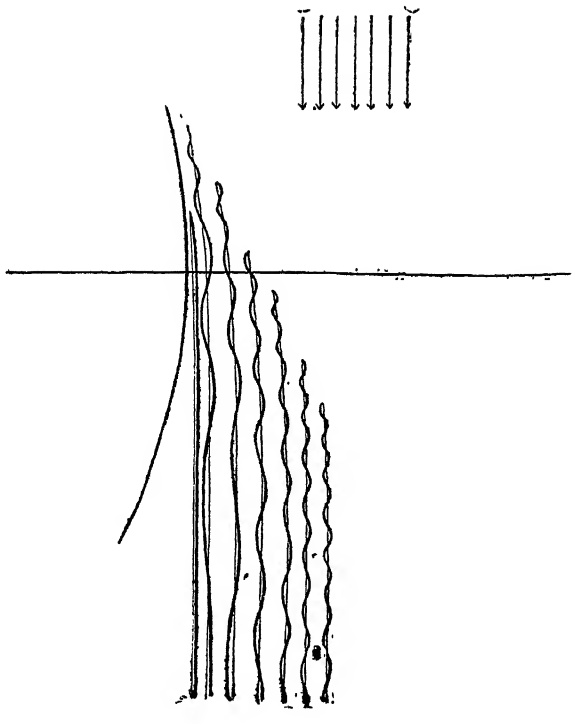
(b)

(c)

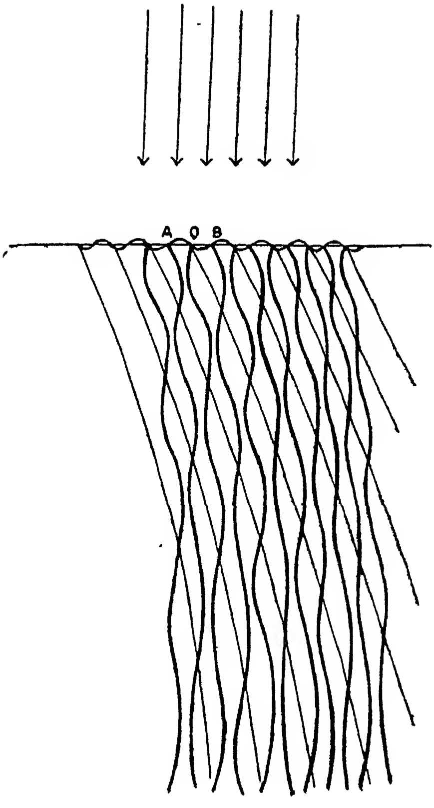
Illumination Curves.



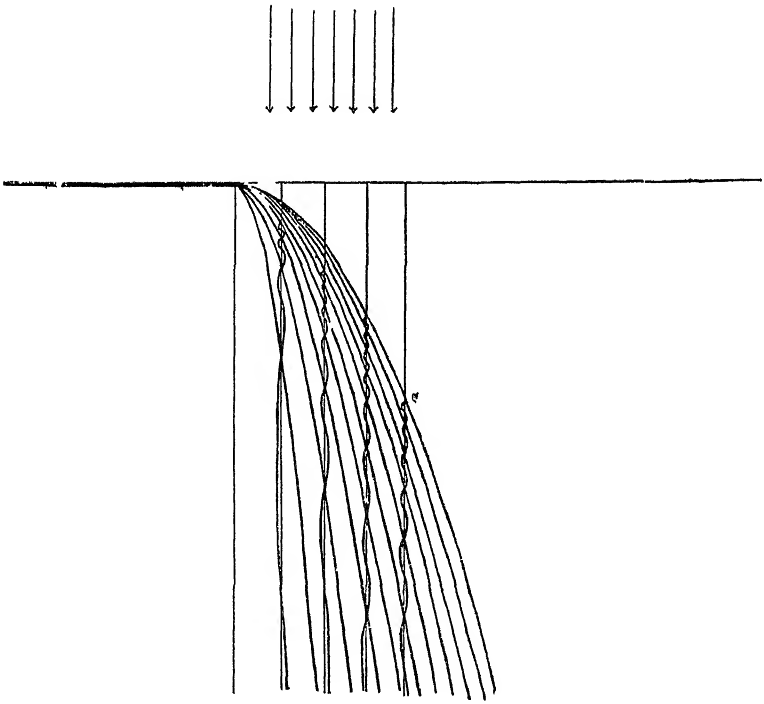
Heavy lines represent the loci of minimum illumination,
thin lines the "mean lines of flow" of energy,
and dotted lines ther eflected rays.



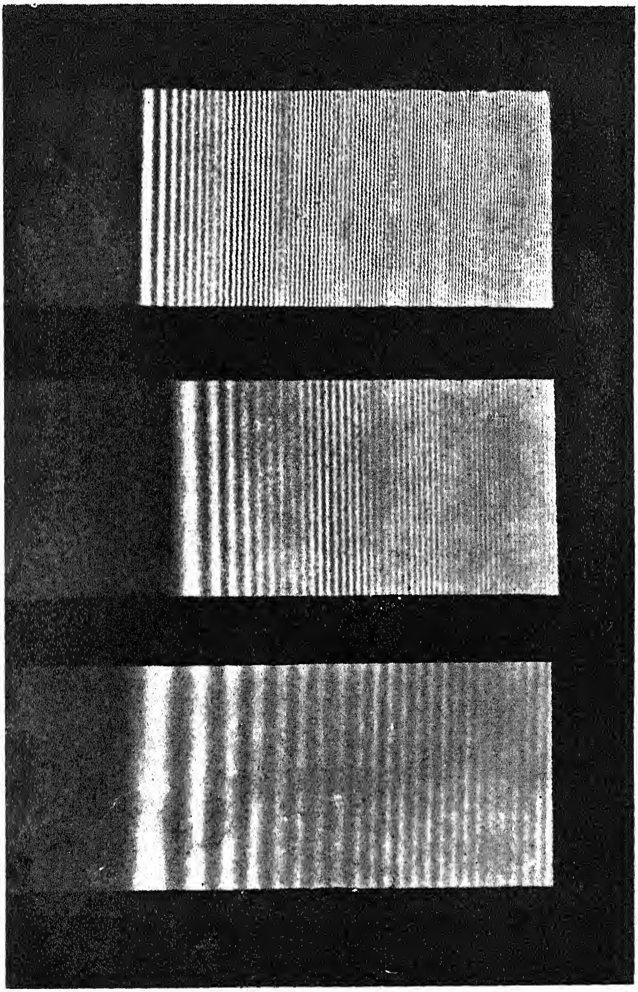
Lines of flow of energy in different parts of the field.




Microscopic structure of the field.



Lines of energy flow in the field due to diffraction at a straight edge



(a)

(b)

(c)

Illustrating the Interferences in the Field surrounding a Reflecting Cylinder, and the Decrease in the Visibility of the Fringes with Increasing Distances from the Cylinder.

PROCEEDINGS

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Vol. III. PART VI.

# On Resonance Radiation and the Quantum Theory.*

By T. K. CHINMAYANANDAM, B.A. (HONS.)

## § 1. *Introduction.*

The most fascinating problem, and the problem that is engaging the attention of the most eminent scientists of the world at the present day, is the problem of atomic structure. As has been expected for over a long time, the study of the radiation from the atom seems to be the best, and perhaps the only, clue to this problem; and already, as we know, the recent developments in the study of X-rays, and other high frequency radiations, have thrown quite a flood of light into this sub-atomic world. But there seems to be one difficulty, that crops up and mars our progress; if we knew the nature of radiation completely, our way would be easy in solving the problem of atomic structure. But some phenomena have come to light, such as the emission of electrons by X-rays, or ultra-violet rays, which seem to defy all attempts at an explanation on the classical spreading-wave

* Read at the Annual Science Convention, 23rd November, 1917.

theory of radiation. To explain these phenomena, which grow larger in number every day, the theory of radiation has to be entirely recast, and one such attempt and certainly the most important and promising attempt at such a recasting, is the Quantum theory of radiation. Thus the present day position comes out to be that these two problems are closely intertwined, and so are the solutions of those problems; and any promising attempt at the solution of either must be in very close relation with the other.

The Quantum theory of radiation postulates a discontinuity in the absorption and emission of radiations by a substance, and suggests, in fact, that the absorption or emission of a mono-chromatic radiation of frequency $\nu$ can take place only in discrete bundles or Quanta as they are called, of energy of amount $h\nu$ where $h$ is Plank's universal constant. In a series of papers in the Philosophical Magazine for 1913, Dr. Bohr has applied this theory to develop a conception of atomic structure, and has shown how the laws of spectral series can be accounted for in the case of Hydrogen, Helium, and also in a general way for other elements. He assumes an atomic model suggested by Sir E. Rutherford, *viz.*, a system with a nucleus of extremely small linear dimensions, and of posititive charge $Ne$ (where N is the atomic number of the element and $e$ the electronic charge) and N electrons revolving in concentric rings round the nucleus. Radiation is due to the reformation of a system, which has lost one or more electrons. This binding of the electrons, Bohr assumes, cannot take place in a continuous manner, but in a series of sudden

discontinuous jumps. If the orbits are for simplicity considered to be circular, his main assumption may be regarded as an atomicity of angular momentum, that is to say, the angular momentum of the electron must always be an ~~exact~~ multiple of $\frac{h}{2\pi}$ where $h$ is Plank's constant. The orbits which satisfy this condition are assumed to be non-radiating orbits, and the electron is supposed to emit a homogeneous radiation in passing from one such orbit to another, or from one "Stationary State" to another as they are called. If $W_1$, $W_2$ be the energy of the electrons in two such orbits, it can be shown that the amount of energy emitted $= W_2 - W_1$; and this Bohr equates to $h\nu$ where $\nu$ is the frequency of the radiation. In a simple case like Hydrogen where there is only one electron, $W_1$, $W_2$ can be easily calculated on the principles of ordinary mechanics; Bohr has thus got the formula

$$V = K\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}$$

where $n_1$, $n_2$ are integers and $K = \frac{2\pi^2 me^4}{h^3}$. This is of course the Balmer series for Hydrogen, and it may be pointed out that the calculated value of K agrees very closely with the observed value of the Rydberg constant, an agreement on which, in fact, depends much of the success of Bohr's theory. Bohr has not considered, in any great detail, the case of other elements, where the atom has a number of electrons, the case being complicated by the fact that the other electrons will have an effect, besides the nucleus, on the radiating electron. But he has suggested that in the permanent configuration of the atom, or

the configuration of maximum energy, the electrons will all be arranged in a few concentric rings round the nucleus. This is the broad outline of Bohr's theory of spectral series. It may also be pointed out, that Sommerfeld has in a recent paper in the Annalen Der Physik for 1916 has carried the theory further and has shown, how by a consideration of the ellipticity of some of the orbits of the radiating electron, we can explain the doublet and triplet structure of the lines in the spectral series of the elements.

It appeared to me a very important and fascinating problem, to try and apply the foregoing ideas, to the phenomena of Resonance Radiation discovered by Prof. R. W. Wood. The ideas developed by Bohr apply naturally to stimulation of the substance by electric discharge through its vapour; and as Prof. Wood remarked soon after his discovery of the phenomena, we have in Resonance Radiation, an entirely new method of stimulating a substance to emit radiation, and it would be highly interesting to consider whether Bohr's ideas would apply *en bloc* to this case as well. Some peculiar qualitative features of the phenomena, such for example, as the transformation of the Resonance spectrum into a band spectrum under certain conditions, seem to promise very much in throwing light on the problem of atomic structure.

### § 2. *On the Law of Spacing of the Resonance Lines.*

In a recent paper* in the Philosophical Magazine an attempt has been made by Dr. Silberstein, to explain the phenomena of Resonance Radiation, on the prin-

* Dr. Silberstin on *Fluorescent vapours and their magneto-optic properties* Phil., Mag., Sept. 1916.

ciples of classical mechanics. Dr. Silberstein considers, that the electron which emits the radiation, is a non-Hookean resonator *i.e.*, a system in which the restitutive force is not simply proportional to the displacement, the case being analogous to that of combinational tones in sound. Dr. Silberstein follows the analogy, and introduces a small term, depending not upon the square as in the case of sound, but upon the $p^{\text{th}}$ power of the displacement, where $p$ is nearly but not exactly equal to one, and thus writes his equation of motion as

$$\ddot{y} + n^2 y + \alpha y^p = e^{int}$$

The consequence of this assumption he finds to be, that the Resonance series should be characterized by constant frequency intervals. He gives in his paper a table, prepared from Wood's data, and thinks that his conclusions from the theory are supported by facts. But a critical examination of his own figures shows that the frequency intervals have a most decided tendency to decrease on the long wave-length side. There are indeed some irregularities, but it is probably due to the fact that he works out successive differences. Adopting a method of calculation, in which the effect of experimental errors is better minimised, I have prepared a table from Wood's data* for the frequencies of the components of the Resonance series of Iodine vapour, and I have found that it is not the frequencies themselves, but their square roots, that have constant intervals in this series.

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* Wood. Phil. Mag., **24**, p. 684.

TABLE.

| 1<br>Serial No.<br>$n$. | 2<br>Frequency.<br>$\nu_n$ | 3<br>$\frac{\nu_0 - \nu_n}{n}$ | 4<br>$\frac{\nu_0^{\frac{1}{2}} - \nu_n^{\frac{1}{2}}}{n}$ |
|---|---|---|---|
| | $10^{10}\times$ | $10^{10}\times$ | $10^{3}\times$ |
| 0 | 54937·5 | | |
| 1 | 54279·0 | 658·5 | 141·0 |
| 2 | 53646·9 | 645·3 | 138·5 |
| 3 | 53013·0 | 641·5 | 137·0 |
| 4 | 52386·0 | 637·8 | 137·7 |
| 5 | 51759·0 | 635·7 | 137·6 |
| 6 | 51144·0 | 632·3 | 137·3 |
| 7 | 50523·0 | 630·7 | 137·4 |
| 8 | 49911·0 | 628·2 | 137·3 |
| 9 | ... | ... | ... |
| 10 | 48696·0 | 624·1 | 137·2 |
| 11 | 48096·0 | 621·9 | 137·1 |
| 12 | 47493·0 | 620·4 | 137·2 |
| 13 | 46902·0 | 618·1 | 137·2 |
| 14 | ... | ... | ... |
| 15 | 45726·0 | 614·1 | 137·1 |
| 16 | 45147·0 | 611·9 | 137·0 |
| 17 | 44562·0 | 610·3 | 137·1 |
| 18 | 43983·0 | 608·6 | 137·4 |
| 19 | 43419·0 | 606·2 | 136·9 |
| 20 | 42855·0 | 604·1 | 136·9 |

I have shown in columns 3 and 4 the intervals of the frequencies and the intervals of their square roots respectively; the former are by no means constant, decreasing systematically by about 9% down the column, while the latter are, expcept for the first few lines, remarkably constant. The attempt to explain the phenomena, on the principles of classical mechanics is apparently, therefore, not quite successful even as regards the spacing of the Resonance lines, not to mention other aspects of the question not considered by Silberstein. I propose in the present paper to suggest an explanation of the phenomena on the Quantum theory.

## § 3. *On the Theory of the Stationary States.*

It may be pointed out at the very outset, that we cannot account for the law of spacing of the components in the Resonance series, on Bohr's simple theory of spectral series. For the law that is characteristic of his theory, is of the Balmer type, the variable factor being $\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ and the frequency interval between two successive components in the series will be proportional to $\left\{\frac{1}{n_2^2}-\frac{1}{(n_2+1)^2}\right\}=\frac{2}{n_2^3}$ approximately. This quantity varies with $n_2$, and the series will not represent anything like the resonance series, unless $n_2$ were as large as 400 or 500, in which case the orbit of the radiating electron must be of dimensions enormously large compared with molecular dimensions, being of the order $10^{-3}$ cm. It is apparent, therefore, that we must take into account the effect of the other electrons in the system on the radiating electron and consider

in greater detail the theory of the possible frequencies of radiations from an atomic system having a large number of electrons.

Bohr regards Light radiations to be due to electrons falling into a system which has lost one or more electrons from the outer ring. And as in the course of the reformation of the system it is extremely unlikely that two of the falling electrons are at any instant at the same distance from the nucleus, our problem becomes practically equivalent to determining the "stationary states" and the frequencies of the radiations of an electron in a system consisting of a few electrons revolving one in each ring round the rest of the molecular system. Let us also assume that any one of the electrons, not necessarily the outermost one, has a series of "stationary states" throughout the system, by passage from one to another of which, it can absorb or emit radiation of appropriate frequency.

On these general assumptions, we can derive a formula for the frequencies of the possible radiations from the system. Leaving out the radiating electron itself from reckoning, let us suppose that it lies at any instant between the orbits of the $(\tau_0-1)^{th}$ and the $\tau_0^{th}$ electrons (counted from the outer surface of the system). Then since an electron revolving in a circle, produces very little effect at a point inside its orbit, while for a point outside, the effect is nearly the same, as if it were placed at the centre of its orbit, the radial force on the electron, may be supposed to be due to an equivalent nucleus of charge S$e$ where

$$S=\tau_0+\phi(r) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$\phi(r)$ being the correction factor, which is a function

of $r$ the radius of the orbit. The function will be definite, if the configuration of the other electrons is given. The force on the electron may hence be written

$$F=\frac{S}{r^2}e^2 \qquad \dots \qquad \dots \quad (2)$$

Let $v$ be the velocity of the electron in its orbit. Then

$$\frac{mv^2}{r}=F=\frac{Se^2}{r^2} \qquad \dots \qquad \dots \qquad \dots \quad (3)$$

and since the angular momentum of the electron is a multiple of $\frac{h}{2\pi}$, $mvr=\frac{\tau h}{2\pi}$, where $\tau$ is an integer. From these equations, we get,

$$r=\frac{\tau^2 h^2}{4\pi^2 m}\cdot\frac{1}{Se^2}=\frac{k\tau^2}{S} \text{ (say).} \qquad \dots \quad (4)$$

If W be the energy of the electron in any of its "stationary states"

$$W=\tfrac{1}{2}mv^2=\frac{2\pi^2 e^4 m}{h^2}\cdot\frac{S^2}{\tau^2} \qquad \dots \quad (5)$$

so that the possible frequencies of the radiations from the system are given by

$$\nu=\frac{2\pi^2 e^4 m}{h^3}\left\{\frac{S_1^2}{\tau_1^2}-\frac{S_2^2}{\tau_2^2}\right\} \qquad \dots \qquad \dots \quad (6)$$

Even without knowing the exact nature of the function $\phi(r)$ in equation (1), it is obvious that the quantity S will increase as $r$ increases, for the repulsion of the inner electrons becomes less, and that of the outer increases. Hence it is evident that, since

$$\frac{S_{\tau_2}}{\tau_2^2}-\frac{S_{\tau_2+1}}{(\tau_2+1)^2}<S_{\tau_2}\left\{\frac{1}{\tau_2^2}-\frac{1}{(\tau_2+1)^2}\right\},$$

closer groups of lines are possible on this theory than on a simple series of the Balmer type. We may now consider what is likely to happen when light of a definite frequency $\nu$ excites the system; it will be

absorbed or, in general, will affect the system, if any one of the electrons in the system requires an amount of energy $h\nu$ to pass from any one of its stationary states to any other. Suppose for definiteness, that for the mercury green radiation this condition is satisfied by an electron in a stationary state between the orbits A and B of its neighbours (fig. 1), if it passes to

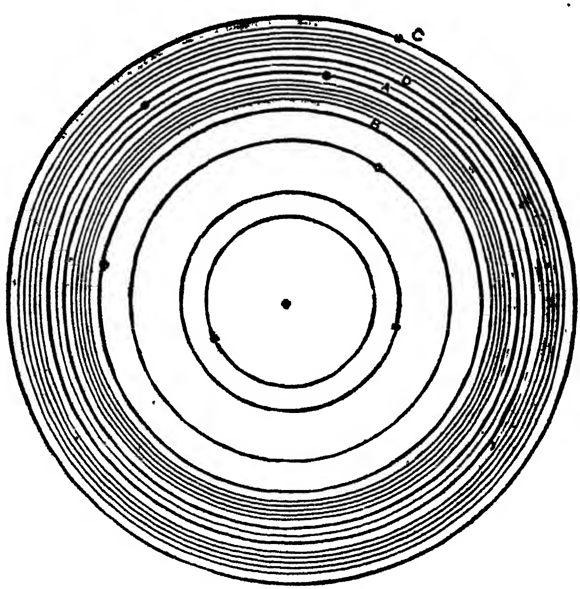

Fig. 1,

a stationary state between C and D orbits, of two other consecutive electrons. Then as soon as the light falls on the system, it will jump from the former to the latter state, absorbing the incident radiation. It then jumps back, emitting radiation. If it jumps exactly to its original orbit, the frequency of the emitted radiation will be the same as that of the incident one; but when it jumped, the configuration must have changed, so

that it is not likely that it jumps exactly to its original path. Let us assume that it jumps to one of the series of stationary orbits; then it is clear, that in the resulting radiation, there will be as many different components as the number of orbits. The frequency intervals of these components will obviously depend upon the difference in the function $\frac{S^2}{\tau^2}$, between the consecutive orbits. To determine the spacing of the lines in the emitted radiation, therefore, we must know more definitely the nature of the function $\frac{S^2}{\tau^2}$. We shall now work out, as an illustrative case, the nature of that func-

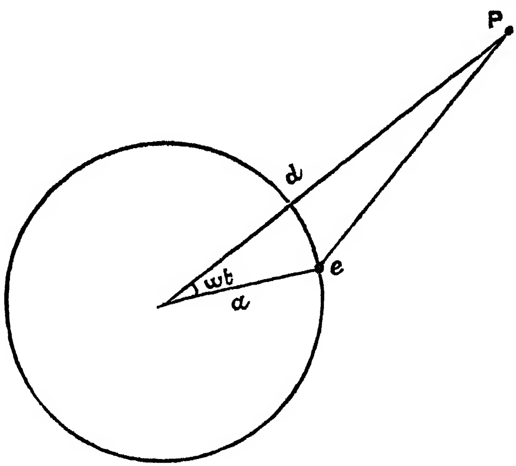


Fig. 2.

tion, on the assumption that the force exerted by a revolving electron is correctly represented by its time-mean value. Thus, if $f$ be the mean force due to an electron revolving in a circle of radius $a$, at a distance $d$ from its centre,

$$f=\frac{1}{T}\int_0^T \frac{e\,(d-a\cos\omega t)}{(a^2+d^2-2ad\cos\omega t)^{\frac{3}{2}}}\cdot\delta t.$$

If $d<a$

$$f=\frac{e}{2\pi}\frac{\partial}{\partial d}\int_0^{2\pi}\frac{1}{a}\left[P_0(\cos\theta)+\frac{d}{a}P_1(\cos\theta)+\ldots\right]\delta\theta$$

$$=-\frac{e}{d^2}\left(\frac{1}{2}\frac{d^3}{a^3}+\frac{9d^5}{16a^5}+\ldots\right)$$

If $d>a$

$$f=\frac{e}{a^2}\left(1+\frac{3}{4}\cdot\frac{a^2}{d^2}+\frac{45}{64}\cdot\frac{a^4}{d^4}+\ldots\right)$$

The function $\phi(r)$ becomes

$$\phi(r)=\sum_{\tau_0-1}^{1}\left(\frac{1}{2}\frac{r^3}{a^3}+\frac{9}{16}\cdot\frac{r^5}{a^5}+\ldots\right)$$

$$-\sum_{\tau_0}^{N}\left(\frac{3}{4}\cdot\frac{a^2}{r^2}-\frac{45}{64}\cdot\frac{a^4}{r^4}+\ldots\right).$$

where the summation extends in the first term over all the electrons outside the orbit of the radiating electron, and in the second term over the electrons inside it. Neglecting powers higher than the square, we may write

$$S=\tau_0-\sum_{\tau_0}^{N}\left(\frac{3}{4}\cdot\frac{a^2}{r^2}\right)$$

But from equation (4), $r=\frac{k\tau^2}{S}$, so that

$$S=\tau_0-\sum_{\tau_0}^{N}\frac{3}{4}\cdot\frac{a^2S^2}{k^2\tau^4}.$$

Denote $\sum_{\tau_0}^{N}\frac{3}{4}\cdot\frac{a^2}{k^2}$ by $c$.

Then $\frac{Sc^2}{\tau^4}+S-\tau_0=0.$

Hence $S=-\frac{\tau^4}{2c}+\left\{\frac{\tau^8}{4c^2}+\frac{\tau_0\tau^4}{c}\right\}^{\frac{1}{2}}$

$$S^2=\frac{\tau_0\tau^4}{c}+\frac{\tau^8}{2c^2}-\frac{\tau^4}{c}\left\{\frac{\tau^8}{4c^2}+\frac{\tau_0\tau^4}{c}\right\}^{\frac{1}{2}}$$

now $c=\Sigma\frac{3}{4}\frac{a^2}{k^2}$ and will be of the order $\Sigma\frac{3}{4}\cdot\frac{\tau_m{}^4}{m^2}$, if $\tau_m$ corresponds to the orbit of the $m^{th}$ electron, and

unless the electrons in the periphery of the atom are revolving in orbits extremely close to one another, $c$ will hence be smaller than $\frac{\tau^4}{\tau_0^2}$. The first term under the square-root will be larger than the second, and we may write

$$S^2 = \frac{\tau_0\tau^4}{c} + \frac{\tau^8}{2c^2} - \frac{\tau^8}{2c^2}\left(1 + \frac{2c\tau_0}{\tau^4} - \frac{2c^2\tau_0^2}{\tau^8} + \frac{4c^3\tau_0^3}{\tau^{12}} - \cdots\right)$$

$$= \tau_0^2 - \frac{2c\tau_0^3}{\tau^4} + \cdots$$

$$\frac{S^2}{\tau^2} = \frac{\tau_0^2}{\tau^2} - \frac{2c\tau_0^3}{\tau^6} + \cdots = y \text{ (say)}.$$

Now the frequency intervals between two successive components of the series of radiations must be proportional, on the present theory, to the difference in the value of this function $y$ in any two consecutive "stationary states." We can approximately obtain the latter by differentiating $y$ with respect to $\tau$. Thus

$$\frac{dy}{d\tau} = -\frac{2\tau_0^2}{\tau^3} + \frac{12c\tau_0^3}{\tau^7}.$$

Let us reckon from a line in the series which corresponds to the path $\tau_1$ and write for the successive lines $\tau = \tau_1 + n$. Then if $\nu_{n-1}$, $\nu_n$ be the frequencies of two successive lines in the series of radiations.

$$\frac{1}{K}\left(\nu_{n-1} - \nu_n\right) = \frac{2\tau_0^2}{\tau_1^3} - \frac{12c\tau_0^3}{\tau_1^7} - \frac{n}{\tau_1}\left(\frac{6\tau_0^2}{\tau_1^3} - \frac{84c\tau_0^3}{\tau_1^7}\right) - \frac{n^3}{\tau_1^2}\left(\ldots\right) \quad \ldots \quad \ldots \quad (7)$$

and can be written in the form

$$\nu_{n-1} - \nu_n = \alpha + \beta n + \gamma n^2 + \ldots$$

If the coefficient $\alpha$, $\beta$, $\gamma$ etc. are all exactly zero, the resonance series will be characterized by constant frequency intervals. It does not seem possible, without

making further assumptions to determine their magnitude absolutely. But though it is unlikely that $\alpha$, $\beta$, $\gamma$, all vanish, it can be seen that the successive coefficients will decrease rapidly. In the case of Iodine vapour, it was pointed out in the previous section that $\nu_{n-1}^{\frac{1}{2}}-\nu_n^{\frac{1}{2}}$ seemed to be constant. This relation is equivalant to

$$\nu_{n-1}-\nu_n=2\bar{\nu}_0\delta-2n\delta^2,$$

where $\delta=\nu_{n-1}^{\frac{1}{2}}-\nu_n^{\frac{1}{2}}$. Thus $\nu_{n-1}-\nu_n$ is of the form $(\alpha+\beta n)$. It is also possible to show that the quantities $\alpha$, $\beta$ given by equation (7) are of the right order. Thus in the case of Iodine vapour $\frac{\alpha}{K}$ is about $2\times10^{-3}$ and $\frac{\beta}{K}$ about $10^{-5}$. Hence we shall have

$$\frac{2\tau_0^2}{\tau_1^3}-\frac{12c\tau_0^3}{\tau_1^7}=2\times10^{-3}.$$

$$\frac{1}{\tau_1}\left\{\frac{6\tau_0^2}{\tau_1^3}-\frac{84c\tau_0^3}{\tau_1^7}\right\}=10^{-5}$$

Putting $\tau_0=2$, we get $\tau_1$ to be about 13, and the diameter of the orbit of the electron will be of the order of $10^{-6}$ cm., which does not seem improbable, when account is taken of the fact that the vapour must be at a very low pressure to emit Resonance radiation.

It may be pointed out here, however, that since the equation $\nu_{n-1}-\nu_n=\alpha+\beta n$ is more general than the equation $\nu_{n-1}^{\frac{1}{2}}-\nu_n^{\frac{1}{2}}=\delta$, it seems only accidental, on the present hypothesis, that the latter formula holds very approximately in the case of Iodine vapour. In the case of Sodium vapour it has been found by the writer, that while there is no approach to constant frequency intervals, even the expression $\nu_{n-1}^{\frac{1}{2}}-\nu_n^{\frac{1}{2}}$ gives

a systematic deviation on one side, so that this form has perhaps no special significance.

One point still remains to be noticed. In a Resonance series ordinarily as many as 20 lines are observed, and we cannot certainly assume that there are as many orbits between two consecutive electrons of the system, to which it can jump. This difficuity is easily got over, if we assume that it need not jump from the same orbit every time, but from one of a series of orbits between C and D (fig. 1). It is obvious, then, that we can account for as many as 25 lines in the resonance series with only 5 orbits in each series; suppose, for example, the interval of the function $\frac{S^2}{T^2}$ is constant over successive orbits in each series, but the interval in one series is 5 times that in the other; then, we shall have 25 lines in the resonance spectrum at constant frequency intervals. It seems at first sight unlikely that the resonance series of Iodine vapour is made up of smaller series patched up like this; for, if it is so formed, the patching up must be very nicely adjusted, so as to make the series indistinguishable from a single continuous series. But certain observations of Wood*, regarding the Resonance radiation of Sodium vapour, seem to bear out the suggestions that have been put forth here. Wood has found that, as we go away from the resonance line, new lines appear, which do not belong to the old series, and new series start from where the old ones leave off. This is exactly what should be expected, if in the above illustration considered, the interval of

* Prof. R. W. Wood. Phil. Mag., **25**, p. 588.

$\frac{S^2}{\tau^2}$ in one series of orbits is not an exact multiple of that in the other series. As a matter of fact, generally only 4 or 5 lines can be made out as forming one continuous series ; and the whole resonance spectrum is made up of bits like these. Even in the case of Iodine vapour, a grouping suggested by Wood* from a consideration of the structure of these lines, seems to give additional evidence to the suggestions offered here.

§ 4. *Some Characteristics of Resonance Radiation.*

Coming back to the mechanism of resonance radiation, the main point in the hypothesis suggested here is that soon after the external radiation has begun to excite the system, if not even before, in the permanent configuration of the system itself, the electrons in the periphery of the atom are revolving one in each ring round the nucleus, and one particular electron absorbs and emits light by passing from one of a series of "stationary states" between two consecutive electrons to another series of states between another pair of consecutive electrons. The frequency intervals between successive components in the emitted radiation are proportional to the intervals of the function $\frac{S^2}{\tau^2}$ over consecutive "stationary states" in the same series, and, as is evident from the analysis given in the previous section, they will depend upon the configuration of the other peripheral electrons, both relative to themselves and relative to the nucleus. It is not to be expected, *a priori*, that this configuration will be the

* Wood. Phil. Mag., **26**, p. 841.

same for all the molecular systems, and hence each molecular system may, in general, be expected to give its own Resonance series. We can more easily conceive, therefore, of the system giving a band-spectrum than a line-spectrum. Experimental evidence seems generally to support this conclusion, Resonance spectra being exhibited by very few substances, and by these, only under particular conditions of temperature and pressure ; and they are either destroyed altogether, or give place to the band-spectrum under slight disturbing causes. But the fact that it does occur under some conditions, points to the conclusion that there are some stable or quasi-stable configurations which are characteristic of the substance and hence assumed by the falling electrons in all the systems at once. All configurations very near these quasi-stable configurations lead to these latter configurations, though a configuration very different from them may not do so, the case being perhaps something like that of the configurations that may be assumed by a system of floating magnets.

On this view, we find an easy interpretation of the fact that the resonance spectrum of Iodine vapour, is transformed into a band-spectum, on rarefying the vapour; that is to say, that the same Iodine vapour, when it is at a lower preasure, gives under the same monochromatic excitation, an almost continuous spectrum, instead of the characteristic line spectrum. Wood*, who discovered this phenomenon, gives, however, a different explanation. He thinks that the band-spectrum is always produced, but is absorbed by the fluorescent gas itself when it is dense. He cites

* Wood. Phil. Mag., Nov. 1913.

as an evidence for this, an experiment tried by him, which was to pass the radiation from the rarefied vapour, through another tube containing cool iodine vapour. He found that the band-spectrum was completely absorbed, the resonance lines alone being left out. I will show presently, that the result of this experiment is not inconsistent with the views I have tried to develop. It may be pointed out, meanwhile, that apart from some objections that may be raised against his conclusions, his hypothesis does not lead us very far, and leaves it for further explanation—why from an almost continuous spectrum, only certain lines should be left out unabsorbed. Wood has indeed realised the difficulty, and has raised the question whether the same molecule emits both the resonance and the band-spectra, or only one, and whether in the vapour there are simultaneously systems emitting the two spectra independently. But the essential point about it seems to me to be, that it does not throw any light on the occurrence of the band-spectrum under apparently totally different conditions. Wood and Frank* have discovered that the resonance spectrum of Iodine vapour is transformed into a band-spectrum by the introduction of the gas Helium at a very low pressure. Indeed, the discovery was prior to the one previously mentioned; and Wood says nothing more on it than that it is due to the collision of the Helium molecules with the Iodine molecules. He regards the phenomena as a new effect of molecular collisions on the radiations from an atomic system, and has left it as a problem for the theoretical

* Wood and Frank. Phil. Mag., Feb. 1911.

physictsts to solve. The problem, so far as I know, has not been solved; but Wood has tried to find out whether the fact that Helium has a small affinity for electrons, has anything to do with its behaviour. The facts that have been observed, may be briefly set forth here and an explanation then suggested. Electro-negative gases, that is, gases which have strong affinity for electrons, completely destroy the fluorescence. Gases which have a weak affinity for electrons, like Helium, transform the line-spectrum into a band-spectrum, but this effect does not vary inversely as the affinity for electrons, for Neon, which has a much smaller affinity for electrons than Helium, shows scarcely so marked an effect as Helium in transforming this spectrum. The explanation is briefly this. Electro-negative gases pull out the outer electrons in the Iodine molecule (which are of course those responsible for the resonance spectrum), pull them either into their own molecules or at least so far away from the nucleus as to destroy the condition of absorption of the incident light. Gases like Helium with less affinity for electrons, do not pull these outer electrons so far as to destroy the absorption of the incident light, but far enough to destroy the quasi-stable configurations described above. Gases with absolutely no affinity for electrons, apparently do not produce any effect on the Iodine molecules or on the radiation from them.

I may now explain why the resonance lines are not absorbed, while the band-spectrum is absorbed * when passed through a tube containing cool Iodine vapour.

* See Wood. Phil. Mag., Nov. 1913, p. 838.

The radiation of the resonance lines depends upon a particular quasi-stable configuration assumed by the system, under the action of the exciting light, and naturally, on our theory, can be absorbed by the system only when in that configuration. On the other hand, the band-spectrum is due to molecules in varying configurations, more or less distributed at random, and the vapour in its ordinary state will also generally contain such systems with no unique configuration for all the molecular systems. It is only to be expected, therefore, that the band-spectrum should be absorbed. This explanation is indeed analogous to that given by Bohr* for the fact that Hydrogen does not exhibit absorption lines corresponding to all its radiation frequencies.

We may now pass on to consider the structure of these resonance lines, and also the companion lines that are found in large number distributed throughout the spectrum. As it has been pointed out, it seems very unlikely that under any conditions the configurations of all the molecular systems are the same, though in the case of certain quasi-stable configurations at least most of the molecules must have the same configurations if the substance can emit a line spectrum. But even in the latter case, one would naturally expect some systems to remain "out of tune," and it is suggested here that the companion lines observed are due to these systems which are "not in tune" with the rest. A great point in favour of this suggestion is that most of these lines are faint in comparison with the other lines, which is just what

* Bohr, Phil. Mag., July 1913.

we should expect, since only a few systems are supposed to be emitting them. Further, even in the other systems, if the adjustment is not perfect we should expect the resonance lines to be not quite homogeneous. Wood has found that the structure of these resonance lines is in fact very complex, the number of components in each line observable seeming to depend only on the resolving power used. He considers this to be due to the complex structure of the exciting mercury green line, but as he himself says, the structure of the resonance lines is much more complex, so that each exciting line gives rise to more than one line in the resonance lines or line-groups, as we may call them. Indeed, some of his observations on the complicated effects of changing the structure of the lines, lead us to doubt whether the phenomena of resonance radiation can ever be satisfactorily explained on the principles of classical mechanics.

Reference may now be made also to the point, in what way, if at all, resonance radiation is simpler than radiation excited by other means. Since we use a mono-chromatic exciter, Wood thought that in resonance excitation, we were, as we may call it, striking a single key in the key-board of the atom; and the comparatively simple structure of the resonance spectrum seemed to give weight to this conjecture. But we now know that the emitted radiation is so simple, only under certain conditions; while, under other conditions, the same mono-chromatic exciter gives rise to a band-spectrum. On the views set forth in this paper, the simplicity of the resonance

spectrum is due not to the nature of the exciter but to the nature of the system, to the fact that it has some quasi-stable configuration of the nature described already. In fact, the assumption of such configurations is necessary, even in the case of electrical excitation; Hick's* four "Sequences," for example, require four such definite configurations.

## § 5. *Röntgen Spectra.*

We may now pass on to consider another set of phenomena, which we may term also resonance radiation; when X-rays fall upon a substance, they excite it to something like resonance, when their frequencies are near certain values characteristic of the substance. The resulting radiation from the substance is termed, as we know, the Characteristic radiations, and consists of at least three series of lines, the K, L and M series as they are called. In an epoch-making discovery, Moseley has shown that the characteristic radiations from different elements were related in a simple manner to their relative positions in the periodic table of elements, and more definitely that the frequencies of the K-Characteristic radiations of the elements are represented by the equation

$$\nu = K(N-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

where N is the atomic number and K is the Rydberg constant for Spectral series. He wrote it in this form to bring out the analogy with the Balmer formula, and tried to interpret the formula on Bohr's theory by supposing that the radiations were due to the vibra-

* Hicks, Proc. Roy. Soc., **83**, p. 226.

tions of a ring of four electrons, immediately surrounding the nucleus. But it has been pointed out by Nicholson * and Bohr †, that this is not the correct interpretation, since we will have to assume that several quanta are emitted at the same time. Ishiwara‡ has pointed out that the lines in the Röntgen spectra can be accounted for, if Bohr's spectral formula be altered to

$$\nu = K\left\{\frac{(N-e_1)^2}{(n_1+\mu_1)^2} - \frac{(N-e_2)^2}{(n_2+\mu_2)^2}\right\}.$$

Remembering that the region of excitation in the molecular system is now the innermost ring of elec-

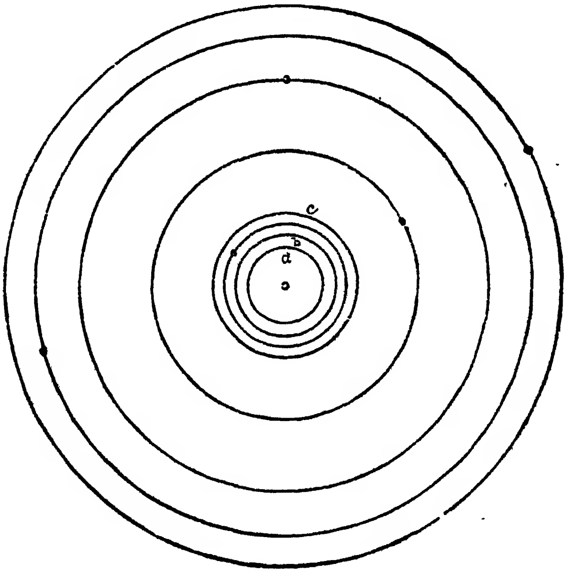


Fig. 3.

* Nicholson, Discussion on the Structure of the Atom, March 19, 1914. Proc. Roy. Soc., vol. 90.

† Bohr, On the structure of the Atom and the Quantum Theory of Radiation, Phil. Mag., Sept. 1915.

‡ Jun Ishiwara, Proc. of the Tokyo Math. Phys. Soc., Ser. 2 vol. 9, p. 160. (July 1917).

trons instead of the outermost, this form can be derived from the general formula obtained in this paper for the frequencies of the radiations from an atomic system, by giving a suitable value to the function $\phi(r)$. A detailed conception of the mechanism of emission of these radiations is also possible. The electron which emits the radiations (very probably the first from the nucleus) is supposed to be in one of its stationary states *b* (fig. 3) inside the orbit of the second electron. Under proper excitation, it is supposed to jump out to an orbit *c* just outside the second electron absorbing energy, and then back again to an orbit near its previous one emitting radiation. The $L_\alpha$ radiation, I suggest, is emitted when it jumps from orbit *c* to orbit *b*; the $K_\alpha$, from orbit *b* to orbit *a*, and the $K_\beta$, from orbit *c* to orbit *a*. Then it evidently follows

$$\nu_1 = \nu_2 + \nu_3$$

if $\nu_1, \nu_2, \nu_3$, be the frequencies of the $K_\beta$, $K_\alpha$, and $L_\alpha$ radiations of an element, respectively. This is, of course, Kossel's relation. It is also seen, if we assume that the electron is in orbit *b* to begin with, why $L_\alpha$ can be emitted without the K-radiations at the same time being emitted, while $K_\alpha$ cannot be emitted without $K_\beta$; and also why the K-radiations are necessarily accompanied by L-radiations. The other lines in the Röntgen spectrum can similarly be accounted for by considering some more orbits of the radiating electron.

The secondary corpuscular radiation, which is found to invariably accompany the emission of Characteristic radiations, may be supposed to be due to the collision

of the radiating electron with the second electron in one of its passages across its orbit, and the consequent dislodgment of the latter. One remarkable result will follow from this supposition, that the velocities of the corpuscular rays should be the same for both the K and L Characteristic radiations. This has really been found to be the case by Barkla and Shearer*, who have noted it as a remarkable fact in view of the general supposition, that the radiations have their origin in two different rings of electrons.

## § 6. *The general law of Spectral Series.*

Before concluding the paper, I will show you how from the atomic structure suggested in this paper a general formula for spectral series can be obtained taking into account all possible regions of excitation. As it has been shown, the frequencies of radiation from an atomic system are given by

$$\nu = K\left\{\frac{S_1^2}{\tau_1^2} - \frac{S_2^2}{\tau_2^2}\right\}$$

Now $S = \tau_0 + \phi(r)$ where $r$ is the radius of the orbit of the radiating electron, and

$$r = \frac{k\tau^2}{S}.$$

so that

$$S = \tau_0 + \phi\left(\frac{k\tau^2}{S}\right),$$

on solving which, we may express S as an explicit function of $\tau$ of the form

$$S^2 = \psi_r(\tau)$$

the suffix being introduced to take account of the fact

* Barkla and Shearer, "On the Velocity and electrons expelled by X-rays," Phil Mag, Dec. 1915.

that the function will change in passing from between one pair of consecutive electrons to another. Thus

$$\nu = K\left\{\frac{\psi_r(\tau_1)}{\tau_1^2} - \frac{\psi_s(\tau_2)}{\tau_2^2}\right\}.$$

This is of the same form as Ritz and Rydberg's general formula, though, as has been indicated, it also represents lines in high frequency spectra.

One point seems worthy of notice. An examination of any general law of spectral series shows us that the functions $\psi_r$ and $\psi_s$ are in actual cases found to be different; in Hicks law of spectral series, for instance, the lines in the spectra are never got from the same "sequence," but from two different sequences, the integral parameter in one of which is given a constant value while that in the other is allowed to vary. This would indicate that the radiations are due to an electron jumping into an orbit between two consecutive electrons in the system from one of a series of orbits outside the two electrons; and it may be remembered that we made the same assumptions in explaining Resonance radiation. It is indeed difficult to explain the fact stated here, if we consider light radiation to be due to the outermost electron, or rather to an electron which remains always the outermost in the system.

In conclusion, it may be pointed out that only the broad outlines of the subject have been touched upon in this paper, so as just to show what light the hypothesis suggested here throws upon the observed phenomena of radiation. The further development, especially with reference to quantitative relationships, is a matter for subsequent investigation.

*December 15, 1917.*

PROCEEDINGS

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Vol. III. PART VII.

# Equilibrium between Copper salts and Mercury in presence of Chloridion and Bromidion.

BY JNANENDRA CHANDRA GHOSE, M.Sc.

The replacement of one metal by another in acqueous solutions is capable of exact theoretical treatment. Here the potential difference between the metal and the solution which is given by the equation $-\frac{RT}{nF}\log\frac{C}{c}$ is the determining factor in the reaction. $-\frac{RT}{nF}\log C$ is constant for each metal and is called the normal electrode potential $E_n$. A metal, *A* replaces, is in equilibrium with, or is replaced by another metal *B* according as

$$(E_n)_A-(E_n)_B >=< \frac{RT}{F}\left[\frac{(\log c_a}{n_a}-\frac{\log c_b}{n_b}\right]$$

This conception can also be extended to the action of oxidising agent on metals. $(E_n)_B$ in the above equation means, in this case, the potential difference existing between the oxidising agent and its products of reaction, when the concentration of each of these is normal; and the term $\log C_b$ is $\Sigma n \log C$, *i.e.*, log of the ratio of conc. of the oxidising agent to that of its pro-

duct of reaction. In general cases, the reaction proceeds entirely in one direction for $(E_n)_A - (E_n)_B$ is often large enough to make one of the terms $C_a$ or $C_b$ vanishingly small in comparison with the other.

It is well known that copper salts in presence of hydrochloric acid is reduced by mercury[1]. It has however been found out that in the case of the reaction

$$2CuCl_2 + 2Hg = Hg_2Cl_2 + 2CuCl$$

the cupric salt is not completely reduced to the cuprous state but an equilibrium sets in at a definite measurable concentration of cupric ion. Applying the Law of Mass action to the reaction

$$2\dot{\dot{C}u} + 2Hg = \dot{\dot{H}g_2} + 2\dot{C}u \text{ we get}$$

$$K = \frac{[C\dot{C}u]^2}{C_{Hg_2} \times [C\ddot{C}u]^2} = \frac{[C\dot{C}u]^2 \times [C_{Cl'}]^4}{S_1 \times S_2^2} = \left[\frac{[C\dot{C}u] \times [C_{Cl'}]^2}{\sqrt{S_1} \times S_2}\right]^2$$
$$= \frac{K_1^2}{[\sqrt{S_1} \times S_2]^2}$$

where $S_1$ is the solubility product of $Hg_2Cl_2$ and $S_2$ the solubility product of CuCl.

In the reaction between mercury and cupric salt in presence of bromidion an equilibrium also sets in at a measureable concentration of cupric salt and here also we expect the equation

$$K = \frac{K_2^2}{[\sqrt{S_1'} \times S_2']^2} = \left[\frac{[C\ddot{C}u] \times [C_{Cl'}]^2}{\sqrt{S_1'} \times S_2'}\right]^2$$

where $S'_1$ is the solubility product of $Hg_2Br_2$ and $S'_2$ that of CuBr, to hold good.

Experimental procedure :—Samples of pure $CuSO_4$, CuBr, KCl and KBr were prepared in the laboratory. $\frac{N}{10}$ solutions of KCl and KBr were prepared, as also

(1). Trans. Chem. Soc. 1911, P. 1415 by Borrar.

stock solutions of $CuSO_4$, $CuCl_2$ and $CuBr_2$ whose strength was measured by means of a standard thio-sulphate solution. A mixture of [$CuCl_2$ − KCl], [$CuBr_2$ − KBr], [$CuSO_4$ − KCl] or [$CuSO_4$ − KBr], as the case may be, of exactly known composition was poured into a stout clean bottle having a capacity of 125 c.c. A suffiicent amount of pure mercury was then put into the bottle and the whole vigorously shaken. In preliminary experiments, the reaction was carried out with air inside the bottles, but as will be shown in Table 2, values of K obtained from several experiments did not agree with one another. It was thought possible that in the presence of an inert gas like $CO_2$ better results could be obtained. Pure dry $CO_2$ was, therefore, bubbled through the solution in the bottle for about 15 minutes before mercury was poured in. The bottles were then quickly stoppered with a rubber cork and sealed by means of wax. They were vigorously stirred in an electrically driven shaker for a period of $2\frac{1}{2}$ to $3\frac{1}{2}$ hours. The bottles were then taken out, and after the precipitate had completely settled down, 25 c.c. of the solution were drawn out and poured into a strong solution of KI. The solution was then titrated with the standard thio-sulphate solution. The concentration of the halogen ion was determined by titration with $AgNO_3$ solution. The colour of $\dot{\dot{Cu}}$ ion does not interfere with the determination of the end-point of titration in dilute solutions. For very small concentration of halogen ions, direct determination of concentration is rather difficult. It is possible however to control the concentration of halogen ion thus obtained, by calculating its values

from the concentration of $\dot{\dot{Cu}}$ ions[1]. Duplicate samples were prepared in all cases, one of which was shaken for at least an hour longer than the other; and the results were accepted only when the two samples gave concordant results.

It is of fundamental importance to ascertain whether a real equilibrium takes place. That we get a constant value for the expression $\Sigma n \log C$ when the reaction proceeds from either side is the only convincing proof that a real equilibrium has set in. To determine this, a large quantity of pure $CuCl_2$ solution was shaken with a sufficient amount of Hg for several hours and the final strength of $\dot{\dot{Cu}}$ and Cl′ ion determined. Here we have the reaction proceeding from left to right. The supernatant liquid was then decanted off and a sufficient amount of pure water put inside the bottle, which now contained Hg, $Hg_2Cl_2$ and CuCl. This bottle was vigorously shaken and the final strength of $\dot{\dot{Cu}}$ and Cl′ ion was found to be the same.

For $\dot{\dot{Cu}}$ we get 0·0242 gram mol. per litre.

For Cl′ we get 0·0484 gram mol. per litre.

Determination of the cuprous-cupric potential:—According to theory, in the reaction $2\dot{\dot{Cu}} + 2Hg = \dot{\dot{Hg_2}} + 2\dot{Cu}$, a real equilibrium exists only when the reacting ions have assumed such concentrations that the cuprous-cupri electrode potential becomes

(1) The calculations are made thus:—

If x and y be the initial molar contentrations of Cü and Cl′ ions and if x′ be the final concentration of Cu ions, then the amount of Cl ions that has passed out of solution is given by $2(x-x')$. Therefore the final concentration of Cl′ ion is $y-2(x-x')$.

equal to the potential difference existing between mercurous ion and mercury. It was thought interesting to verify this relation experimentally. A potential vessel containing a large platinised platinum electrode was complelely filled with a clear solution of copper salt in equilibrium with a mixture of CuCl, $Hg_2Cl_2$ and mercury, and its electrode potential against that of a deci-normal calomel electrode was measured by means of the potentiometer. The mercurous-mercury electrode potential could easily be calculated if we know the concentration of chloridion from the following equation :—

$$E = -\frac{RT}{nF}\log\frac{C}{c} = -\frac{RT}{nF}\log\frac{C.(C_{Cl'})^2}{S_1}$$

where $S_1$ is the solubility product of $Hg_2Cl_2$. We have, when $C_{Cl'} = 0 \cdot 1$ N, $E = +0 \cdot 6226$ Volt at 30°C. The results are given in Table I.

TABLE I.

| | Concentration of chloridion. | Observed cuprous-cupric potential. | Calculated mercurous-mercury potential. |
|---|---|---|---|
| 1 | 0·0361 N | +0·6055 Volt | +0·6059 Volt |
| 2 | 0·0484 N | +0·6026 „ | +0·6020 „ |
| 3 | 0·0601 N | +0·5990 „ | +0·5992 „ |
| 4 | 0·0755 N | +0·5960 „ | +0·5963 „ |
| 5 | 0·0980 N | +0·5922 „ | +0·5929 „ |

The agreement is fair.

### TABLE II.

Results obtained when mixtures of $CuCl_2$ and KCl are shaken with mercury in presence of air.

| | Molar Concentration of Cü ion. | Concentration of Cl′ ion. | $[C_{Cü}] \times C_{Cl'}]^2 = K_1$ |
|---|---|---|---|
| 1 | 0·01694 | 0·07276 | $8{\cdot}768 \times 10^{-5}$ |
| 2 | 0·0099 | 0·7244 | $1{\cdot}21 \times 10^{-4}$ |
| 3 | 0·0053 | 0·1676 | $1{\cdot}49 \times 10^{-4}$ |
| 4 | 0·03 | 0·058 | $1{\cdot}02 \times 10^{-4}$ |

It will be at once seen that the value of $K_1$ varies a good deal.

### TABLE III.

Results obtained when mixtures of $CuCl_2$ and KCl are shaken with mercury in presence of $CO_2$

| | Molar Concentration of Cü ion. | Concentration of Cl′ ion. | $K_1$. |
|---|---|---|---|
| 1 | 0·02114 | 0·05174 | $5{\cdot}66 \times 10^{-5}$ |
| 2 | 0·01985 | 0·05286 | $5{\cdot}54 \times 10^{-5}$ |
| 3 | 0·01824 | 0·05536 | $5{\cdot}58 \times 10^{-5}$ |
| 4 | 0·01689 | 0·05746 | $5{\cdot}57 \times 10^{-5}$ |
| 5 | 0·01438 | 0·0637 | $5{\cdot}73 \times 10^{-5}$ |
| 6 | 0·01209 | 0·0692 | $5{\cdot}79 \times 10^{-5}$ |
| 7 | 0·6100 | 0·0755 | $5{\cdot}70 \times 10^{-5}$ |
| 8 | 0·006 | 0·0981 | $5{\cdot}75 \times 10^{-5}$ |
| 9 | 0·0011 | 0·2400 | $5{\cdot}66 \times 10^{-6}$ |

TABLE IV.

Results obtained when mixtures of $CuSO_4$ and KCl are shaken with mercury in presnce of $CO_2$ gas

| | Molar Concentration of Cü ion. | Concentration of Cl′ ion. | $K_1$. |
|---|---|---|---|
| 1 | 0·04566 | 0·0361 | $5·65 \times 10^{-5}$ |
| 2 | 0·03937 | 0·03813 | $5·72 \times 10^{-5}$ |
| 3 | 0·03614 | 0·04013 | $5·70 \times 10^{-5}$ |
| 4 | 0·03024 | 0·04367 | $5·76 \times 10^{-5}$ |
| 5 | 0·0264 | 0·04644 | $5·68 \times 10^{-5}$ |

I will thus be seen that in presence of $CO_2$ gas, fairly constant values of $K_1$ are obtained.

The mean $= 5·65 \times 10^{-5}$.

TABLE V.

Results obtained when mixtures of $CuSO_4$ and KBr are shaken with mercury in presence of $CO_2$ gas.

| | Molar Concentration of Cü ion. | Concentration of Br′ ion. | $K_2$. |
|---|---|---|---|
| 1 | 0·00216 | 0·00432 | $4·03 \times 10^{-8}$ |
| 2 | 0·00834 | 0·00222 | $4·10 \times 10^{-8}$ |
| 3 | 0·0223 | 0·00133 | $3·92 \times 10^{-8}$ |
| 4 | 0·0323 | 0·00113 | $4·12 \times 10^{-8}$ |
| 5 | 0·04586 | 0·000929 | $3·96 \times 10^{-8}$ |
| | | | Mean $= 4·05 \times 10^{-8}$ |

Results obtained and their discussion :—

The reaction is ionic and to obtain the exact value of the equilibrium constant it is necessary to determine the ionic concentration of Cü, Cl′ or Br′ ions. The titration values give the total copper and halogen content of the solution. To eliminate the effects due to incomplete dissociation, the concentrations of both copper and halogen ions were not allowed to exceed ·1 normal.

It is well known that CuCl and CuBr dissolve in KCl and KBr solutions respectively[1], to from complex $CuCl'_2$ ions. For dilute solutions however the effect due to formation of complex salt may be neglected.

The value of the real equilibrium constant

$$K=\left[\frac{K_1}{\sqrt{S_1}\times S_2}\right]^2 \quad \cdots \quad \cdots \quad \cdots \quad (1)$$

$$=\left[\frac{K_2}{\sqrt{S_1'}\times S_2'}\right]^2 \quad \cdots \quad \cdots \quad \cdots \quad (2)$$

The mean value of $K_1=5{\cdot}65\times 10^{-5}$; $S_1$ the solubility product of $Hg_2Cl_2$ is $3{\cdot}5\times 10^{-18}$. $S_2$ and $S'_2=1{\cdot}2\times 10^{-6}$ and $4{\cdot}5\times 10^{-8}$ (Bodlander. Ziet. Electrochem. 1202. **8**, 514-515); $S'_1=1{\cdot}37\times 10^{-21}$ (Sherril. Ziet. Phys. Chem. **43**, 735, 1903). $K'_2=4{\cdot}1\times 10^{-8}$.

The values of K obtained from equation (1) and (2) are $6{\cdot}35\times 10^{20}$ and $7{\cdot}19\times 10^{20}$ respectively. The agreement is very good.

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1. Bödlander—Ziet Electrochem. 1902, 8, 514.

# Notes on some Fish Teeth from the Tertiary Beds of Western India.

By Hem Chandra Das-Gupta M.A., F.G.S.

(with Plate I.)

The specimens that are to be described in this short note were obtained from a place called Hathab in Kathiwar. They were obtained when I had an opportunity of visiting this area in charge of a party of students from the Presidency College, Calcutta, in 1914.

The fossil-bearing locality is situated on the sea opposite to the small island of Perim, so well known for its mammalian remains. Besides Fedden's monograph[1], a few other contributions, *e.g.*, by Evans[2], Chapman [3] and Adye [4] have been published dealing with the geology of Kathiwar, but it is only in the first paper that reference has been to Hathab. Fedden gave an account of the petrology of the locality and also mentioned the fossils found by him[5]. But in course of our search in the 'rag' bed of the place we obtained a richer material including the sharks' teeth to be described here. It may be pointed out that though sharks' teeth were not met with in this area previously, the Perim list includes vertebræ of shark[6].

1. *Hemipristis serra*, Agas :—Two teeth of this species have been obtained, one of medium size and

(1) Mem. Geol. Surv. Ind., vol. XXI, pp. 73-136.
(2) Quart. Journ. Geol. Soc. (Lond.), vol. 56, pp. 559-583.
(3) *Ibid.* vol. 56, pp. 584-589.
(4) Mem. Economic Geology of Navanagar State.
(5) *Op-cit.* p. 111.
(6) *Op. Cit.* p. 117.

another very small. The species has been recorded from Burma by Dr. Noetling[7] and Dr. Stuart[8] and the occurence of it in the tertiary beds of Western India is of considerable interest. The larger specimen is figured (Plate I, figs. 1, 2). The tooth is a typical upper one and, though not completely preserved, is exactly like what has been described and figured by Dr. Woodward[9] from the Parana formation (Argentine Republic).

2. *Charcharias (Prionodon) egertoni*, Agas [10] :— This species is also represented by two teeth, one of which is very small (Plate I, figs. 2, 4). It has a great similarity with *C. (Prionodon) Similis*, Probest [11]. The teeth described by Probest have, however, a root which is rather thin while those described by Agassiz have a very thick and deep root. Besides this difference in the nature of the root there appears to be another important point of distinction between the 2 species in as much as the species of Agassiz has got the teeth prominently serrated, while those described by Probest 'stehen auf der Wurzelbasis aufrecht, sind symmetrisch, gazahnett, gegen die Basis verliert sich die Zahnelung.' The Hathab teeth are prominently serrated up to the base and they can be safely referred to *C. (Prionodon) egertoni*, Agas. It may be mentioned here that this

(7) Pal. Ind., New Ser., vol. I, Part 3, pp. 374-5, PL. XXV, figs., 9, a-e, 10.

(8) Rec. Geol. Surv. Ind., vol. XXXVIII, pp. 273, 274, 293, 297, Pl. 25, figs. 7 and 8.

(9) Ann. Mag. Nat. Hist., Ser. 7, vol. VI, P. 5, Pl. 1, figs. 11, 11a.

(10) Poiss Foss., vol. III, p. 228, Pl. XXXVI, figs. 6, 7.

(11) Württ. Jahresh., vol. 34, pp. 125-127, Taf. I, Figs. 12-19.

species has also been recorded from the tertiary beds of Burma by Dr. Stuart [12].

3. *Oxyrhina Feddeni* n. sp :—The teeth are very narrow, triangular and stand at right angles to the root. The crown is very high and very markedly curved inwards at the base and outwards towards the top. The root is fairly robust with two elongated branches diverging at an acute angle and with a very prominent vertical fissure. The present species is established on one complete tooth (pl. I figs 5, 6, 7). It has a very marked resemblance with *O. exigua* Probest[13] but differs from it in the nature of the root. It may be further added that the lower part of *O. exigua* is cylindrical while the corresponding portion in *O. Feddeni* is rather quite flat. This species has also got some resemblance with *O. spallanzani* Bon [14] but can be distinguished from it by the fact that *O. Feddeni* is comparatively narrower and the branches of the root divulge at a small angle.

PLATE I.

Figs 1-2—*Hemipristis serra*, Agas.
Figs 3-4—*Carcharias* (*Prionodon*) *egertoni*, Agas.
Figs 5, 6, 7—*Oxyrhina Feddeni* n. sp.
All natural size.

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(12) Op. cit. p. 295.

13. Wurtt. Jahresh., vol. 35, pp. 135-137, Taf. II, figs 20-25 Foldt. Kozl., vol. XXXIII, p. 156, figs 24 a-f.

14. Pal. Ind., New. Ser., vol. I, Pt. 3, pp. 372-373, pl. XXV, figs. 4, 5 a-e, 6 a-e.

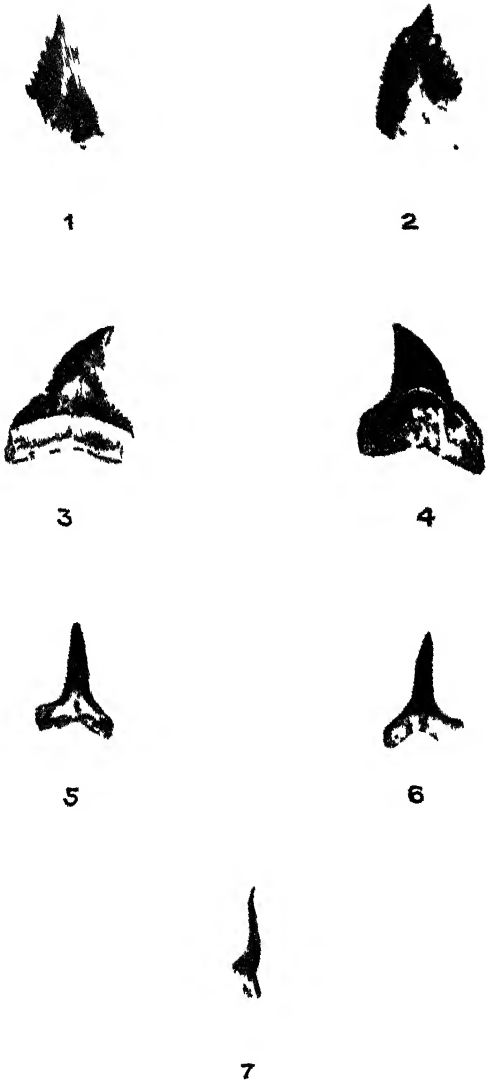

PHOTO BY B. MAITRA

FIGS. 1 2 *Hemipristis Serra*, Agas.
FIGS. 3 4 *Carcharias (Prionodon) egertoni*, Agas.
FIGS. 5 7 *Oxyrhina Feddeni* n. sp.

# PROCEEDINGS

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CONTENTS.

PROCEEDINGS

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Vol. IV. PART I.

# Projective Properties of Regular Solids.

By F. HALLBERG.

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## § 1. *Introduction.*

The importance of accurate perspective drawings for the successful study of projective geometry of three dimensions has, apparently, not yet been realized. Even the great geometrician Jacob Steiner was of opinion that such representations of the properties of space would firstly be hardly practicable, and secondly lack elegance*. In fact, Steiner has systematically avoided almost all figures in three-dimensional geometry, which is the more to be regretted, as he was a master of illustrating his results in plane geometry. His first objection is, no doubt, well founded, but the difficulty alone of a task should never be sufficient reason for not undertaking it. And his second objection is, as may easily be seen, groundless. Not only is it most difficult to communicate a complicated theorem of three-dimensional geometry to a layman without an illustration, but the success of original research depends to a great extent on the facilities at hand to

* Systematische Entwicklung der Abhängigkeit geometrischer Gestalten von einander. Th. 1, Kap. 2. § 26.—Herausgeg. von A.J.v. Oettingen, Ostwald's Klassiker, Nr. 82.

represent results already known. If any of these facilities be neglected, the mind has to make an unnecessary effort, which cannot fail to influence the result.

In addition, later research has shown, that a certain group of important theorems of plane projective geometry require for their proofs the assumption of the existence of a third dimension. A notable example is the theorem

Harm. (ABCD) implies Harm. (BCDA),

which requires Desargues' theorems of perspective triangles. The latter theorems depend, however, on the assumption of a three-dimensional space, since it has been proved (first by Peano, and subsequently by Hilbert, and others) that if geometry be restricted to two dimensions, the ordinary axioms of plane geometry together with the contradictory of Desargues' theorems form a consistent system of axioms*. This shows that a kind of higher order exists in three-dimensional space, and we may conclude, not only that a number of complicated theorems of plane geometry may be found (and proved) in an easier way by the aid of figures in space, but that a group of such theorems can only be found in this manner. The simplest process is then not to build up a system of axioms for plane geometry, involving the assumption of a higher space, but to project the ready-made figures in three dimensions on the plane. This latter process has been used throughout the present paper. It gives us, however, no answer to the question, which of the theorems for the plane are due to properties in space. This can only be decided by means of a discussion of the fundamental axioms.

## § 2. *Method employed.*

In the present paper, projective systems in space, associated with the regular solids, and some of their point-plane intersections, are discussed.

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* See A. N. Whitehead, The axioms of projective geometry (Cambr. Math. Tracts, No 4), where references will be found.

The method is everywhere the same. A certain regular solid (or a group of such solids) is considered. Such a system always is in a definite relation to a system of points and lines in the plane at infinity, which relation is carefully analysed. All proofs in this connection have been omitted for the sake of brevity, and should not present any difficulty to the reader. Any (1, 1)-correspondence between the points of space is then established, transforming: (1) the plane at infinity into a finite plane, called the "leading plane"; (2) the centre of the solid into some other point, spoken of as the "leading point."

In this way, a group of "existence theorems" for the system is established, but no method of construction is obtained, except the trivial one of actually performing the projective transformation. Where convenient, more direct methods of construction are indicated. Lastly the system obtained is projected on a plane, which process gives the corresponding existence theorems for plane geometry. Some remarks on the independent construction of the plane system are given, but this question has not been treated in detail.

For the sake of simplicity, I speak of a system, thus obtained, as a "solid" or "plane complex," and give it the name of the regular solid (or group of such solids) from which it is derived. It is to be noted, however, that this distinction is largely arbitrary, since the different systems pass into one another in a variety of ways.

An application to determinants has been indicated, and performed for a special case (see 1.C.)

### § 3. *Remarks on the figures.*

Considerable care has been taken to provide accurate illustrations, the reason for this being sufficiently indicated in the introduction. Wherever possible, the leading plane has been included in the figures, since it must be considered the most interesting part of each complex — at least before

any equivalent plane (in projective sense) has been found. In this way the drawing, no doubt, is deprived of some of its æsthetic value, because the figure in the plane in question is different from the rest of the complex, and appears out of place. But this is more than compensated by the gain in mathematical interest, as has been pointed out by v. Oettingen (*ibid.* p. 107). In fact, from the discussion of the leading plane, we may generally infer, that the system, with which we started, is only a part of another system with a higher degree of symmetry. There is, however, a practical consideration to be taken into account. A solid complex may be considered merely as a group of points, lines and planes, connected with each other in some definite manner. This view is the most general one, and therefore the most valuable. But a drawing gains much in clearness, if it really represents a solid, in other words, if the regular solid, from which we have departed, is not too much distorted. Consequently, although this would not be absolutely necessary, I have followed the latter principle, being convinced, that the study of the figures thus is made much easier. But for this reason, it has been necessary to omit the leading plane in some cases, as for instance in the drawings of the dodeca-icosahedral complex.

## § 4. *Notation.*

Except where a statement is made to the contrary, all elements of a complex are named entirely by means of its points. This is done partly to secure uniformity, and partly to avoid introduction of unnecessary symbols, which would only result in a confusion of the figures.

Points are denoted by capital letters : A, B, C, etc.

Straight lines are named after any group of two (or more) points on them, and the expression obtained enclosed in round brackets : (AB), (ABC), etc,

Planes are named after any group of three (or more) points on them, and the expression obtained enclosed in straight brackets: [ABC], etc.

Similarly, the expression for a system in three dimensions is enclosed in crooked brackets: {ABCD}, etc.

Equivalent systems, that is: systems with identical projective properties, are denoted by the same symbol, and their number by an index: $\{A^4\}$, $[(AB)^3]$, $\{[A^3]^4\}$, etc., the meaning of a combination of brackets being evident.

# 1. THE TETRAHEDRAL COMPLEX.

## A. In three dimensions.

### 1.1. *The fundamental system.*

1.11. Consider the points $\{A^4\}$, determining the lines $\{(A^2)^6\}$ and the planes $\{[A^3]^4\}$, the system thus forming a tetrahedron. Select the leading point O not in any $[A^3]$.

1.111. Construct the planes $\{[A^2O]^6\}$, determining:
the points $\{B^6\}$, one on each $(A^2)$, forming an octahedral complex;
the lines $\{(AB)^{12}\}$, three in each $[A^3]$;
the points $\{C^4\}$, one in each $[A^3]$, forming a tetrahedral complex.

Through each B thus pass two (ABC), through each C three (ABC).

1.112. Construct the planes $\{[B^3]^4\}$, one for each group $\{(A^2)^3\}$ through an A. They cut each $[A^3]$ in three lines $[(B^2)^3]$.

1.113. There exist three planes $[B^4O]$, and hence also three lines $(B^2O)$.

1.114. There exists further a leading plane, containing a point-system $[D^6]$, such that:

| | | |
|---|---|---|
| through every D pass | one | $(A^2)$, |
| | two | $(B^2)$, |
| | one | $(C^2)$; |

the points D lie on a system of four lines $[(D^3)^4]$. three on each;

through every D pass two $(D^3)$.

1.115. There exists further an octahedral complex $\{E^6\}$ such that

on each $(C^2D)$ there is one E;
on each $(B^2O)$ there are two E;
through every D pass two $(E^2)$.

Thus O is the leading point and $[D^6]$ the leading plane for the two systems $\{B^6\}$ and $\{E^6\}$.

1.12. Join each D to—the point B, which lies on the particular $(A^2)$ not meeting the $(A^2)$ through D. There are six such lines (BD). They meet in four points F, $\{F^4\}$ forming a new tetrahedral complex. Through each F pass three lines $(BDF^2)$.

1.121. In each of the four planes $[F^3]$, draw the three lines (BF), which meet in a point H. There exists thus a tetrahedral complex $\{H^4\}$, related to $\{F^4\}$ as $\{C^4\}$ to $\{A^4\}$.
In particular:
there is one F and one H on every (AOC);
there is one D and one E on every $(H^2)$.

1.122. The two systems $\{C^4\}$ and $\{H^4\}$ form together a hexahedral complex with O as leading point and $[D^6]$ as leading plane, (see 2.11, where the meaning of D is unchanged). Hence the twelve lines (CH), not containing O, meet by fours in three points in $[D^6]$, any two of which lie in a straight line with two of the points D. As will be seen below, these three points are projectively equivalent to O. We have therefore the following fundamental theorem:

1.13. *If a tetrahedral complex $\{A^4\}$ and any point O (not in an $[A^3]$) are given, another tetrahedral complex $\{O^4\}$ is completely determined, such that the vertices of one complex are projectively equivalent with regard to those of the other.*

Evidently the condition, that O shall not lie in an $[A^3]$, may be excluded by means of a simple limit-

ing process. It follows that if one O lies in an $[A^3]$, so do the others. We have now the following very general theorem :

1.14. *If the tetrahedral complex* $\{O^4\}$ *moves in any manner (the complex* $\{A^4\}$ *being fixed), then the curves or surfaces, determined by any equivalent points, lines or planes in the moving system, are projective.*

Many interesting consequences may be deduced from the last theorem, particularly with regard to the space curves of third order. I do not propose to go into details here, since I intend to deal with this question in a special memoir.

1.15. The two systems $\{A^4\}$ and $\{F^4\}$ form together another hexahedral complex, which has the same relation to $\{O^4\}$ as the hexahedral complex $\{C^4H^4\}$. (This follows immediately from 1.13., since, by 1.121., four lines (AF) pass through one of the vertices O.)

1.2. *Two intersecting systems.*

(In the figures 1.2. the leading plane, with its point-system $[D^6]$, lies to far away for any points D to be represented. This causes the systems to appear more regular.)

1.21. Consider a new tetrahedral complex $\{G^4\}$, similar to $\{F^4\}$ in all respects, except that its lines $\{(G^2)^6\}$ do not contain the system $\{B^6\}$. The complex $\{G^4\}$ thus satisfies, in particular, the conditions:
there is one G on every (AOC);
there is one D on every $(G^2)$.

1.211. The complex $\{G^4\}$ determines a point-system, $\{P^{12}\}$, such that:
there are two P on every $(A^2)$;

there are three P, and hence also three $(P^2)$ in every $[G^3]$;

there is one D on every such $(P^2)$;

through every D pass two such $(P^2)$.

1.212. The system $\{P^{12}\}$ may be called a double tetrahedral complex of the general type. There is an infinity of such systems, each determined by any one of its points P. (Construct the plane $[PD^3]$, determining two other points P and three points G on three lines (AOC). Now the fourth G lies on the fourth (AOC), and on three lines (GD)).

1.213. Among these systems there is, in particular, one, having a $(P^2)$ passing through a C. In this case, there are three lines $(P^2)$ passing through every C, and each figure $[P^6]$ in an $[A^3]$, always being a hexagon of Pascal's type, since three pairs of lines $(P^2)$ always meet on a $(D^3)$ (by 1.114, 1.211), is now in addition a hexagon of Brianchon's type.

It follows, that this particular system $\{P^{12}\}$ must be richer than any of the others in projective properties. It has therefore been selected for representation, and a few of its properties are enumerated below. Some of these belong, of course, to any $\{P^{12}\}$ of the general type, but I have not thought it necessary to make any distinction.

1.214. There are three lines (GP) in every $[G^3]$, meeting in a point K on the corresponding (AOG). The lines (GP) meet the lines $(P^2)$ in a point-system $\{L^{12}\}$, which is a new double tetrahedral complex (of the general type), having the leading point O and associated in the usual manner with the system $[D^6]$ in the leading plane. There is one L on every (ABC).

1.215. There exists an octahedral complex $\{M^6\}$, also with O as leading point and $[D^6]$ as leading plane, such that:
every $(G^2)$ contains one M;
through each M pass two (GP);
through each D pass two $(M^2)$.

1.216. There exists further a double tetrahedral complex $\{N^{12}\}$ (of the general type), also with O as leading point and $[D^6]$ as leading plane, such that:
on every (ABC) there is one N;
on every $(P^2)$, belonging to a $[G^3]$, there are two N;
on every $(G^2)$ there are two N;
through every N pass two $(P^2)$.

1.217. Six more lines (GP) may be drawn in every $[G^3]$. They meet, two at a time, in three points on the lines (GP) of 1.214. These points, twelve in all, constitute another double tetrahedral complex (of the general type).

1.22. Consider the hexagon $[P^6]$ in an $[A^3]$. The points P may be joined by fifteen lines $(P^2)$, of which we have already discussed the nine lines $(P^2D)$. The remaining six meet each other in six points S, and determine on the three lines $(A^2)$ six points R.

1.221. There are two points S on every (ABC). The points R, obtained from two adjacent hexagons on any $(A^2)$ coincide, two and two, so that:
there are two R on every $(A^2)$;
through each R pass two $(P^2)$.
The system $\{R^{12}\}$ is again a double tetrahedral complex (of the general type) with O as leading point and $[D^6]$ as leading plane.

1.222. There exists a tetrahedral complex $\{Q^4\}$, such that:
on every (AOG) lies one Q;
on every (GQ) lies one R;
through every Q pass three (GR).
The systems $\{G^4\}$ and $\{Q^4\}$ form together a hexahedral complex, associated with $\{O^4\}$ in the manner of $\{C^4H^4\}$, $\{A^4F^4\}$. In particular:
there are six lines $(Q^2)$, of which one passes through every D;
the points, in which a $(Q^2)$ cuts a $(G^2)$, six in all, lie on the lines $(B^2O)$, two on each, and form an octahedral complex with O as leading point and $[D^6]$ as leading plane.

1.223. There exist in the leading plane twelve points, three on each $(D^3)$, through each of which pass:

one (ABC),
one (BF),
one (GK),
two $(P^2S^2)$.

1.23. Through each D pass two $(S^2)$. There are in each $[A^3]$ six such lines, meeting one another in six points T, two of which lie on every $(P^2C)$. The lines $(S^2)$ further determine on every $(A^2)$ two points U, and two points V, such that:
through every U pass two $(S^2)$;
through every V pass two $(S^2)$.
(In reality, the points U and V are equivalent in projective sense, but in order not to overload the figures, and to be able to perform different constructions on the two different sets of points, I have called those between the points PP, U, and those outside, V. It should also be noticed, that $\{S^{24}\}$, is not a complete set of projectively equivalent points).

1.24. There are, in every $[A^3]$, three more lines $(U^2)$, each containing one S. The lines $(V^2)$ determine on each edge $(P^2)$, which is not also on edge of $\{A^4\}$, two points W, such that:
in each triangular face $[P^3]$, $[W^6]$ is a hexagon of Pascal's as well as of Brianchon's type, $(D^3)$ being a Pascal line, and K a Brianchon point;
in each hexagonal face $[P^6]$, the three additional lines $(W^2)$ meet the three additional lines $(U^2)$ in six points X, two of which lie on every $(P^2\ C)$. There are 24 lines (AWTP), six in each $[A^3]$.

This discussion of the surface lines of the system could of course be continued to any degree of complexity, but without any apparent purpose.

The systems $\{U^{12}\}$, $\{V^{12}\}$ are, as may easily be seen, two double tetrahedral systems (of the general type) with O as leading point and $[D^6]$ as leading plane.

## B. In two dimensions.

### *1.1.* *The fundamental system.*

1.11. Consider the points $[A^4]$ in a given plane, determining the lines $[(A^2)^6]$. Select a point O.

1.111. Draw the lines $[(AO)^4]$.

Take a point C on one of the lines (AO), and construct the lines $[(AC)^3]$.

Then three points B are determined, and hence also three lines (AB), which again determine the remaining three points C.

The theorems 1.112—1.222 and 1.15 are now valid for our plane complex $[A^4]$, with the following modifications: all symbols { } should be replaced by symbols [ ], since now the whole system

belongs to one plane; for the same reason, all references to different planes should be excluded as trivial.

1.13. This theorem remains valid, if we add the condition of the arbitrary point C of 1·111.

1.14. *There is no equivalent theorem in plane projective geometry.*

1.2. *Two combined systems.*

The whole section remains valid, the above modifications being made. The construction of the system $[G^4]$ may be carried out by the aid of the system $[D^6]$.

C. Application to determinants.

Theorems of the above type admit of an immediate application to determinants.

In the present article, let

$$K_\nu = (k_{1\nu},\ k_{2\nu},\ k_{3\nu},\ k_{4\nu})$$

denote a set of four numbers, forming a row (or column) in a fourth order determinant. Two such sets, the elements of which differ only by a constant factor k, are regarded as identical:

$$K_\nu = (k.k_{1\nu},\ k.k_{2\nu},\ k.k_{3\nu},\ k.k_{4\nu}).$$

A set $K_\nu$ now defines a point in a rectangular co-ordinate system, if we take

$$k = \frac{1}{k_{4\nu}},$$

and consider the values

$$\frac{k_{1\nu}}{k_{4\nu}},\quad \frac{k_{2\nu}}{k_{4\nu}},\quad \frac{k_{3\nu}}{k_{4\nu}},$$

as co-ordinates for the point in question, which we

may denote by the same symbol $K_\nu$.

Let further

$$\left| K_\alpha, K_\beta, K_\gamma, K_\delta \right|$$

denote a determinant, the rows (or columns) of which are given by means of four sets K.

Knowing, that the necessary and sufficient condition for the equation

$$\left| K_\alpha, K_\beta, K_\gamma, K_\delta \right| = 0$$

is that the four points K, corresponding to the four given sets K, lie in the same plane, we obtain from Art. 1. A:

1.11. Let four sets $A_\nu$ ($\nu=1,2,3,4$) be given, such that

$$|A^4| = |A_1, A_2, A_3, A_4| \neq 0.$$

Select a set $O_1$, such that, if any $A_\nu$ in the determinant A be replaced by $O_1$, the new determinants formed are all $\neq 0$.

1.111. There are then exactly six sets $B_{\alpha\beta} = B_{\beta\alpha}$ ($\alpha=1, 2, 3, 4$; $\beta=1, 2, 3, 4$; $\alpha\neq\beta$), such that

$$\left|A^3B\right| = \left| A_\alpha, A_\beta, A_\nu, B_{\alpha\beta} \right| = 0;\ \nu\neq\alpha;\ \nu\neq\beta \text{ (12 dets.)};$$

$$\left|A^2OB\right| = \left| A_\mu, A_\nu, O_1, B_{\alpha\beta} \right| = 0;\ \mu\neq\nu;\ \mu,\nu\neq\alpha,\beta \text{ (6 dets.)}$$

There are further exactly four sets $C_\nu$ ($\nu=1,2,3,4$) such that:

the four dets., obtained by replacing any $A_\nu$ in $|A^4|$ by the corresponding $C_\nu$, are all $=0$;

the twelve dets., obtained by replacing the set $A_\nu$ in a det. $|A^3B|$ by the set $C_\mu$, are all $= 0$;

the eighteen dets, obtained by replacing any $A_\mu$, $A_\nu$, or both, in a det. $|A^2OB|$, by the corresponding $C_\nu$, $C_\mu$, or both resp., are all $=0$.

1.113. There are then in addition three dets.

$$\left|B^4\right| = \left| B_{\alpha\beta},\ B_{\mu\nu},\ B_{\alpha\nu},\ B_{\beta\mu} \right| = 0,$$

each of which retains its zero value, when an element B is replaced by $O_1$ (12 dets.).

If we now take into account the leading plane, as well as the three remaining points O, the results become very complicated.

In a similar manner, we may translate the theorems of Art. I. B. into a determinant notation. In fact, each point in a plane may be represented by a set of three elements, and this set may be taken as a row (or column) in a third order determinant, which thus is associated with a group of three points. If these points lie on a straight line, the determinant in question vanishes.

There is no difficulty in applying the above process to the following sections. I have, however, not done this, on account of the extreme complexity of the theorems, which would result.

# 2. THE HEXAHEDRAL COMPLEX.

## A. In three dimensions.

### I.

2.1 *The fundamental system.*

(The hexahedral complex has been represented in the figures by means of a solid: a truncated, quadrangular pyramid. The construction of the system is thus apparent.)

2.11. There exists a system of eight points $\{A^8\}$, lying in twelve planes $\{[A^4]^{12}\}$, and joined by sixteen straight lines $\{(A^2)^{16}\}$, which pass, four at a time, through four points. Of the latter, one point O is, at least for the present, to be considered as the leading point, while the other three define the leading plane $[B^3]$.

2.111. Construct the lines $\{(BO)^3\}$. They determine with six of the planes $[A^4]$, which we may call the "faces" of the complex, an octahedral complex $\{C^6\}$, one point C in each face. The system $\{C^6\}$ has O as leading point and $[B^3]$ as leading plane.

2.112. Construct the "diagonals" $\{(A^2)^{12}\}$, two of which lie in each face, passing through its C-point. These lines determine a point system $[D^6]$ in the leading plane with the following properties:
on each $(A^2C)$ there is one D;
on each $(B^2)$ there are two D;
through each D pass two $(A^2)$,
two $(C^2)$.
The points D lie, three at a time, on four lines $[(D^3)^4]$, two of which pass through every D.

2.113. The lines $\{(A^2O)^4\}$ determine a point system $[E^4]$ in the leading plane with the following properties:

on every $(A^2O)$ there is one E ;
on every $(E^2)$, six in all, there are one B and one D.

2.12. Construct the lines $\{(BC)^{12}\}$, of which four pass through each B and two lie in each face $[A^4]$

2.121. These lines determine a point-system $\{F^{12}\}$ with the following properties :
there are two F on each (BC) ;
there is one F on each $(A^2B)$ ;
through every D pass five $(F^2)$, one of which contains O.

2.122. The system $\{F^{12}\}$ may be represented by a solid figure with twelve vertices, twenty-four edges, eight triangular and six quadrilateral faces, and with four diagonal planes through its leading point, each containing six edges and one line $(D^3)$, so that the hexagon $[F^6]$ is both of Pascal's and Brianchon's type.

The system may be considered as the intersection between a hexahedral and an octahedral complex in a unique relation to each other, but as no new arbitrary construction has been made, the system being completely determined by the given complex alone, we may also consider it as fundamental.

2.2. *Combination of a hexahedral and an octahedral complex.*

2.21. Instead of considering the complex $\{F^{12}\}$, we may discuss the complex determined by any other point K on an edge $(A^2B)$, thus introducing an arbitrary element.

2.211. There exists a point-system $\{K^{24}\}$ with the following properties :
there are two K on each $(A^2B)$, both simultaneously

approaching the point F on the edge ;
through every B pass eight ($K^2$), not counting any ($A^2K^2B$) ;
through every D pass ten ($K^2$) ;
through every C pass four ($K^2$) ;
there are twelve lines ($K^2O$).

2.212. The system $\{K^{24}\}$ may be represented by a solid figure with twenty-four vertices, thirty-six edges, eight tringular and six octagonal faces.
Evidently the system may be considered as the intersection between a hexahedral and an octahedral complex in a definite (but not unique) relation to each other. The complex is completely defined by means of one of its points K, the complex $\{A^8\}$ being given.

2.213. The surface-lines in any face [$A^4$] intersect in a very complicated manner, producing a multitude of allied sytems, which may be studied by the aid of the combinations of the diagonals of a regular octagon. As an example, consider the system $\{N^{24}\}$, some of the properties of which are :
there are four N in every face [$A^4B^2$] ;
there are two N on every ($A^2C$) ;
through every N pass two ($K^2B$) ;
there are twelve lines ($N^2O$) ;
there are twelve lines ($N^2B$) not in any face [$A^4B^2$], four of which pass through every B ;
there is a system of lines $\{(N^2D)^{48}\}$, none of which belong to any face [$A^4B^2$], and eight of which pass through every D.

2.22. As an example of a different type, consider the system $\{L^{24}\}$ with the following properties :
there are four L in every face [$A^4B^2$];

there are two L on every (BCF$^2$) ;
through every L pass two (K$^2$D) ;
there are twelve lines (L$^2$O) ;
there are twelve lines (L$^2$B) not in any face [A$^4$B$^2$], four of which pass through every B.

Since this system is more interesting than that described in 2.213, I will examine it in detail.

2.3. *Discussion of the complex* $\{L^{24}\}$.

2.31. The system $\{L^{24}\}$ may be represented by a solid figure with twenty-four vertices thirty-six edges, six quadri-lateral and eight hexagonal faces.

It may be obtained from a certain octahedral complex $\{H^6\}$, the solid angles of which are truncated by means of the faces of the given hexahedral complex $\{A^8\}$.

2.311. The hexagonal faces are of Pascal's type, each of the lines $[(D^3)^4]$ being a Pascal line for two systems [L$^6$]. In general, the systems [L$^6$] will not be of Brianchon's type, but this will happen for two particular fundamental systems in a unique relation to each other. Since we have constructed our complex $\{L^{24}\}$ by the aid of an arbitrary $\{K^{24}\}$, it must be of the general type.

2.312. There is then in each hexagonal face [L$^6$] three diagonals (L$^2$), each containing a D belonging to a (D$^3$). They intersect in three points M, thus producing a point system $\{M^{24}\}$ with the following properties:
there are twelve lines (M$^2$O) ;
there are thirty-six lines (M$^2$D) not connecting points M in the same hexagon, six such lines passing through every D;

there are thirty-six lines $(M^2B)$, twelve of which pass through every B.

We are thus able to identify the system $\{M^{24}\}$ as a complex of the same type as $\{K^{24}\}$.

2.32. The vertices of the octahedral complex $\{H^6\}$ lie on the lines $[(OB)^3]$, two on each line. Through every H pass four $(L^2)$, intersecting a certain $[A^4B^2]$ in four points $[L^4]$. Each triangular face $[H^3]$ contains six points L. The complex has O as leading point and $[B^3]$ as leading plane. Through every D pass two $(H^2)$

2.4. *A different combination.*

2.41. It is easy to establish several other octahedral systems, equivalent to $\{H^6\}$. In order to demonstrate a few more properties, I construct a complex $\{P^{24}\}$, of the same type as $\{L^{24}\}$, starting, as in 2.22., from the complex $\{K^{24}\}$.

2.411. Consider the four lines $(K^2)$ in an octagonal face of $\{K^{24}\}$, which are also sides in triangular faces. The four points P, determined by these lines, belong to the required system $\{P^{24}\}$.

2.412. The properties of this complex are almost identical with those of $\{L^{24}\}$, and any line through O cuts corresponding lines or planes of the two systems in projective points.

We note in particular:

The system $\{P^{24}\}$ determines an octahedral complex $\{G^6\}$, entirely equivalent to $\{H^6\}$.

2.42. Of the twelve lines $(P^2)$ through any D, two contain two points G each, and eight others lie in the faces $[G^3]$ of the octahedral complex $\{G^6\}$. The remaining two lie in three diagonal planes $[G^4]$

The last mentioned lines $(P^2)$ form a system of twelve lines $\{(P^2)^{12}\}$, which meet, four at a time, in the six points S, forming an octahedral complex $\{S^6\}$ with O as leading point and $[B^3]$ as leading plane.

It follows from the above, that also the system $\{L^{24}\}$ determines a similar octahedral complex.

## II.

We may now make the following important generalisation.

In I. 2.11. we selected one point out of four as the leading point. This was done in order to bring out the connection between our complex and a cube—— the regular representative for the system——as well as to obtain a convenient notation. This process, however, involves the loss of many fundamental properties of the system, above all that of symmetry. We will now reconsider part of the theorems under section I., giving them in a more symmetrical form, at the same time pointing out, which systems are equivalent. For the latter purpose, the sign $\infty$ is used.

2.11. There exists a system of eight points $\{A^8\}$, lying in twelve planes $\{[A^4]^{12}\}$, and joined by sixteen straight lines $\{(A^2)^{16}\}$, which pass, four at a time, through four points $\{B^4\}$, forming a tetrahedral complex $(O \infty B)$.

2.111. On each line $(B^2)$ there are two points C, such
2.112. that $(D \infty C)$:
the system $\{C^{12}\}$ consists of four octahedral complexes $\{C^6\}$, each associated in the same way with $\{B^4\}$, and each obtained by removing from the complete system, the plane system $[C^6]$, belonging to any $[B^3]$;

there are twelve lines $(A^2C^2)$, two passing through every C;
there are sixteen lines $(C^3)$, four in each $[B^3]$, four passing through every C.

2.113. The four lines $(A^2B)$ through any particular B, determine on the plane $[B^3]$, not containing the first B, four points E. The system $\{E^{16}\}$ has the following properties:
there are twenty-four lines $(BCE^2)$, six in each $[B^3]$, six through every B, two through every C, three through every E.

2.12. We have $F \infty E$; it is thus possible to select four systems $\{E^{12}\}$, disregarding for the moment the four E in a $[B^3]$, each system being associated with a B, as $\{F^{12}\}$ with O. In particular:
there are sixteen planes $[E^6BC^3]$, the hexagon $[E^6]$ in each being both of Pascal and Brianchon type.

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It is not difficult to extend the above process further.

It will be seen, that a further generalisation is possible. In fact, we may consider $B \infty A$. I have not developed this idea for three dimensions, because such a change would make the terminology too complicated. It will, however, be briefly considered in the following section.

## B. In two dimensions.

### 2.1. *The fundamental system.*

2.11. There exists in a plane a system of twelve points A (B,O ∞A), lying on sixteen lines ($A^3$), four of which pass through every A. The figure may be regarded as made up of twelve complete quadrilaterals, each A belonging to six, each ($A^3$) to three quadrilaterals.

2.111. Construct the remaining eighteen lines ($A^2$), each a diagonal in two quadrilaterals, each quadrilateral containing three ($A^2$), three ($A^2$) passing through every A.

2.112. Among the points of intersection in the line-system, [$(A^2)^{18}$], there is a system of twelve points C, such that:
on every ($A^2$) lie two C;
through every C pass three ($A^2$).

2.12. There may now be drawn 72 lines (AC), six of which pass through every A. These lines meet one another, three at a time, in 48 points E, of which there are two on every (AC), three on every ($A^3$).

They also intersect in 48 points J, of which there are four determined by each quadrilateral, two J lying on each (AC), three (AC) passing through every J. There are then 48 lines ($AJ^2$).

In addition, there are 144 lines ($CE^2$), twelve passing through every C, six passing through every E. These lines form a system of hexagons, each with a Pascal line ($C^3$) and a Brianchon point A.

*The theory partly developed above, is associated with that of a Brianchon hexagon.*

In fact, let a hexagon of this type be given $A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$; and let $A_7$ be a point such, that $(A_1A_7A_4)$, $(A_2A_7A_5)$, $(A_3A_7A_6)$ are straight lines. Draw the lines

$(A_1A_2)$, $(A_4A_5)$, intersecting in $A_8$;
$(A_2A_3)$ $(A_5A_6)$, ,, ,, $A_9$;
$(A_3A_4)$, $(A_6A_1)$, ,, ,, $A_{10}$.

Construct further the six lines

$(A_8A_3)$. $(A_8A_6)$; $(A_9A_1)$, $(A_9A_4)$; $(A_{10}A_2)$, $(A_{10}A_5)$; whicn meet, three at a time in two points: $A_{11}$, $A_{12}$, such that $(A_7A_{11}A_{12})$ is a straight line.

The system $[A^{12}]$, thus defined, is the projection on a plane of a hexahedral complex, treated above.

The system $[A^{12}]$ evidently may be treated much more symmetrically, than the system of six lines, touching a conic. The former must therefore be considered as more natural, especially as it is connected with an exactly equivalent system in three dimensions of a still more natural character, from which system it is obtained by projection. No such relation exists in the case of Brianchon's hexagon.

# 3. THE OCTAHEDRAL COMPLEX.

## A. In three dimensions.

### I.

3.1. *The fundamental system.*

3.11. There exists a system $\{A^6\}$ with the following properties:

The points A lie, four at a time in three planes meeting at a point O, each A belonging to two $[A^4]$;
there are thus three lines $(A^2O)$;
the remaining lines $(A^2)$, twelve in number, meet each other, two at a time, in six points B, all lying in one plane $[B^6]$.

3.111. We may represent the complex $\{A^6\}$ by means of a solid figure with six vertices, twelve edges, eight triangular faces, and three quadrilateral diametral sections. A regular octahedron is a solid of this type. The leading point of the complex is O, the leading plane $[B^6]$.

3.112. The points B lie, three at a time, on four lines $(B^3)$, two such lines passing through every B.

Let $[C^3]$ be the diagonal points of the complete quadrilateral $[(B^3)^4]$, so that:
there are three lines $(B^2C^2)$;
through every B passes one $(B^2C^2)$:
through every C pass two $(B^2C^2)$.
Then each $(A^2O)$ passes through a C.

3.113. Construct the system of lines $\{(BO)^6\}$.
Each (BO) intersects two $(A^2B)$, both passing through the second point B on the line $(B^2C^2)$, on which the first B lies.

3.114. There are in all twelve such points of intersection F, having the following properties:
on every ($A^2B$) there is one F;
through every F pass two ($BF^2$);
there are twenty-four lines ($BF^2$), four of which pass through every B.

Hence it is easily seen, that the complex $\{F^{12}\}$ is identical with that of 2.12. of the same notation.

(It follows, that the systems [$B^6$], [$C^3$] in our present notation are identical with the systems [$D^6$], [$B^3$] of section 2.)

3.12. Construct the system of lines $\{(AF)^{24}\}$; each face [$A^3$] in the octahedral complex contains three of these lines, determining a point D, through which they all pass.

3.121. The system $\{D^8\}$ is a hexahedral complex with O as leading point and [$C^3$] as leading plane.

3.122. Consider a certain face [$A^3$], and the corresponding line ($D^3$). Draw the three remaining lines (AB). They determine three points E, two on each (AB.)

3.123. There are, in all, eight points E, forming a hexahedral complex with O as leading point and [$C^3$] as leading plane.

Through every E pass two (AB);
through every E pass three (ADF), the A-points on these three lines belonging to one face [$A^3$];
there are four lines ($D^2E^2O$);
there are twelve lines ($E^2C$), four passing through every C.

II.

We are now able to transform the above theorems in a manner, similar to that employed in the case of the hexahe-

dral complex, at the same time obtaining more symmetrical results.

We observe first, that B $\infty$ A, and hence that the figure consists of twelve équivalent points A, joined by sixteen lines ($A^3$), four of which pass through every A. The points $\{A^{12}\}$ belong to twelve planes [$A^6$], each containing four lines ($A^3$). Six such planes meet in every A.

The indicated generalisation may now easily be carried out.

It should be noticed, that the octahedral complex and the hexahedral complex are polar reciprocal systems in three (but not in two) dimensions.

## B. In two dimensions

### 3.1. *The fundamental system.*

The fact is at once apparent, that the hexahedral and octahedral systems in two dimensions are very similar to each other, the difference being due only to different arrangements of the elements (cf. 2.B.)

3.11. There exists in a plane a system of twelve points A (B $\infty$ A), lying on sixteen lines ($A^3$), four of which pass through every A. The figure may be regarded as made up of twelve complete quadrilaterals, each A belonging to six, each ($A^3$) to three quadrilaterals.

3.111. Construct the remaining six lines ($A^2$), each a diagonal in two quadrilaterals, each quadrilateral containing three ($A^2$), one ($A^2$) passing through every A.

3.112. Among the points of intersection in the line-system $[(A^2)^6]$ there is a system of four points C (O $\infty$ C), such that:
on every $(A^2)$ lie two C;
through every C pass three $(A^2)$.

There is no difficulty in continuing this analysis, as has been done in 2. B.—Evidently, the two systems differ chiefly with regard to their C-points.

## 4. THE DODECA-ICOSAHEDRAL COMPLEX.*

### A. In three dimensions.

4.1. *The fundamental systems.*

*Def.:* A dodecahedral (icosahedral) complex is a system, equivalent in a projective sense to a regular dodeca- (icosa-) hedron. We will speak of the twelve vertices, thirty edges, and twenty triangular faces of the icosahedral, and of the twenty vertices, thirty edges and twelve pentagonal faces of the dodecahedral complex, whether the systems define solids or not.

The two systems are clearly polar-reciprocal (in three dimensions).

We find, that a given complex of one of these two types, completely determines an infinite series of systems of both types.

4.11. There exist an icosahedral complex $\{A^{12}\}$, and a dodecahedral complex $\{B^{20}\}$ in the following relations to one another:
through every B pass two $(A^2)$;
a pentagon $[B^5]$ is determined by the diagonals in another pentagon $[A^5]$, the vertices of which belong to the five edges $(A^2)$ through a certain sixth vertex A.

4.111. There exists a leading point O, such that:
there are six lines $(A^2O)$;
there are ten lines $(B^2O)$.

4.112. Through every $(A^2O)$ pass five diagonal planes, each containing:

* These two systems are conveniently treated together. In the figures to this section the leading plane has been omitted.

two edges ($A^2$);
two edges ($B^2$).

4.113. Each ($A^2O$) meets corresponding face [$B^5$] in a point C. There are thus twelve such points, forming an icosahedral complex {$C^{12}$} with O as leading point. In each [$A^2B^2O$] there are two ($C^2$).

4.114. There are five lines (AB) in each plane [$A^5B^5$], all passing through C.

4.12. The points B lie, six at a time, in twenty planes [$B^6$]. These planes intersect in twelve points D, forming a new icosahedral complex {$D^{12}$}, with O as leading point, and with two ($D^2$) in every [$A^2B^2C^2O$].

4.2. *Non-convex solids in the fundamental system.*

In order to demonstrate a few more properties of the combined systems in the last article, I have selected four partial systems, which, at least when the original complex is regular, appear as non-convex solids, first studied, I believe, by Kepler. In the figures of these systems, only the edges of the solids in question are drawn.

4.21. There exists a solid with thirty-two vertices, ninety edges and sixty triangular faces.

Twelve vertices A, at each of which ten triangles meet, form an icosahedral complex, the edges of which also belong to the system.

Twenty vertices B, at each of which three triangles meet, form a dodecahedral complex.

4.22. There exists a solid with thirty-two vertices, thirty edges, and sixty triangular faces.

It may be considered as made up of a dodecahedron $\{B^{20}\}$, on each of the faces of which stands a pentagonal pyramid with its vertex in a vertix of the icosahedron $\{A^{12}\}$. Each edge $(A^2B^2)$ belongs to six pyramidal faces, and five pyramidal faces $[AB^2]$ lie in the plane of every $[B^5]$.

4.23. Construct the diagonals $(B^2)$ in each $[B^5]$, determining other pentagons $[F^5]$.

Let a star-shaped pyramid with ten lateral edges $\{(AB)^5(AF)^5\}$ stand on each $[B^5F^5]$.

We obtain a solid with triangular faces of the types $[ABF]$, $[B^2F]$. The former are 120 in number, the latter 60.

The solid may be regarded as a kind of combination between 4.21, 4.22.

(In the figure, slightly more than half the number of edges only are drawn).

4.24. There exists a second solid with thirty-two vertices, thirty edges and sixty triangular faces (cf. 4.22.)

It may be regarded as a system of twenty triangular pyramids, the base-planes of which are the faces of the icosahedron $\{D^{12}\}$, while their vertices coincide with those of the dodecahedron $\{B^{20}\}$.

Each edge $(B^2D^2)$ belongs to six triangular faces, and five faces $[BD^2]$ lie in the plane of every $[D^5]$.

4.3. *Intersecting systems (first type.)*

4.31. Construct the face-diagonals $\{[(B^2)^5]^{12}\}$ of the dodecahedron $\{B^{20}\}$.

The diagonals belonging to any $[B^5]$ determine on each of the five edges $[(A^2)^5]$ in the same plane

(cf. 4.114) five points, one of which belongs to the leading plane. Excluding the latter, we have a system of 120 points E, two ($B^2$) passing through every E. (The remaining points form the system [$H^{15}$], discussed in 4.5.)

4.311. There are sixty lines ($E^2O$).

Only sixty points E are marked in our figure. They form, as will be seen, a system, complete in itself, and similar, to that, formed by the remaining sixty points. In the following theorems, only the first mentioned set is considered.

4.312. The system {$E^{60}$} is, when sufficiently "regular," a solid with sixty vertices, ninety edges, twelve pentagonal and twenty hexagonal faces. It may be considered as the intersection between the icosahedron {$A^{12}$} and a certain dodecahedron (not drawn), the latter truncating the solid angles of the former. There is one hexagonal figure [$E^6$] in every face [$A^3$], which is of Pascal's type, a Pascal line being the intersection ($H^3$) between the face [$A^3$] and the leading plane.

4.313. When the "solid angles" of an icosahedral complex are truncated by means of the "faces" of a dodecahedral complex, the two systems having the same leading point and leading plane, the hexagonal faces obtained will in general not be of Brianchon's type. There exists, however, one unique relation between the two systems, for which this is the case.—The hexagons [$E^6$] are not of Brianchon's type. Still, since the system {$E^{60}$} is constructed in a definite manner, it is not of the general type.

4.32. Construct the five diagonals ($B^2$) in each [$B^5$]. They determine a complex {$F^{60}$}, entirely equiva-

lent to the complex $\{E^{60}\}$. In particular, there are twenty planes, each containing six points F. The solid, represented in the figure, may be considered as the intersection between the dodecahedrom $(B^{20})$ and a certain icosahedron (not drawn), the former truncating the solid angles of the latter.

4.321. It is easy to construct the icosahedron in question. For this purpose, produce the five edges, passing through the vertices of a pentagonal face. They all meet in one point, which is a vertex in the icosahedral complex required.

4.322. Through each F passes one (ABC). There are thirty lines $(E^2F^2O)$.

4.323. Any straight line through O, which passes through the point of intersection between two diagonals in one of the faces of the complex $\{E^{60}\}$, also contains another such point in a certain second face, and the same line contains, in addition, two similar points, belonging to two faces of the complex $\{F^{60}\}$.

## 4.4. *Intersecting systems (second type).*

4.41. Construct the five diagonals $(F^2)$ in a pentagon $[F^5]$. Each diagonal $(F^2)$ intersects each of the five edges $(B^2)$, one in a point H in the leading plane, the others in points G outside the leading plane.

4.411. There are 120 points G, four on each edge $(B^2)$ Through every G pass two diagonals $(F^2)$. There are sixty lines $(G^2O)$.

4.412. If the system $\{B^{20}\}$ is sufficiently regular, as in our figure, consider separately the sixty points G lying between the two endpoints B on an edge $(B^2)$.

4.413. The system $\{G^{60}\}$, thus obtained, is a complex with O as leading point, which may be represented by means of a figure with sixty vertices, ninety edges, twenty triangular and twelve decagonal faces.

This solid may be regarded as the intersection between the dodecahedron $\{B^{20}\}$ and a certain icosahedron (not drawn), the latter truncating the solid angles of the former, both having the same leading point and the same leading plane.—It is evident, that such a solid may be obtained in an infinity of ways. Our complex $\{G^{60}\}$ is a unique special case.

4.414. There are five diagonals $(G^2)$ in every face $[G^{10}]$, each containing one F.

4.415. Every point of intersection between two diagonals in a face $[G^{10}]$, is associated with a similar point in a certain other face, the line joining the two points passing through O.

(For other properties of the present complex, see 4.56).

## 4.5. *The system in the leading plane.*

(This article is not illustrated).

4.51. Let the edges $(A^2)$ meet the leading plane in points H.

There are fifteen points H, through each of which pass two edges $(A^2)$.

Through every H pass two $(A^2B^2)$.

Through every H pass four face-diagonals $(B^2F^2)$ in the dodecahedron $\{B^{20}\}$.

4.511. Since the edges ($A^2$) belong to twenty triangular faces, which intersect, two at a time, in lines in the leading plane, it follows, that there are in this plane ten straight lines ($H^3$).
Through every H pass two ($H^3$).

4.512. Since the edges ($A^2$) belong to twelve planes [$A^5$ $B^5$], which intersect, two at a time, in lines in the leading plane, we have, in addition, six lines ($H^5$).
Through every H pass twe ($H^5$).

4.513. Through every H pass two edges ($D^2$) in the icosahedral complex {$D^{12}$}, as well as two other lines ($D^2$).

4.52. There are on every ($H^5$), five points K, through each of which pass two lines (ABC).

4.53. Let the six lines ($A^2O$) intersect the leading plane in the points [$L^6$].

4.531. There are fifteen lines ($H^2$), each containing two points L.
Through every L pass five lines ($H^2$).

4.54. Let the ten lines ($B^2O$) intersect the leading plane in the points [$M^{10}$]

4.541. There are two points M on each of the fifteen lines ($H^2$) of 4.531.
Through every M pass three lines ($H^2$).

4.55. Through every H pass:
four edges ($E^2$) in the system {$E^{60}$} of 4.312, not counting any ($A^2E^2$);
four diagonals ($E^2$), belonging to hexagonal faces [$E^6$]
four diagonals ($E^2$), belonging to pentagonal faces [$E^5$].

Of course, the complex $\{F^{60}\}$ has similar properties.

4.56. The three edges $(G^2)$ belonging to a triangular face $[G^3]$ in the system $\{G^{60}\}$ of 4.412, pass resp. through the points H on a line $(H^5)$.

Of the lines $(G^2)$ in any face $[G^{10}]$, two edges and three diagonals pass through the same H.

4.561. Through every $(H^5)$ pass four planes $[G^6]$ (not counting any $[B^5]$), and two planes $[G^{10}]$.

I have not been able to perform a generalisation in the present complex on the lines of 2.A.II., 3.A.II. It appears unlikely, however, that this should not be possible.

### *B. In two dimensions.*

I have not considered it worth while, to discuss the slight modifications necessary to adapt the theorems of the present section to plane geometry. They are perfectly analogous to those made on previous occasions.

It should be noted, however, that the dodecahedral and icosahedral systems in a plane are not polar-reciprocal.

## § 5. *Reciprocal systems.*

The tetrahedral complex in space is evidently polar-reciprocal to itself, and we have seen, that the remaining four fundamental systems in space are polar-reciprocal in pairs. The same is not true for plane geometry, from which it follows, that we obtain five new groups of theorems, simply by letting the letters in any formula of the plane geometry of a complex denote *lines* instead of *points* and making corresponding modifications in the text. A complete discussion of the results obtained, would, however, lead to unnecessary repetitions of the above, and is for this reason left out.

In particular:

*There is a group of theorems for a Pascal hexagon analogous to those given under* 2.*B. for a Brianchon hexagon.*

## § 6. *Conclusion.*

Summing up the above analysis in a few words, I may call it a contribution to the theory of groups of space-elements, equivalent in projective sense.

Our immediate results are only applicable to the elements: point, line, plane.

We have seen, that certain finite groups of these elements exist, each element of which is equivalent to any other. But these results may easily be extended. It is clear, that if a sufficient number of elements of such a group be associated, each in the same manner, with space-elements of a given type—curves or surfaces—then all the elements of the group will have this property; and for the elements of any other group, belonging to the same complex as the first mentioned, a similar corresponding property may be established. Evidently we have here a vast field of research, as yet almost unexplored.

It is not possible to enter into details here. A few suggestions may, however, not be out of place.

Theorem 1.14. may serve as an illustration. There one part of the system is fixed, and another varies in such a manner, that the motion of one point is unrestricted. If it describes an algebraical space-curve, three other points describe curves of the same type. During this motion, six lines develop equivalent surfaces, and four planes envelop equivalent surfaces, etc. The relation between the elements obtained may now be studied, as well as the point-system, formed by their intersections, etc., also the relation between the old and new systems.

Other systems are rigid, so that the unrestricted motion of four of its points affects the whole complex. Such are the fundamental systems in 2., 3., 4.

The shape of a complex may sometimes determine elements of higher type, notably conicoids.—The systems $\{A^{12}\}$, $\{B^{20}\}$, etc. in section 4. lie on conicoids, completely defined, since, in the regular form of the complex, these systems lie on spheres. Similarly, certain conicoids touch the corresponding systems of face-planes. The leading point is the pole of the leading plane with regard to all these conicoids.—Through the five vertices in any face $[B^5]$ a conic may be constructed, with regard to which a point C is the pole of a line $(H^5)$. These conics, twelve in number, are plane sections of the conicoid determined by $\{B^{20}\}$, and, if the figure be projected on a plane, are in a certain relation to one another.—If the complex moves in a given manner (for instance so that the points A describe straight lines in the most general way), each conicoid will determine a family of such surfaces with certain well defined properties.—Many more problems of this type may be stated, and any other complex may be treated in a similar way.

Similar problems to the above may be stated for plane geometry.—

The present discussion, naturally, does not claim to be complete. I intend to deal with questions of this type in a subsequent paper.

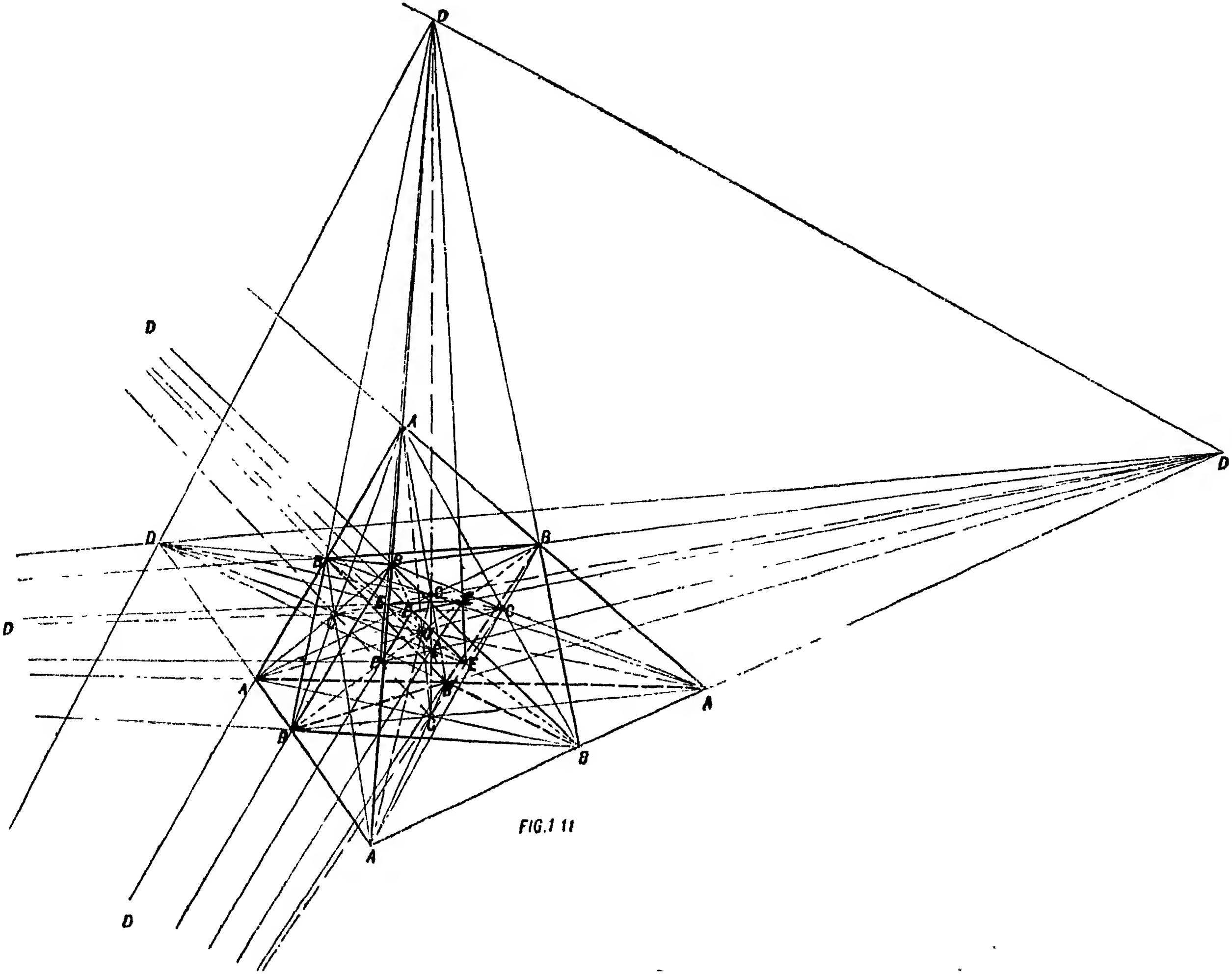
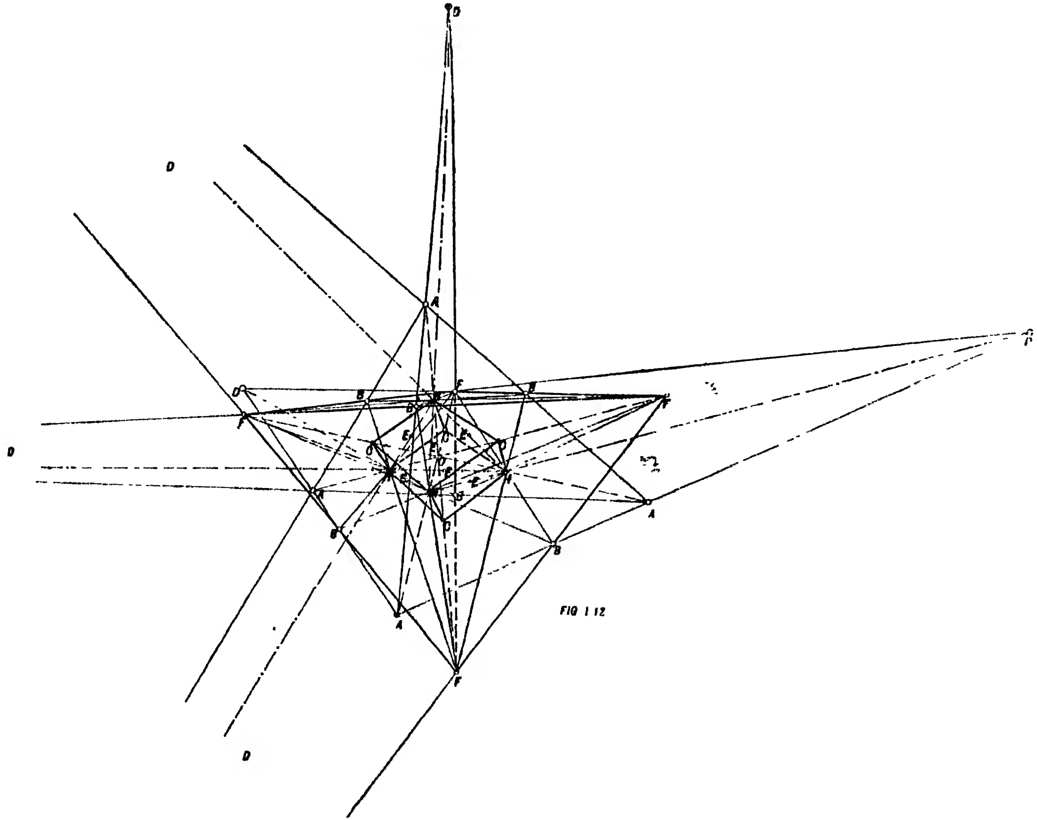



FIG.1 11



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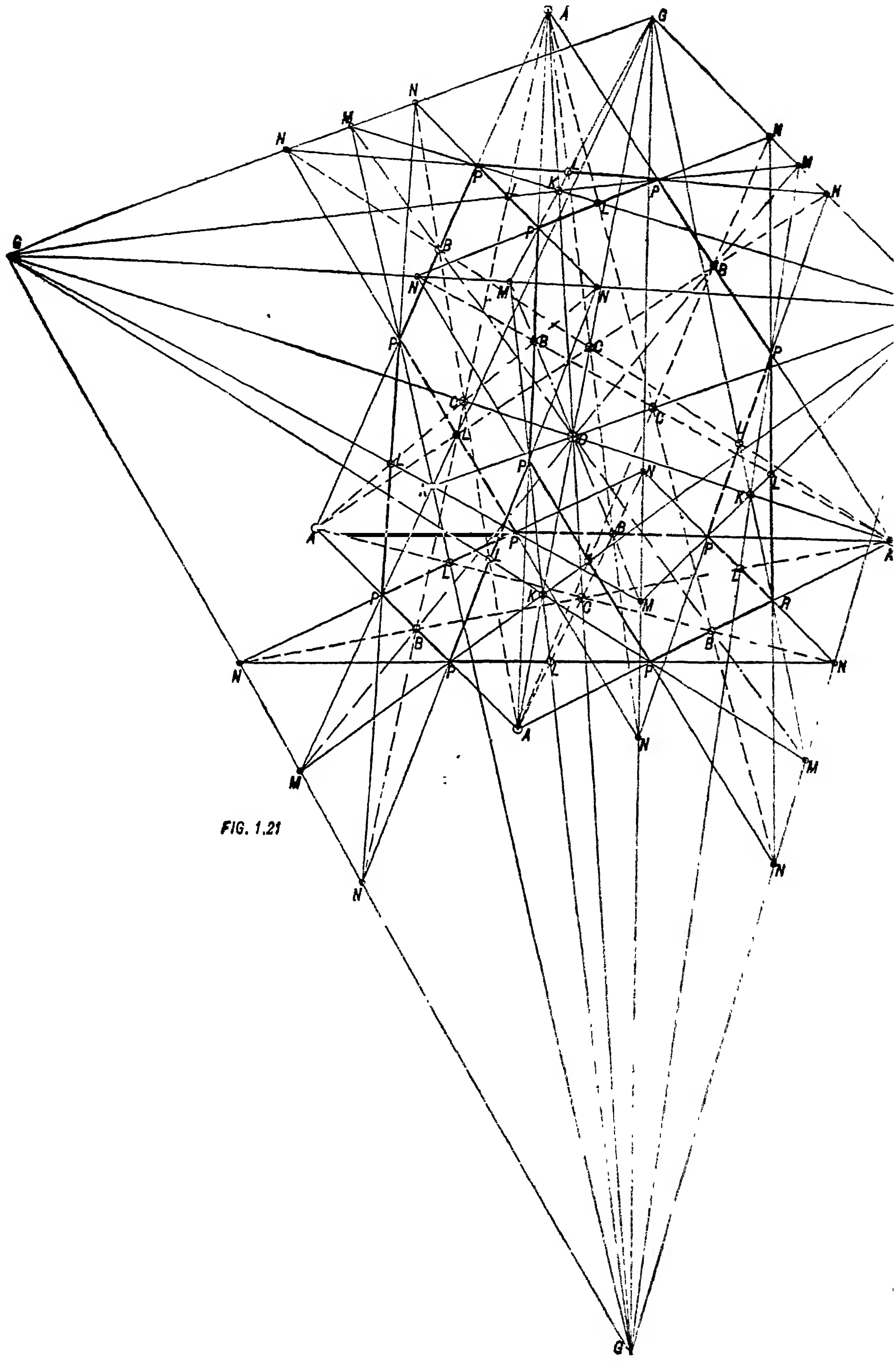
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FIG. 1.21

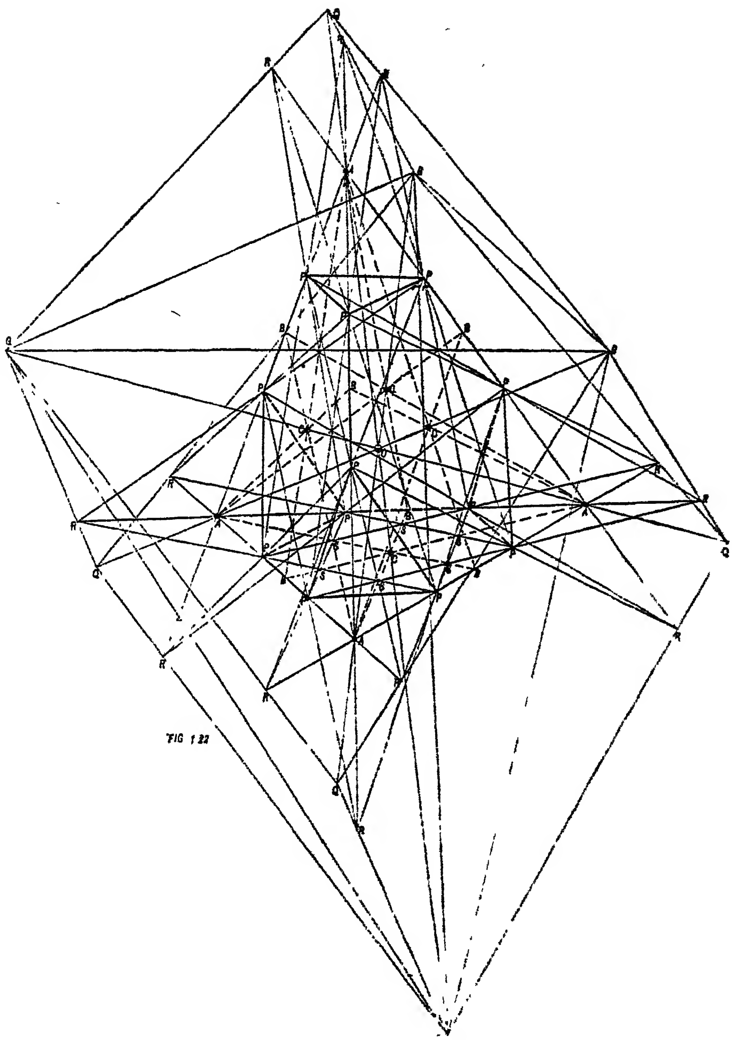

FIG 1 22

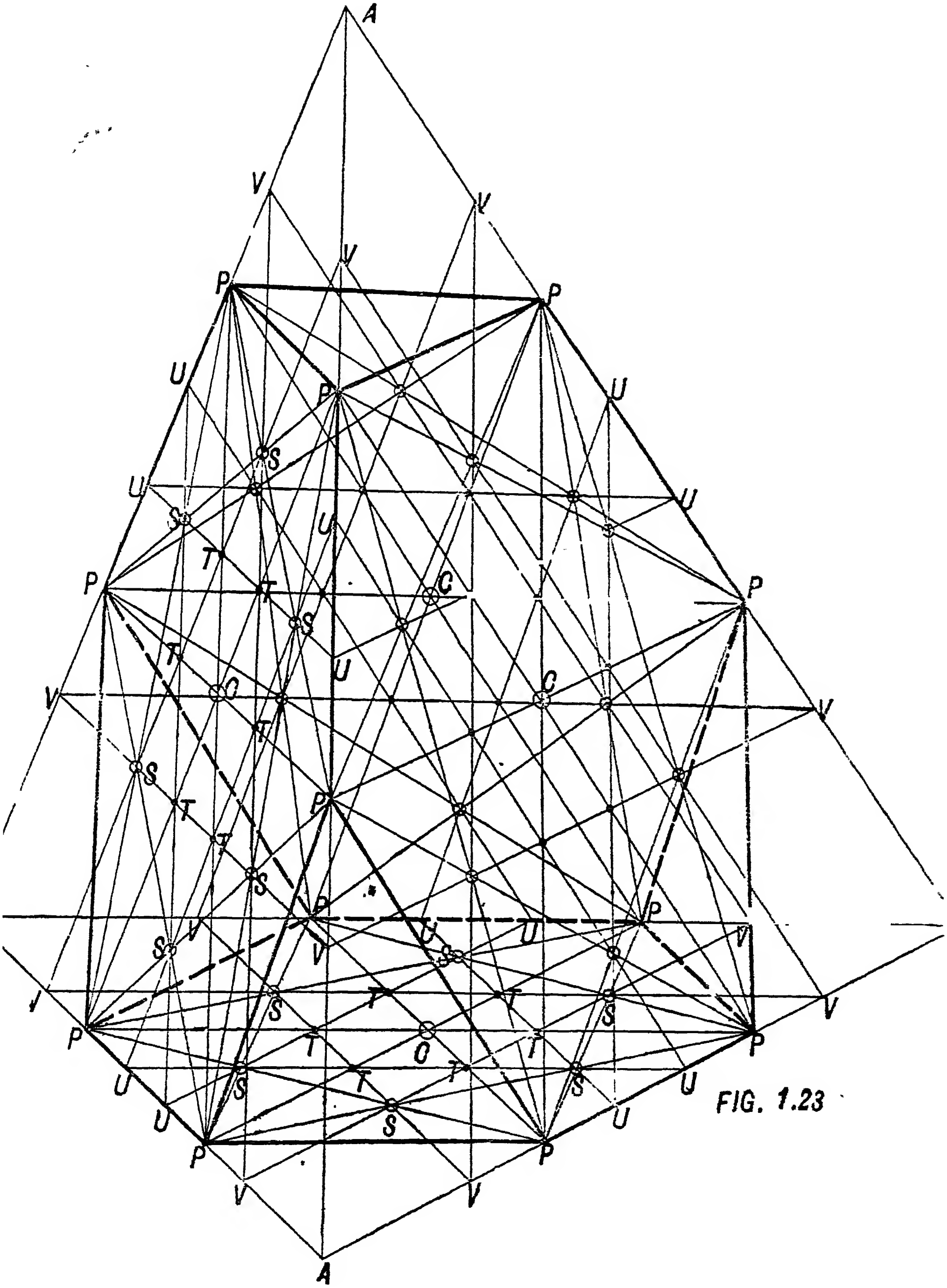


FIG. 1.23

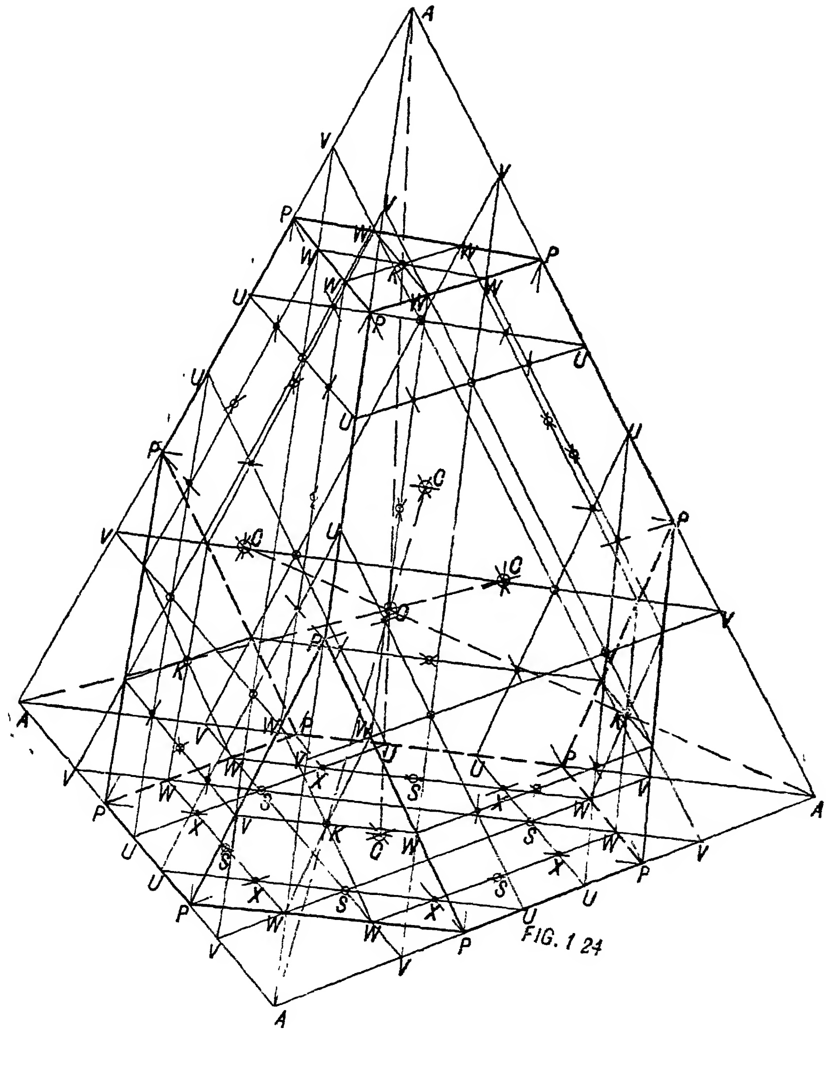
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FIG. 1 24

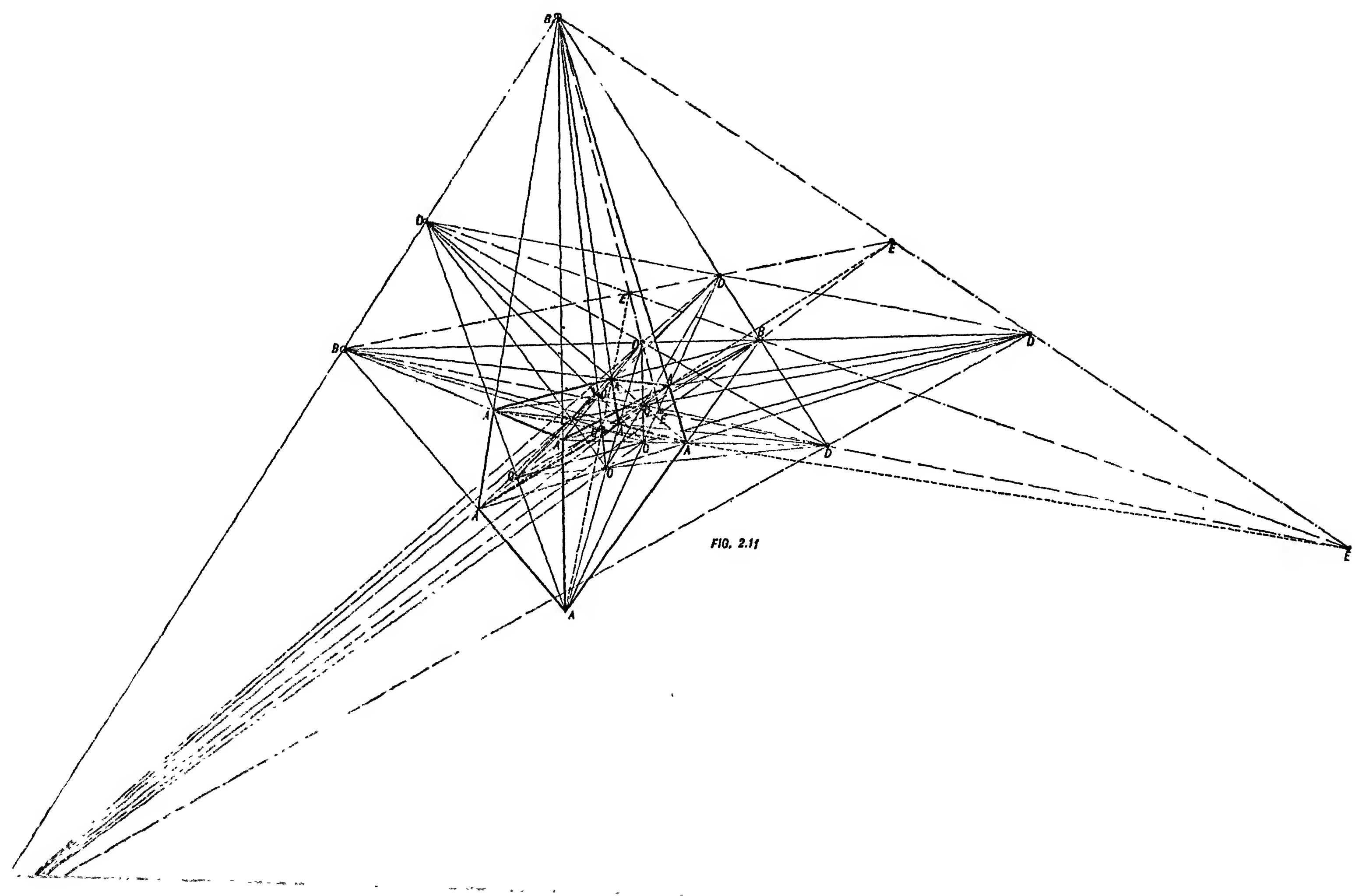



FIG. 2.11

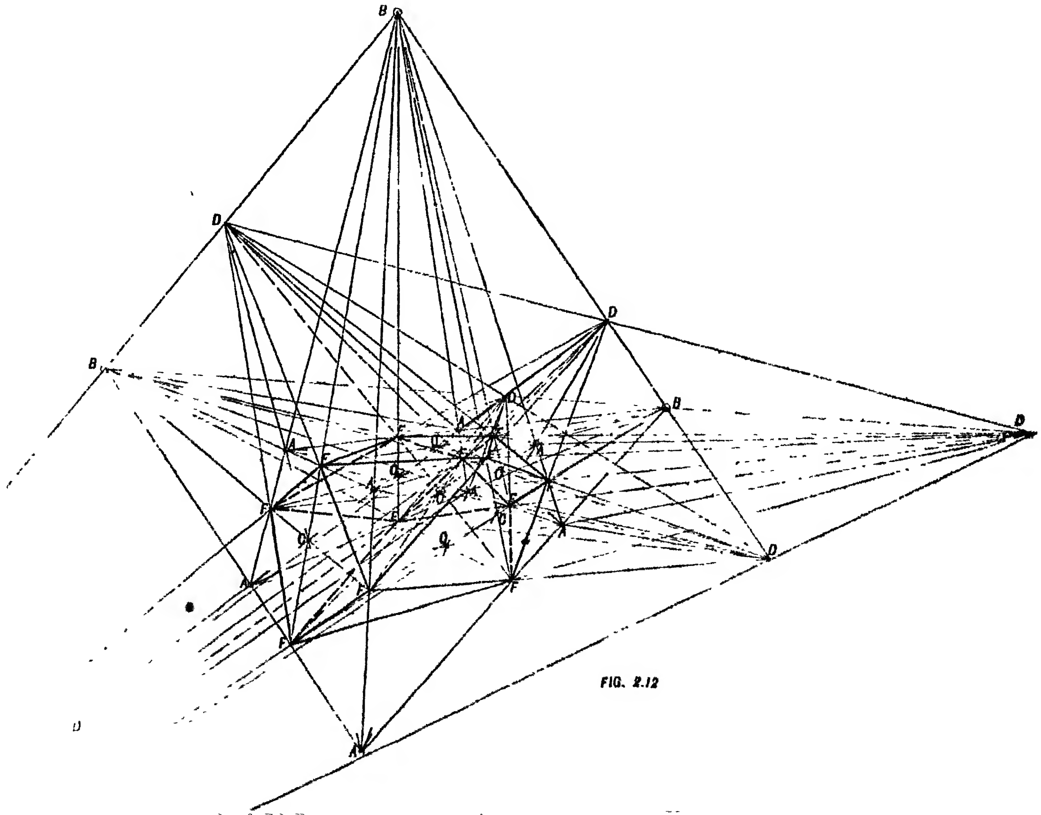



FIG. 2.12

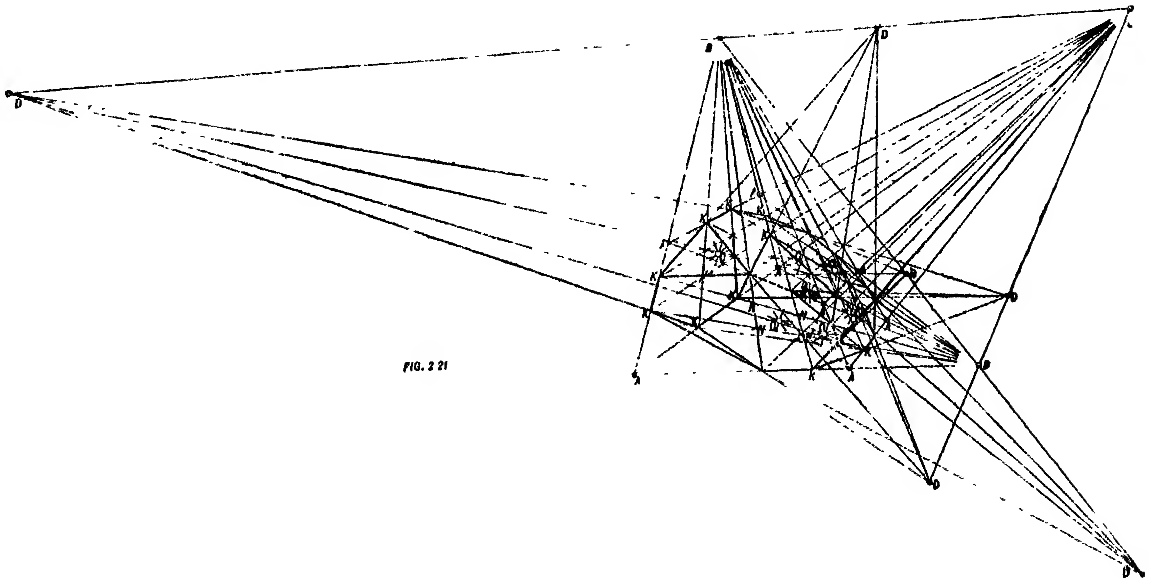


FIG. 2.21

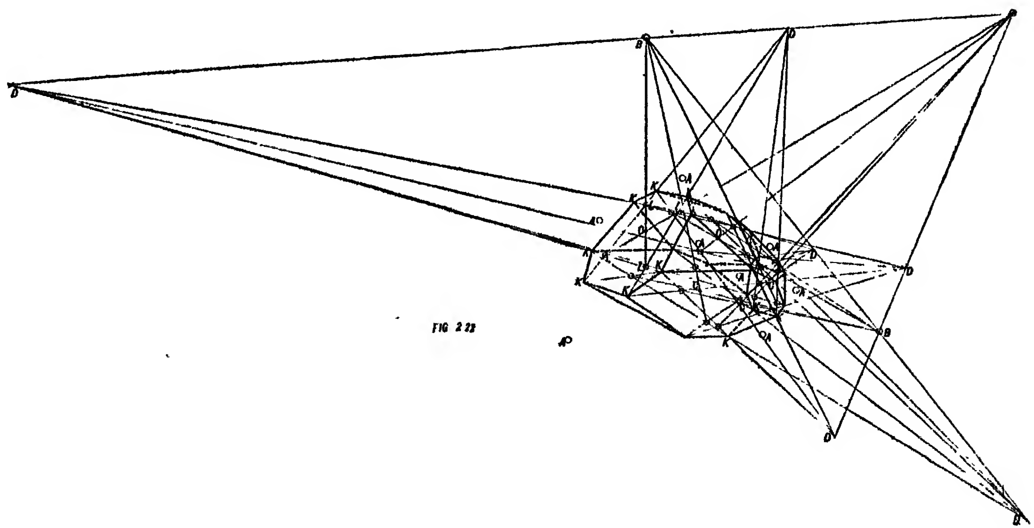


FIG 2 28

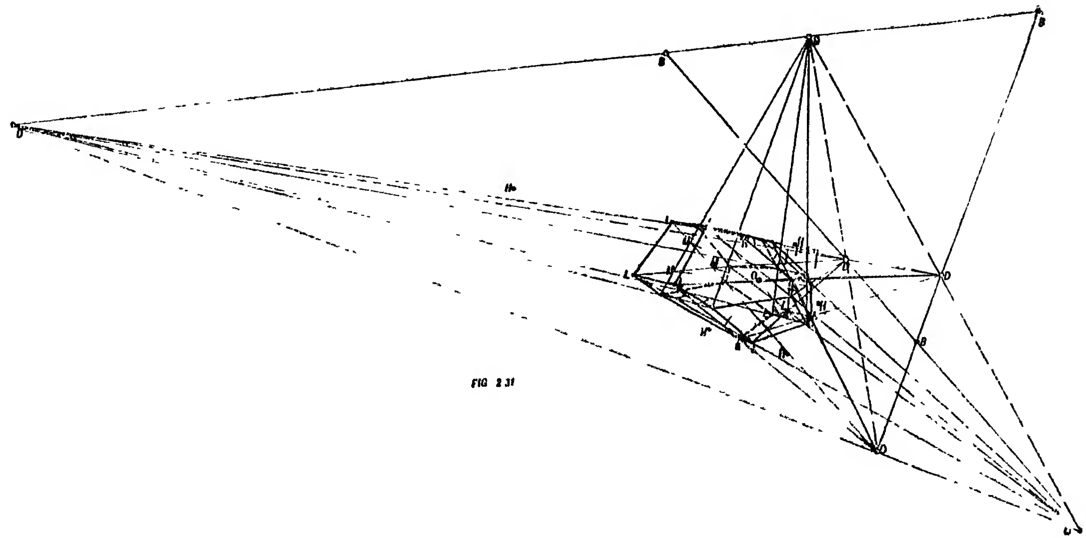


FIG 2.31

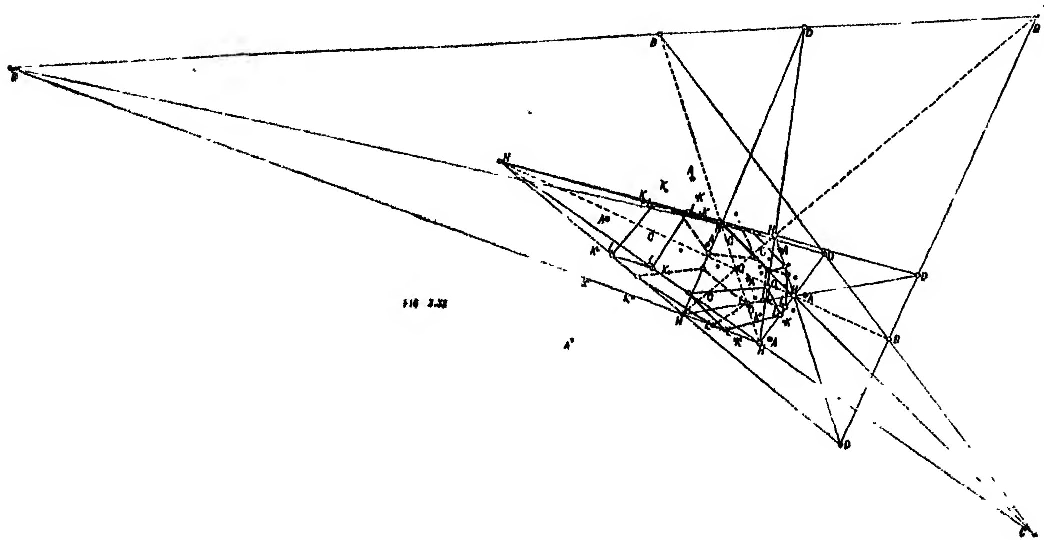


FIG 2.33

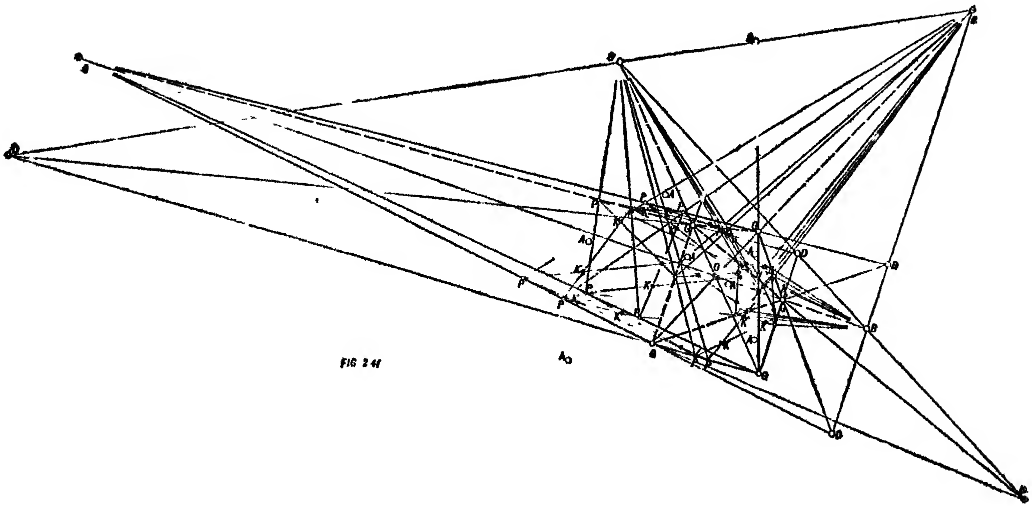


FIG 2 41

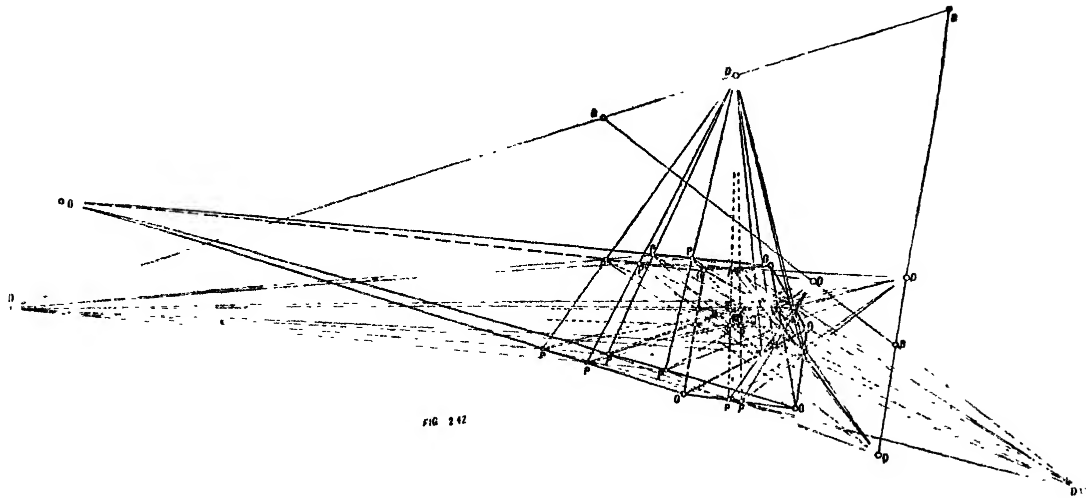


FIG 242

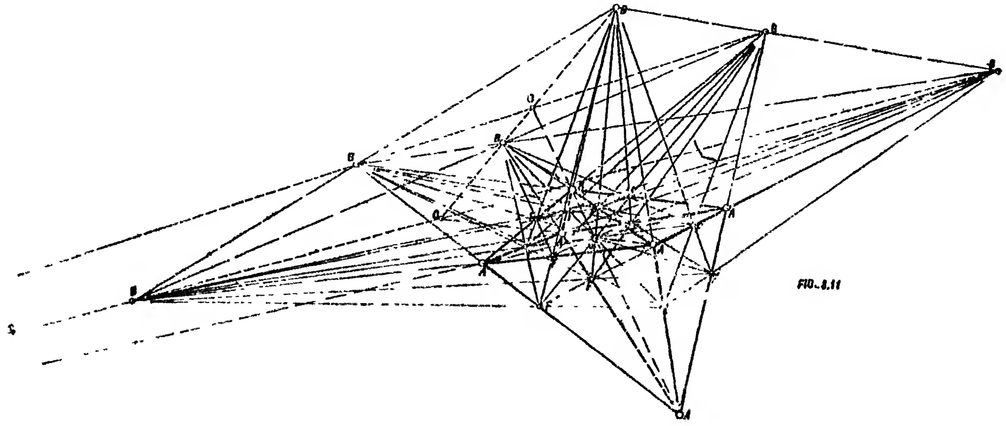


FIG. 8.11

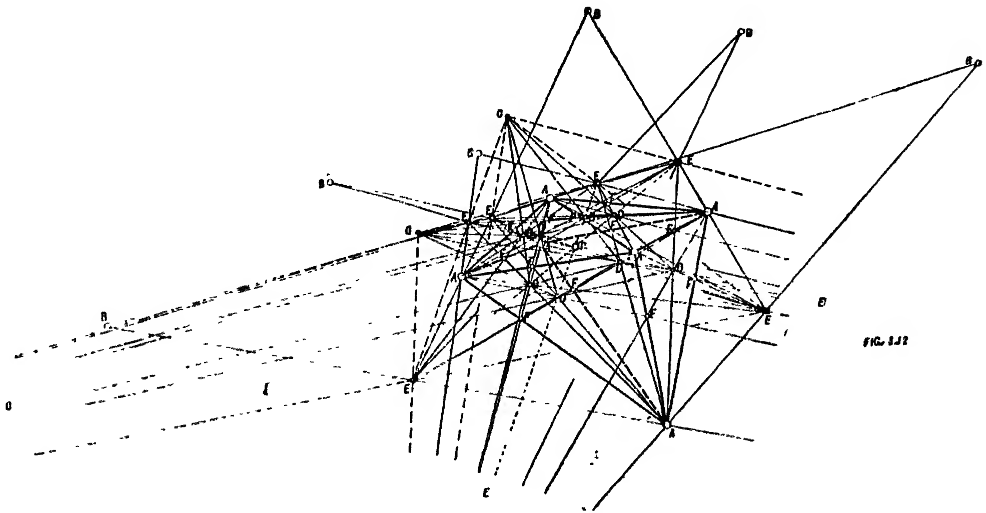


FIG. 3.12

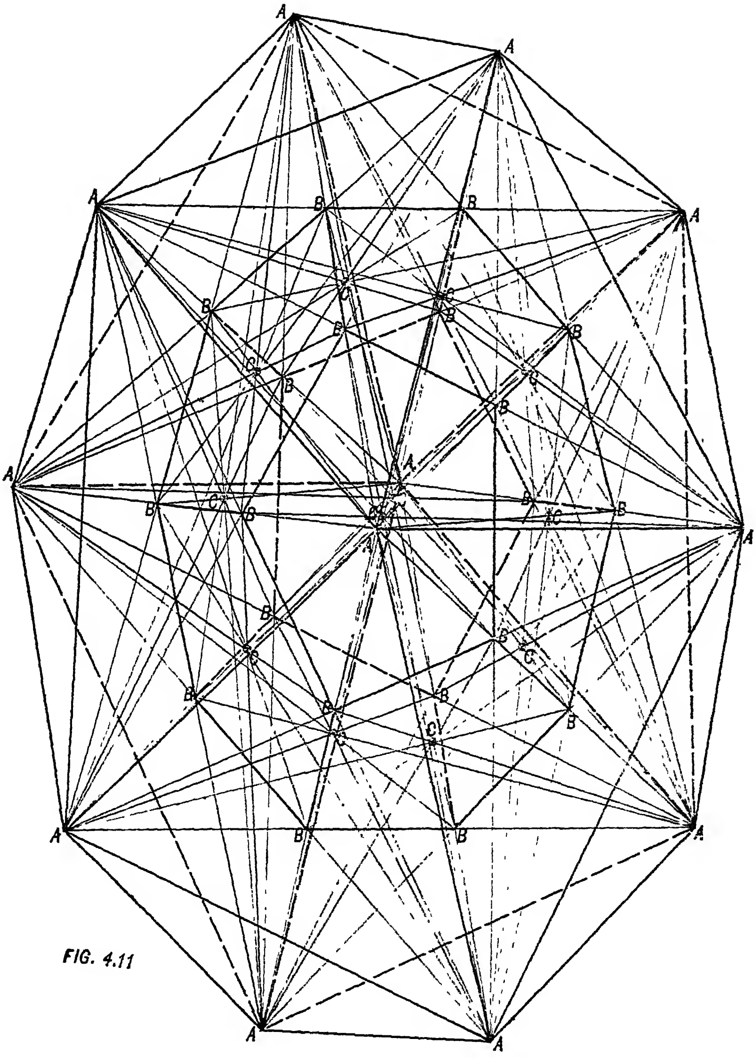



FIG. 4.11

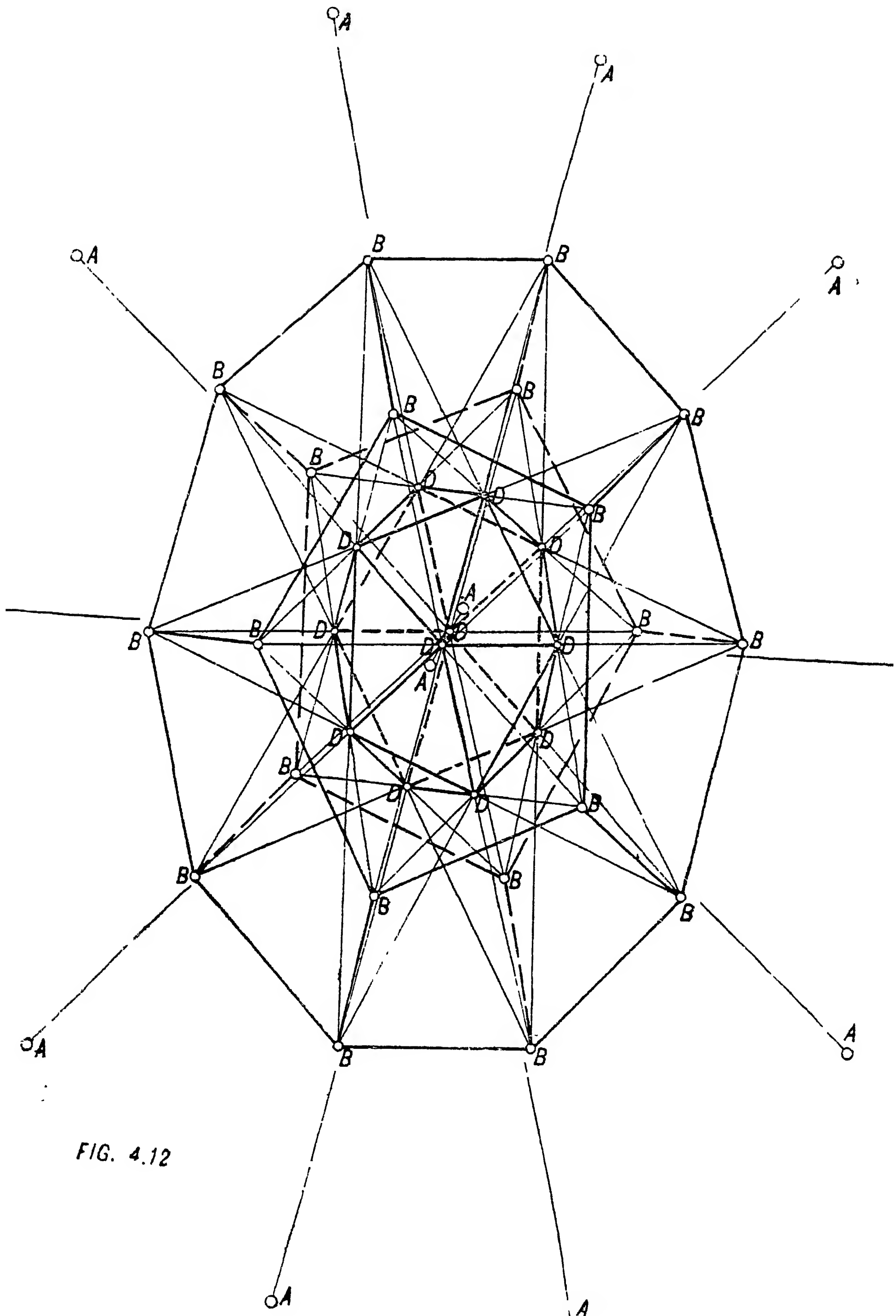



FIG. 4.12

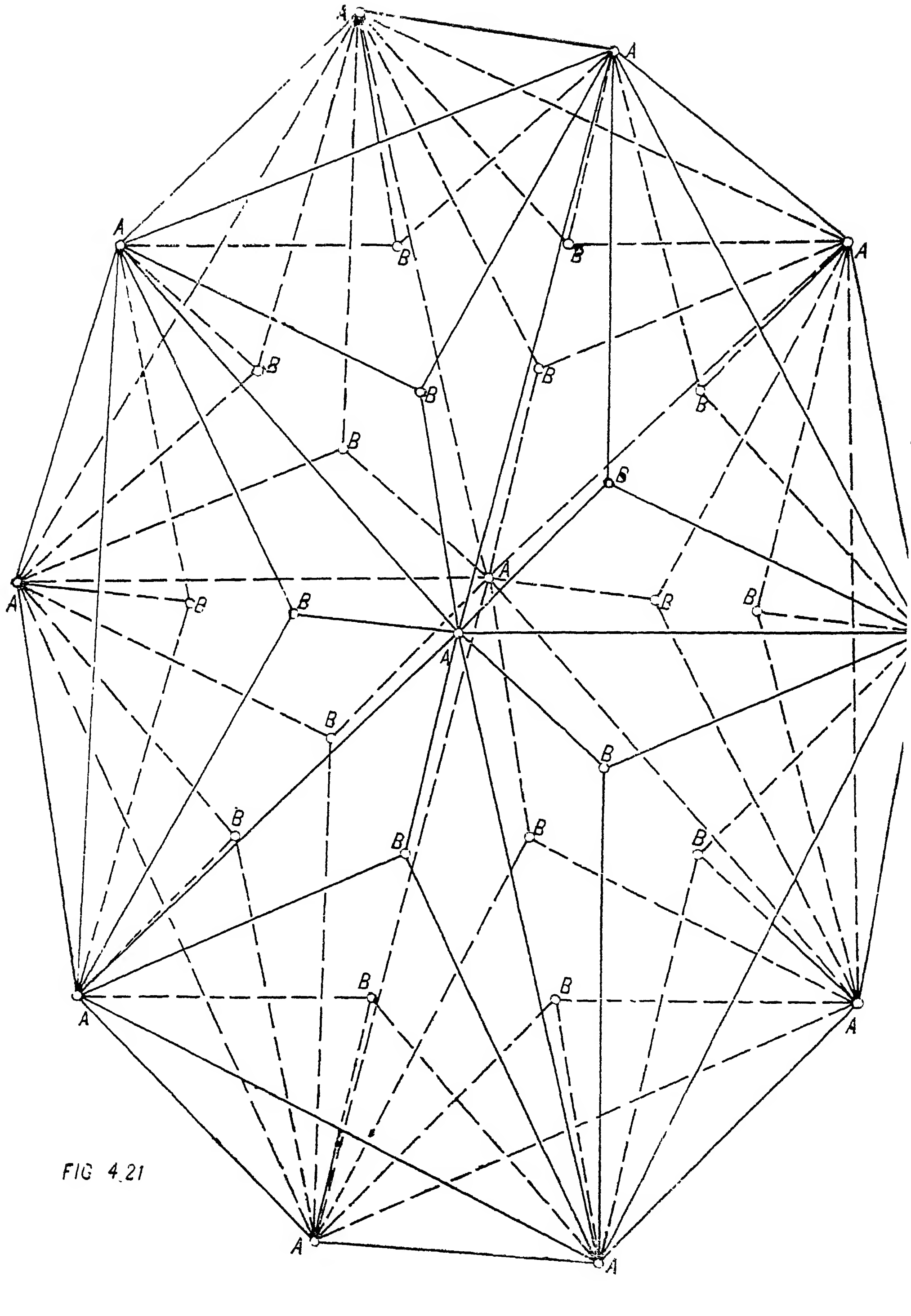



FIG 4.21

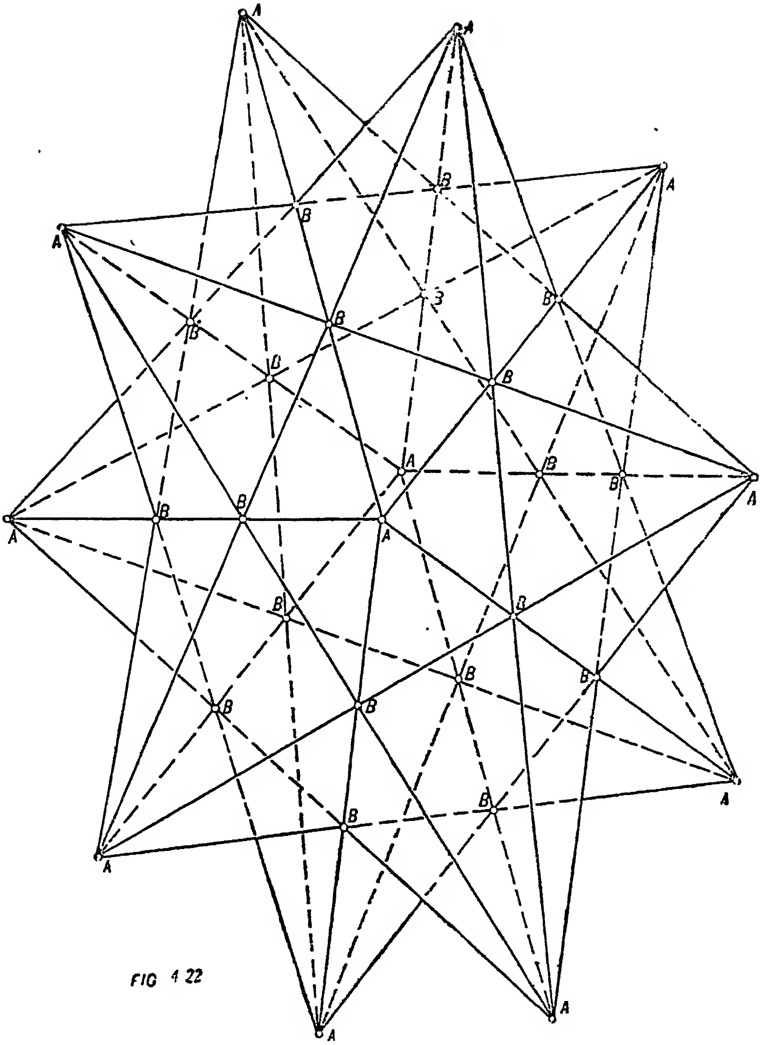
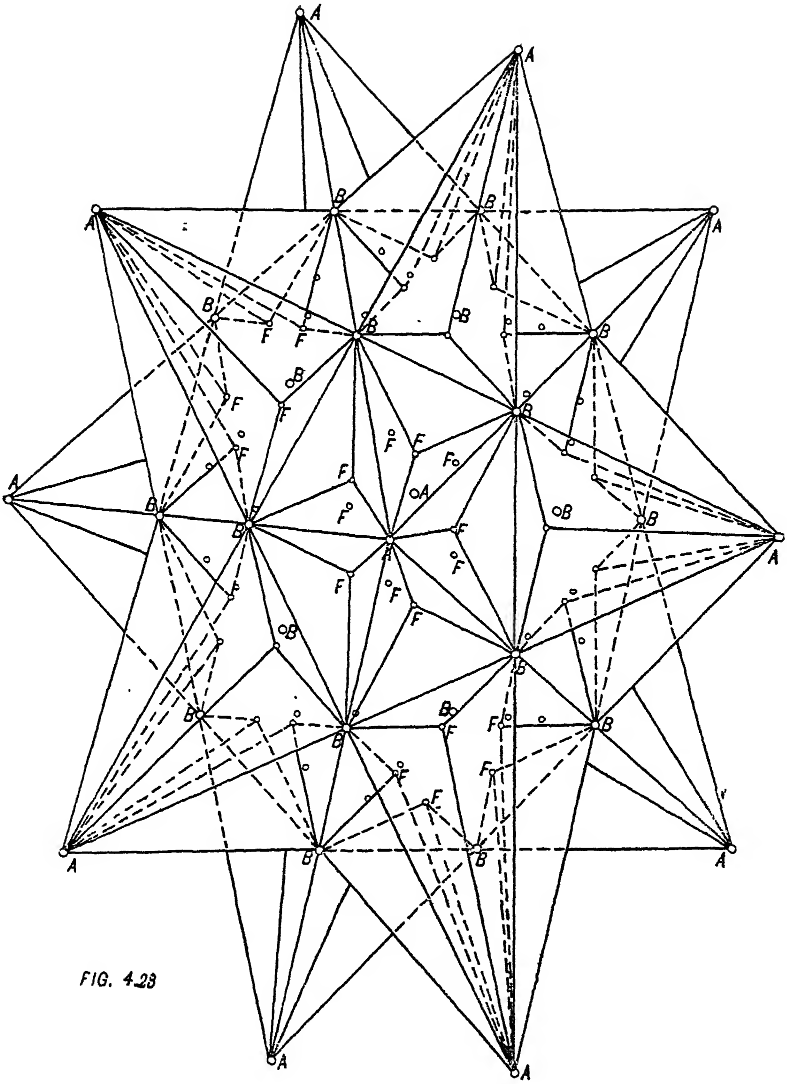



FIG 4 22

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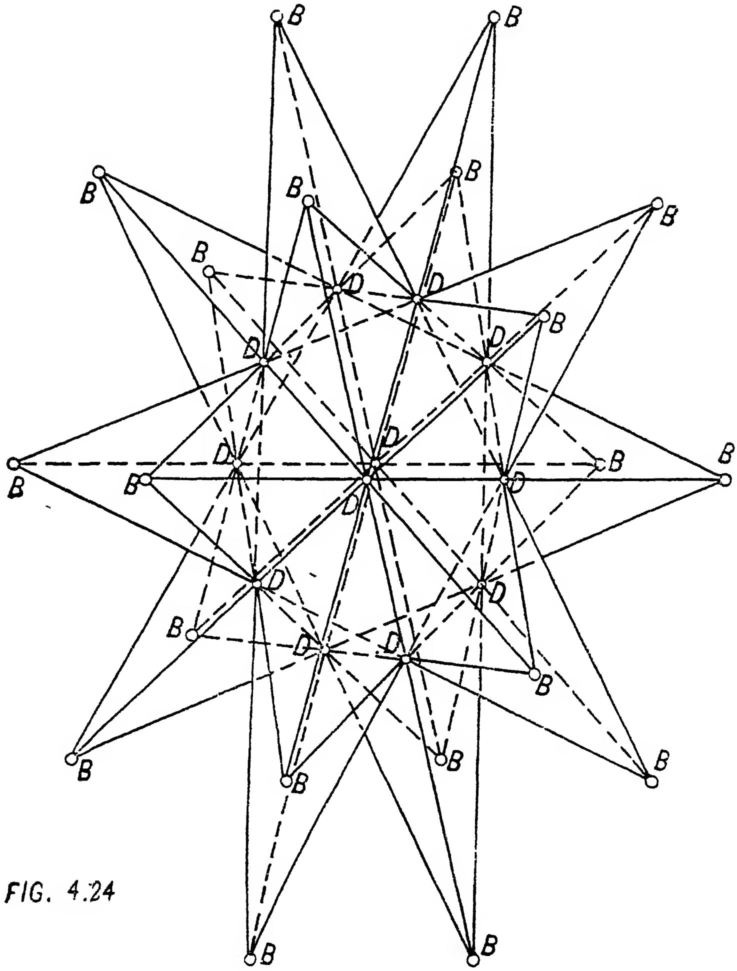


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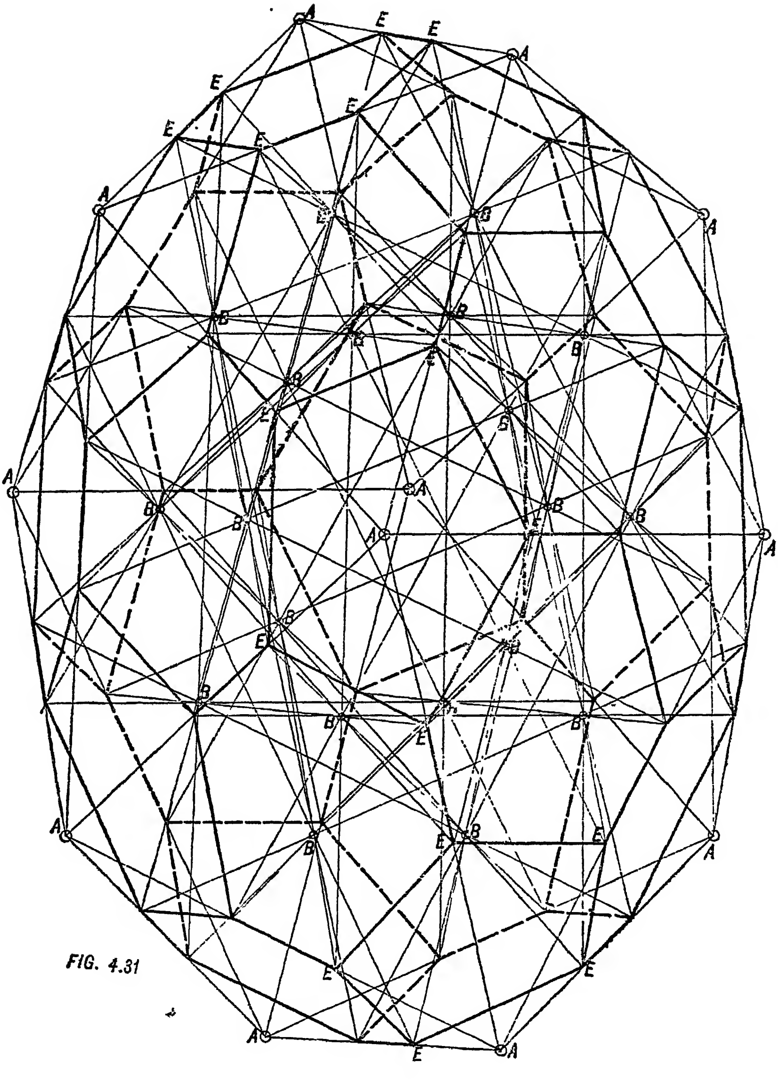


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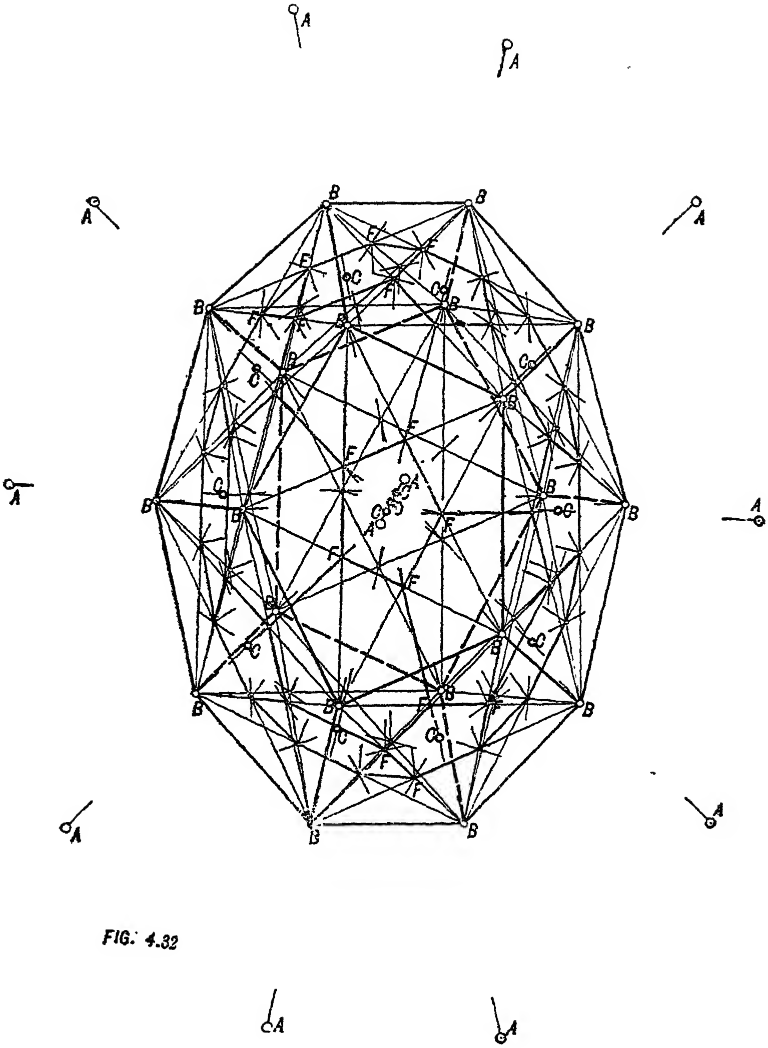

FIG. 4.32

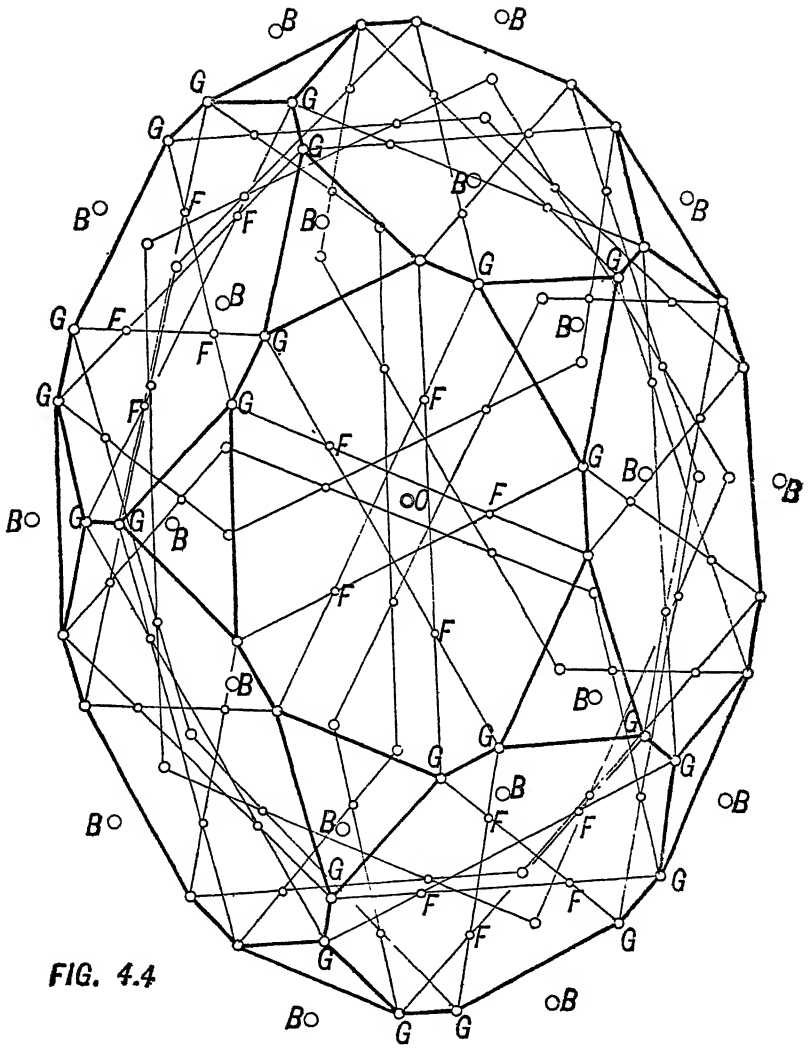
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FIG. 4.4

PROCEEDINGS

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Vol. IV. PART II.

# On the Vibrations of Elastic Shells partly filled with Liquid.

By

SUDHANSUKUMAR BANERJI, M.Sc.

## § 1. *Introduction.*

The problem considered in this paper is chiefly of acoustical interest in relation to the theory of "musical glasses." This class of instrument consists of a series of thin-walled elastic shells whose gravest modes of vibration are tuned to form a musical scale by partially filling them with a liquid and are excited either by striking or by tangential friction on the rims. The principal features of interest requiring elucidation are (1) the dependance of the pitch of the vibration upon the quantity of liquid contained in the vessel and (2) the mode of vibration of the liquid itself. These features are discussed in this paper for the three cases in which the elastic shell is respectively (1) a hemispherical shell, (2) a cylindrical vessel with a flat bottom and (3) a conical cup, these forms approximating more or less closely to those used in practice. The analytical expressions show that the motion of the liquid is very marked near the margin of the vessel and is almost imperceptible near the centre and also at some depth inside the liquid. This feature becomes more and more marked in the case of the higher modes of

vibration of the vessel. Numerical results have also been obtained and tabulated showing the theoretical relation between the quantity of liquid contained in the vessel and the vibration frequency. These show that the rapidity with which the frequency falls on addition of liquid is greatest when the vessel is nearly full, this being specially noticeable in the case of the higher modes of vibration.

The general principle of the analytical method used is similar to that adopted by Lord Rayleigh[1] in treating the two-dimensional case of a long cylinder completely filled with liquid which was studied by Auerbach[2]. This case has also been recently discussed by Nikoloi[3]. The lowering of the pitch produced by the liquid is of course due to the added inertia exactly as in the related case of the vibrations of a bar or a string immersed in a liquid which have been studied by Northway[4], Mackenzie and Kalähne[5].

Musical glasses are sometimes excited by rotating them about a fixed vertical axis, the tangential friction being produced by a rubber kept in a fixed position. No attempt is made in this paper to consider this somewhat complicated case[6], which I hope to be able to deal with on a future occasion.

---

1. Lord Rayleigh, Phil. Mag., XV, pp. 385-389, (1883). [Scientific Papers, Vol. 2, pp. 208-211].

2. Auerbach, Wied. Ann., 3, p. 157, (1878) and also Wied. Ann., 17, p. 964, (1882). Reference may also be made to the papers by Montigny, Bull. del' Acad. de Belg., [2], 50, 159, (1880) and by Kolácek, Wied. Ann. 7, 23, (1879) and also Sitz. math. naturw. cl. Wien, 87, Abth. 2, (1883).

3. Nikoloi, Journ. Russk. Fisik Chimicesk, 41, 5, pp. 214-227, (1909),

4. Northway and Mackenzie, Phys. Rev., 13, pp. 145-164, Sept., (1901)

5. Kalähne, Ann. d. Physik, 46, 1, pp. 1-38, (1914).

6. Reference may be made in this connection to papers by Prof. Love on "The free and forced vibrations of an Elastic Spherical Shell containing a given mass of Liquid," Proc. Lond. Math. Soc., Vol. XIX, where he has studied the case of a rotating spherical shell completely full of liquid, and by Prof. Bryan on "The beats in the vibrations of a revolving cylinder or bell," Proc. Camb. Phil. Soc., Vol. VII, (1892).

## § 2. *Hemispherical Cups.*

The force which a thin sheet of matter subjected to stress opposes to extension is very great in comparison with that which it opposes to bending. From this Lord Rayleigh concluded that the middle surface of a vibrating shell remains unstretched and proposed a theory[7] of flexural vibrations of curved plates and shells in accordance with this condition. As the direct application of the Kirchhoff-Gehring method led to equations of motion and boundary conditions which were difficult to reconcile with Lord Rayleigh's theory, his theory gave rise to much discussion. Later investigations have, however, shown that any extension that may occur must be limited to a region of infinitely small area near the edge of the shell and that the greater part of the shell vibrates according to Lord Rayleigh's type.

Let the radius of the hemisphere be equal to $a$. Let a point whose natural co-ordinates are $a$, $\theta$, $\phi$ be displaced to the position $a+u$, $\theta+v$, $\phi+w$, where $u$, $v$, $w$ are to be treated as small. From the condition of inextension

$$\begin{aligned}(\delta s)^2 &= a^2\,(\delta\theta)^2 + a^2 \sin^2\theta\,(\delta\phi)^2 \\ &= (a+u)^2\,(\delta\theta+\delta v)^2 + (\delta u)^2 + (a+u)^2 \sin^2(\theta+v)\,(\delta\phi+\delta w)^2, \qquad (1).\end{aligned}$$

Lord Rayleigh obtains the three differential equations

$$\begin{aligned}&\frac{\partial v}{\partial\theta} + \frac{u}{a} = 0, \\ &\frac{\partial v}{\partial\phi} + \sin^2\theta\frac{\partial w}{\partial\theta} = 0, \qquad (2) \\ &\frac{u}{a} + \cot\theta.\, v + \frac{\partial w}{\partial\phi} = 0,\end{aligned}$$

which can be integrated in the forms

$$\frac{u}{a} = \begin{matrix}\sin m\phi \\ -\cos m\phi\end{matrix}\left[A_m\,(m+\cos\theta)\tan^m\tfrac{1}{2}\theta + B_m(m-\cos\theta)\cot^m\tfrac{1}{2}\theta\right],$$

$$\frac{v}{\sin\theta} = \begin{matrix}-\sin m\phi \\ \cos m\phi\end{matrix}\left[A_m\tan^m\tfrac{1}{2}\theta - B_m\cot^m\tfrac{1}{2}\theta\right], \qquad (3)$$

7. Lord Rayleigh, Proc. Lond. Math. Soc., Vol. XIII. p. 4, (1881). See also Proc. Roy. Soc., Vol. 45, p. 45 and 443, (1881), Theory of Sound, Vol. I, Chap. XA and Love's Elasticity, Chap. XXXIII.

$$w = \frac{\cos m\phi}{\sin m\phi}\left[A_m \tan^m\tfrac{1}{2}\theta + B_m \cot^m\tfrac{1}{2}\theta\right],$$

$A_m$ and $B_m$ being arbitrary constants. These equations determine the character of the displacement of a point in the middle surface.

Since the pole $\theta=0$ is included, the constant $B_m$ must be considered to vanish and the type of vibration in a principal mode is expressed by the equations

$$\begin{aligned} u &= A_m\, a\, (m+\cos\theta) \tan^m\tfrac{1}{2}\theta \sin m\phi, \\ v &= -A_m \sin\theta \tan^m\tfrac{1}{2}\theta \sin m\phi, \\ w &= \quad A_m \tan^m\tfrac{1}{2}\theta \cos m\phi, \end{aligned} \tag{4}$$

in which $A_m$ is proportional to a simple harmonic function of the time.

The potential energy of bending of the vibrating shell is given by

$$\begin{aligned} V &= \tfrac{8}{3}\pi\mu\frac{t^3}{a^2} m^2 (m^2-1)^2 A_m^2 \int_0^{\frac{\pi}{2}} \tan^{2m}\frac{\theta}{2} \frac{d\theta}{\sin^3\theta} \\ &= \tfrac{2}{3}\pi\mu\frac{t^3}{a^2}(m^3-m)(2m^2-1) A_m^2, \end{aligned} \tag{5}$$

where $t$ = thickness of the shell and $\mu$ = rigidity.

The kinetic energy T is given by the expression

$$\begin{aligned} T &= \tfrac{1}{2}\pi\sigma a^4 t\left(\frac{dA_m}{dt}\right)^2 \int_0^{\frac{\pi}{2}} \sin\theta\{2\sin^2\theta + (\cos\theta+m)^2\} \tan^{2m}\tfrac{1}{2}\theta\, d\theta. \\ &= \tfrac{1}{2}\pi\sigma a^4 t\left(\frac{dA_m}{dt}\right)^2 \int_1^2 \frac{(2-x)^m}{x^m}[(m-1)^2+2(m+1)x-x^2]dx \\ &= \tfrac{1}{2}\pi\sigma a^4 t\, f(m)\left(\frac{dA_m}{dt}\right)^2, \end{aligned} \tag{6}$$

where $\sigma$ represents the density of the shell, and

$$f(m) = \int_1^2 \frac{(2-x)^m}{x^m}[(m-1)^2+2(m+1)x-x^2]\, dx,$$

which can be evaluated for any integral value of m.

Since the types of vibrations of the shell are entirely determined by the geometry of the middle surface of the shell, it is obvious that the types can under no circumstances be altered by the presence of the liquid in the shell. The liquid gives rise to a surface traction and affects only the arbitrary constant $A_m$, that is to say, the amplitude and the frequency of vibration of the shell.

The motion of the liquid will depend upon a velocity potential which satisfies the equation

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{2}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Phi}{\partial\theta}+\frac{\operatorname{cosec}^2\theta}{r^2}\frac{\partial^2\Phi}{\partial\phi^2}=0 \qquad (8)$$

A solution of this differential equation which will correspond to the type of vibration of the shell can be obtained by assuming $\Phi$ to be of the form

$$\Phi=\left(C_m r+\frac{D_m}{r^2}\right)\sin m\phi.\ \Delta_\theta,$$

where $\Delta_\theta$ is a function of $\theta$ only. Substituting in the differential equation we find that $\Delta_\theta$ satisfies the equation

$$\frac{d^2\Delta_\theta}{d\theta^2}+\cot\theta\frac{d\Delta_\theta}{d\theta}+(2-m^2\operatorname{cosec}^2\theta)\Delta_\theta=0.$$

The general solution of this differential equation is

$$\Delta_\theta=E_m\tan^m\tfrac{1}{2}\theta\,(m+\cos\theta)+F_m\cot^m\tfrac{1}{2}\theta\,(m-\cos\theta).$$

Neglecting solutions of the type $\cot^m\frac{1}{2}\theta\,(m-\cos\theta)$, we see that $\Phi$ is of the form

$$\Phi=\left(C_m r+\frac{D_m}{r^2}\right)\sin m\phi\tan^m\tfrac{1}{2}\theta\,(m+\cos\theta), \qquad (9)$$

where $C_m$ and $D_m$ are two arbitrary constants.

Let us first take

$$\Phi=C_m\frac{r}{a}\tan^m\tfrac{1}{2}\theta\,(m+\cos\theta)\sin m\,\phi\cos pt. \qquad (10)$$

The relation between $C_m$ and $A_m$ of (4) is readily found by equating the value of $\frac{\partial\Phi}{\partial r}$, when $r=a$, to $\frac{\partial u}{\partial t}$, both of which represent the normal velocity at the circumference. We get

$$C_m\cos pt=a^2\frac{dA_m}{dt}. \qquad (11)$$

The expression (10) determines the principal mode of vibration of the liquid. The simple character of the fluid motion as determined by this expression will however be a little disturbed on account of the existence of a free surface and we shall have to add a small correction to this expression. The condition to be satisfied at the free surface is

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0, \quad \text{when } z=h.$$

where $h$ denotes the depth of the liquid surface below the centre of the hemisphere. We shall neglect the force of gravity, inasmuch as the period of free waves of length comparable with the diameter of the shell is much greater than that of the actual motion. The condition to be satisfied at the free surface then becomes simply

$$\Phi=0, \quad \text{when } z=h.$$

Hence we must have

$$\Phi=C_m\frac{r}{a}\tan^m\tfrac{1}{2}\theta\,(m+\cos\theta)\sin m\phi\cos pt+f(r,\theta,\phi)\cos pt, \qquad (12)$$

where $f(r, \theta, \phi)$ is a solution of $\nabla^2\Phi=0$ and is such that its differential co-efficient with respect to $r$ vanishes on the spherical boundary and it has the value

$$-C_m\frac{h\sec\theta}{a}\tan^m\tfrac{1}{2}\theta\,(m+\cos\theta)\sin m\phi \qquad (13)$$

on the free surface.

In the particular case, when the shell is completely full of liquid, the differential co-efficient of $f(r, \theta, \phi)$ with respect to $r$ vanishes on the spherical surface and $f(r, \theta, \phi)$ has the value

$$-m\,C_m\frac{r}{a}\sin m\phi \qquad (14)$$

on the surface defined by $\theta=\frac{\pi}{2}$.

For the determination of the function $f(r, \theta, \phi)$, spherical harmonics of the complex degree $-\frac{1}{2}+p\sqrt{-1}$ are extremely suitable. The properties of these harmonics and

their applications to some physical problems have been investigated by Hobson[8]. Solutions of Laplace's equation of the form

$$\frac{1}{\sqrt{r}} \sin (p \log Ar) \begin{matrix}\sin \\ \cos\end{matrix}\, m\phi\ K_p^m (\cos \theta),$$

where $K_p^m (\cos \theta)$ is a harmonic of degree $-\frac{1}{2}+p\sqrt{-1}$ and order m, and is defined by the hypergeometric series

$$K_p^m (\cos \theta) = F\,(m+\tfrac{1}{2}+pi,\ m+\tfrac{1}{2}-pi,\ m+1,\ \sin^2\tfrac{1}{2}\theta),$$

are suitable for our present purpose. These solutions are finite and continuous for all points in the space inside the hemispherical shell (except infinitely near the origin which may be supposed excluded by surrounding it by an infinitely small sphere).

Let us assume

$$f(r.\theta,\phi) = \sum_p B_p \sqrt{a}\, \frac{\sin (p \log r/h)}{\sqrt{r}}\, \frac{K_p^m (\cos \theta)}{K_p^m (\cos \alpha)} \sin m\phi, \tag{15}$$

where $h/a = \cos \alpha$.

Then $\frac{\partial}{\partial r} f(r, \theta, \phi) = 0$, when $r=a$, if the p's are the roots of the equation

$$\tan (p \log a/h) - 2p = 0,$$

and the summation in the above series extends over all the roots of this equation.

The values of the constants $B_p$ have to be obtained from the equation

$$-C_m \frac{\cos\alpha}{\cos\theta} \tan^m\tfrac{1}{2}\theta\,(m+\cos \theta) = \sum_p B_p \sqrt{a}.\frac{\sin [p \log(\sec\theta)]}{\sqrt{h \sec\theta}}\, \frac{K_p^m (\cos\theta)}{K_p^m (\cos\alpha)},$$

which must be satisfied for all values of $\theta$ between the limits $0<\theta<\alpha$.

In the particular case, when the shell is completely full of liquid, the values of the constants $B_p$'s can be obtained in

8. Hobson, "On a class of Spherical Harmonics of Complex degree with application to Physical Problems," Trans. Camb. Phil. Soc., Vol. 14 pp. 212-236, (1889).

a very simple form. Since the origin is a singular point, we exclude the point by surrounding it with a small sphere of radius $\epsilon$, and assume that $f\ (r,\theta,\phi)$ vanishes on the surface of this sphere. Since in this case $\alpha=\frac{\pi}{2}$, we can assume

$$f\,(r,\theta.\,\phi)=\sum_{p} B_p \frac{\sqrt{a}\sin\left(p\log\frac{r}{\epsilon}\right)}{\sqrt{r}}\,\frac{K_p^m\,(\cos\theta)}{K_p^m\,(0)}\ \sin m\phi,$$

$$\text{where } K_p^m\,(0)=\frac{\sqrt{\pi}\ \Pi\,(m)}{\Pi\,(\frac{1}{2}m-\frac{1}{4}+\frac{1}{2}pi)\ \Pi\,(\frac{1}{2}m-\frac{1}{4}-\frac{1}{2}pi)}$$

$$=\frac{\sqrt{\pi}\ \Pi\,(m)}{\left\{\frac{(2m-1)^2+p^5}{2^2}\right\}\left\{\frac{(2m-3)^2+p^2}{2^2}\right\}\cdots}$$

and the summation extends for all values of p which are the roots of the equation

$$\frac{d}{da}\left[\frac{\sin\left(p\log\frac{a}{\epsilon}\right)}{\sqrt{a}}\right]=0\,,$$

that is to say, the equation

$$\tan\left(p\log\frac{a}{\epsilon}\right)-2\,p=0. \tag{16}$$

The constants $B_p$'s have to be determined by the condition that $f\ (r,\theta,\phi)$ must have the value $-m\,C_m\frac{r}{a}\sin m\phi$ on the free surface, which is given by $\theta=\frac{\pi}{2}$. Hence $B_p$'s are given by

$$-m\,C_m\,\frac{r}{a}\ =\sum_{p}\frac{B_p\sqrt{a}\sin\left(p\log\frac{r}{\epsilon}\right)}{\sqrt{r}}$$

Putting $r=\epsilon.\,e^{\lambda}$ it is easy to see that

$$B_p=-\frac{2\,(p^2+\frac{1}{4}).m\,C_m.}{p^2\log\frac{a}{\epsilon}+\frac{1}{2}\,(\frac{1}{2}\log\frac{a}{\epsilon}-1)}\left(\frac{\epsilon}{a}\right)^{\frac{3}{2}}\int_0^{\log\frac{a}{\epsilon}} e^{\frac{3}{2}\lambda}\sin p\lambda d\lambda$$

$$=-\frac{8\,(4\,p^2+1).\ m\,C_m}{4\,p^2\log\frac{a}{\epsilon}+(\log\frac{a}{\epsilon}-2)}\left[\frac{\sin\left(p\log\frac{a}{\epsilon}\right)+p\left(\frac{\epsilon}{a}\right)^{\frac{3}{2}}}{9+4\,p^2}\right]. \tag{17}$$

To obtain an idea of the magnitude of the constant $B_p$, we shall obtain its value when $\frac{a}{\epsilon}$ is a very large quantity. It is easy to see by the method of successive approximation that the roots of the equation (16) are given by

$$p \log \frac{a}{\epsilon} = X - \frac{\log \frac{a}{\epsilon}}{2X} - \left(\log \frac{a}{\epsilon}\right)^2 \left[\frac{1}{4} - \frac{1}{24} \log \frac{a}{\epsilon}\right] \frac{1}{X^3}$$
$$- \left(\log \frac{a}{\epsilon}\right)^3 \left[\frac{1}{4} - \frac{1}{12} \log \frac{a}{\epsilon} + \frac{1}{160}\left(\log \frac{a}{\epsilon}\right)^2\right] \frac{1}{X^5} - \text{etc.},$$

where $X = (s + \frac{1}{2})\pi$, $s$ being any integer.

Now, if we take $\frac{a}{\epsilon} = 10^5$, the roots are $p_1 = \cdot 321$, $p_2 = \cdot 628$, $p_3 = \cdot 960$, $p_4 = 1 \cdot 205$, etc.

Hence we easily find, that the constants $B_{p_1}$, $B_{p_2}$, $B_{p_3}$, etc. have the values $B_{p_1} = -\cdot 09\, C_m$, $B_{p_2} = -\cdot 08\, C_m$, $B_{p_3} = -\cdot 06\, C_m$, etc., from which we infer that the surface correction $f\,(r, \theta, \phi)$ is a small one. The principal mode of vibration of the liquid is therefore expressed by

$$\Phi = C_m \frac{r}{a} \tan^m \tfrac{1}{2}\theta\,(m + \cos\theta) \sin m\phi \cos pt. \tag{18}$$

If $q$ represent the velocity of the liquid as given by this expression, we have

$$q^2 = \frac{C^2_m}{a^2} \tan^{2m-2} \tfrac{1}{2}\theta\ [(m + \cos\theta)^2 (\sin^2 m\phi \tan^2 \tfrac{1}{2}\theta + \tfrac{1}{4} m^2 \cos^2 m\phi \sec^2 \tfrac{1}{2}\theta)$$
$$+ \{\tfrac{1}{2} m\,(m + \cos\theta,\ \sec^2 \tfrac{1}{2}\theta - \tan \tfrac{1}{2}\theta \sin\theta\}^2 \sin^2 m\phi] \cos^2 pt. \tag{19}$$

Since $q$ is independent of $r$, the velocity of the liquid at any point in a given radius vector is constant. We see that the velocity varies as $\tan^{m-1}\frac{1}{2}\theta$. Hence if we move along any given meridian, the velocity increases from a zero value at the pole at first very slowly then rather abruptly to a large value at the surface, the abruptness of rise being greater the larger the quantity m, that is to say, the higher the mode of vibration of the liquid. Since the velocity of the liquid is constant along any given radius vector, we see that if we consider the motion of the liquid on the surface of a cone of semi-vertical angle $\theta$, and trace the motion of the liquid as a whole as $\theta$ increases, the velocity remains small as $\theta$ increases and

assumes a large value only at or near the surface. It is obvious therefore that in every case when the cup is not quite filled to the brim, the velocity of the liquid is of very large value near the margin of the vessel and is almost imperceptible near the centre and at some depth in the liquid. In the particular case when the shell is almost filled to the brim, the velocity of the liquid as given by this expression at a point near the centre and also near the free surface is not small. But in this case the free surface correction $f(r, \theta, \phi)$ to the expression for the velocity potential becomes of some importance near the centre and has a sign opposite to it. Consequently the velocity of the liquid near the centre always remains very small. These indications of theory are all confirmed by experiment.

To calculate the kinetic energy of the liquid, we have to integrate $\Phi \frac{\partial \Phi}{\partial n}$ over the whole boundary of the fluid. At the free surface $\Phi = 0$. We have therefore only to consider the spherical surface.

Therefore

$$T = \tfrac{1}{2}\rho \iint \Phi \frac{\partial \Phi}{\partial n}\, dS$$

$$= \tfrac{1}{2} a\rho \cos^2 pt \int_0^{2\pi}\int_0^{\alpha} [C_m \sin m\phi \tan^m \tfrac{1}{2}\theta\, (m + \cos\theta) + f'(a, \theta, \phi)]$$

$$\times C_m \sin m\phi \tan^m \tfrac{1}{2}\theta\, (m + \cos\theta) \sin\theta\, d\theta\, d\phi$$

$$= \frac{\pi}{2} a\rho \cos^2 pt\, C_m^2 \int_0^{\alpha} \tan^{2m} \tfrac{1}{2}\theta\, (m + \cos\theta)^2 \sin\theta d\theta$$

$$+ \frac{\pi}{2} a\rho \cos^2 pt\, C_m \sum_p B_p \frac{\sin (p \log a/h)}{K_p^m (\cos\alpha)} \int_0^{\alpha} \tan^m \tfrac{1}{2}\theta\, (m + \cos\theta)\, K_p^m (\cos\theta) \sin\theta d\theta,$$

$\rho$ being the density of the liquid. (20)

Since the liquid is supposed to be incompressible, the potential energy is zero.

The sum of the kinetic and potential energies of the solid and liquid together must be independent of the time. Thus we get

$$\left[a^5\rho\int_0^{\alpha}\tan^{2m}\tfrac{1}{2}\theta(m+\cos\theta)^2\sin\theta d\theta+a^5\rho K+a^4t\,\sigma f(m)\right]\frac{d^2A_m}{dt^2}$$
$$+\tfrac{4}{3}\mu\frac{t^3}{a^2}(m^3-m)\ (2m^2-1)\ A_m=0$$

where

$$K=\sum_p\frac{B_p}{C_m}\frac{\sin(p\log a/h)}{K_p{}^m(\cos\alpha)}\int_0^{\alpha}\tan^m\tfrac{1}{2}\theta\ (m+\cos\theta)\ K_p{}^m(\cos\theta)\sin\theta d\theta,$$

which is a small quantity.

If $A_m$ varies as $\cos(pt+\epsilon)$, we get

$$\left[a^4\rho\int_0^{\alpha}\tan^{2m}\tfrac{1}{2}\theta\ (m+\cos\theta)^2\sin\theta d\theta+a^4\rho K+\frac{4}{3\pi}M\,f(m)\right]p^2$$
$$=\frac{4}{3}\mu\left(\frac{t}{a}\right)^3(m^3-m)\ (2m^2-1),$$

where $M$ = mass of the shell.

This equation gives the frequency of vibration of the shell with different quantities of liquid.

If we put

$$F(\alpha,m)=\int_0^{\alpha}\tan^{2m}\tfrac{1}{2}\theta\ (m+\cos\theta)^2\sin\theta\ d\theta$$
$$=\int_{1+\cos\alpha}^{2}\left(\frac{2-x}{x}\right)^m(m-1+x)^2\,dx,$$

the above expression can be written in the form

$$[a\rho\ F(\alpha,m)+t\sigma f(m)+a\rho K]\ p^2=\frac{4}{3}\frac{\mu}{a^3}\left(\frac{t}{a}\right)^3(m^3-m)\ (2m^2-1). \quad (21)$$

The fall of pitch for the three gravest tones given by m = 2, m = 3 and m = 4 for a brass hemispherical shell of 10 cm. in radius, 2 mm. in thickness and of density 8·6 with different quantities of liquid are shown in Table I.

## TABLE I.

| $\alpha$ | Quantity of water in the shell. | m=2. | | | m=3. | | | m=4. | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | F ($\alpha$, m) | $f$ (m) | p × const. | F ($\alpha$, m) | $f$ (m) | p × const. | F ($\alpha$, m) | $f$ (m) | p × const. |
| 90° | $\pi a^3 \times \cdot 667$ | 1·114 | 1·53 | 1·80 | 1·580 | 1·88 | 4·63 | 2·030 | 2·296 | 8·76 |
| 80° | $\pi a^3 \times \cdot 494$ | ·570 | „ | 2·24 | ·641 | „ | 6·50 | ·479 | „ | 14·51 |
| 70° | $\pi a^3 \times \cdot 338$ | ·291 | „ | 2·75 | ·125 | „ | 9·53 | ·097 | „ | 19·42 |
| 60° | $\pi a^3 \times \cdot 208$ | ·123 | „ | 3·29 | ·032 | „ | 10·57 | ·009 | „ | 21·45 |
| 50° | $\pi a^3 \times \cdot 133$ | ·058 | „ | 3·61 | ·003 | „ | 10 95 | ·002 | „ | 21·64 |
| 40° | $\pi a^3 \times \cdot 034$ | ·026 | „ | 3·81 | ·003 | „ | 11·14 | ·000 | „ | 21·68 |
| 30° | $\pi a^3 \times \cdot 014$ | ·015 | „ | 3·88 | ·001 | „ | 11·22 | ·000 | „ | 21·69 |
| 20° | $\pi a^3 \times \cdot 003$ | ·013 | „ | 3·90 | ·000 | „ | 11·23 | ·000 | „ | 21·69 |
| 10° | $\pi a^3 \times \cdot 001$ | ·011 | „ | 3·91 | ·000 | „ | 11·23 | ·000 | „ | 21·69 |
| 0° | 0 | 0 | „ | 3·93 | 0 | „ | 11·24 | 0 | „ | 21·69 |

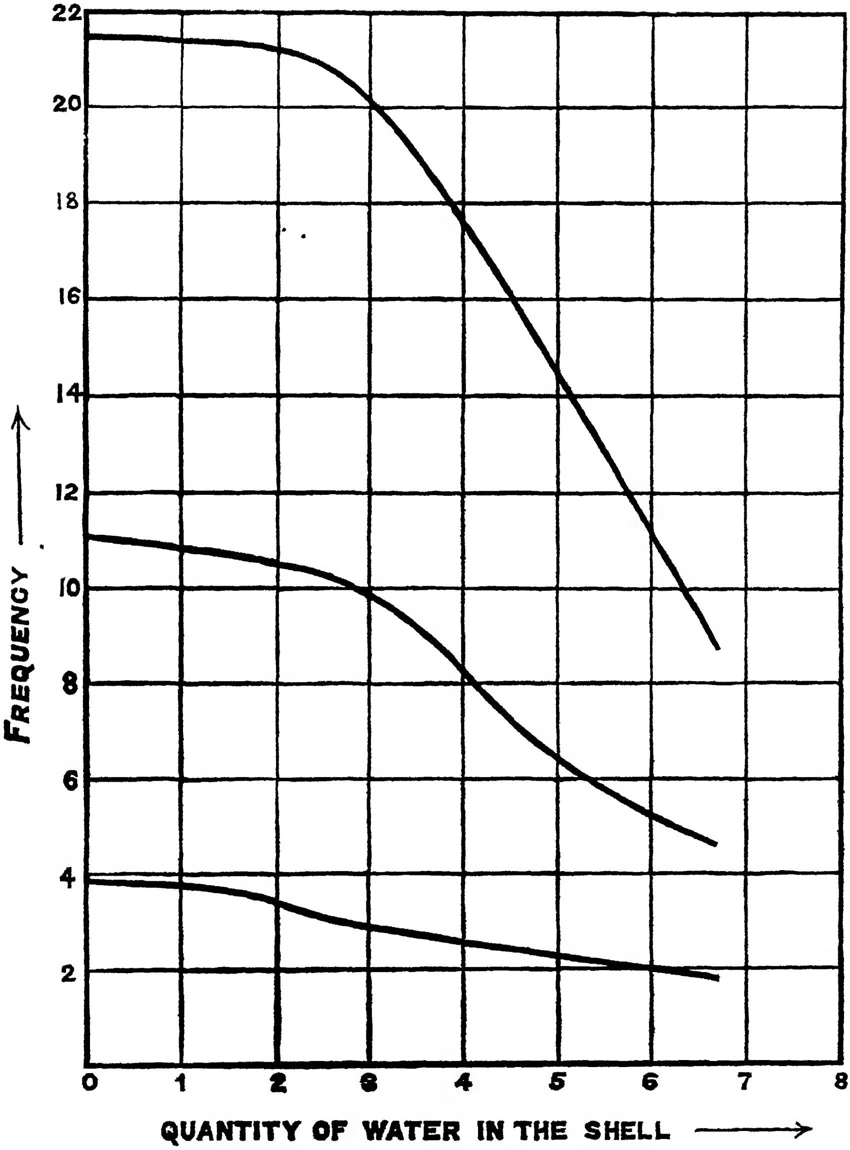



Fig. 1.

In fig. 1, the frequencies have been plotted against the quantity of water in the vessel for these three modes of vibrations.

The frequencies of a brass hemispherical shell of about the same radius and thickness loaded with different quantities of water have also been determined experimentally by a photographic method. The results showed a general agreement with the calculated values. As a shell of uniform thickness and of uniform elastic properties throughout could not be procured, and the one that was used was very much deficient in these respects, the slight discrepancy, that was noticed between the calculated and the observed values of the frequency, was probably due to these defects.

## § 3. *Cylindrical Cups.*

The problem of the flexural vibrations of a cylindrical shell is considered in Lord Rayleigh's *Theory of Sound*, Vol. I, § 235 c. If the displacements at any point $a, \theta, z$ of the cylinder be $\delta r, a\delta\theta, \delta z$, then

$$\begin{aligned} \delta r &= -n\,(A_n a + B_n z)\sin n\theta, \\ a\delta\theta &= (A_n a + B_n z)\cos n\theta, \\ \delta z &= -n^{-1} B_n a \sin n\theta. \end{aligned} \tag{22}$$

Supposing now that the cup has been formed by an inextensible disk being attached to the cylinder at $z=0$, the displacements $\delta r$, $a\delta\theta$ must vanish for that value of $z$. Hence $A_n=0$, and

$$\delta r = -n\,B_n z \sin n\theta,\ a\delta\theta = B_n z \cos n\theta,\ \delta z = -n^{-1} B_n a \sin n\theta, \tag{23}$$

the constant $B_n$ is proportional to a simple harmonic function of the time, say, $\cos pt$.

Since the displacements $\delta r$ and $a\delta\theta$ are proportional to $z$ and the displacement $\delta z$ is independent of $z$, it is obvious, that when $z$ is large the displacements $\delta r$ and $a\delta\theta$ are also very large compared to $\delta z$, that is to say, near the free end of the shell, the displacement $\delta z$ is very small compared to $\delta r$ or $a\delta\theta$. But at the bottom of the shell, the displacements $\delta r$ and $a\delta\theta$ vanishes and $\delta z$ remains constant. We conclude, therefore, from the law of continuity that the disk at the

bottom must have a small normal vibration. If $w$ denote the normal displacement of the disk, it is well-known that $w$ satisfies the differential equation

$$\frac{\partial^2 w}{\partial t^2}+c^4\nabla^4 w=0 \tag{24}$$

where $\nabla^2=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}+\frac{1}{r^2}\frac{d^2}{d\theta^2}$ and $c$ is a certain constant.

If $w \propto \cos(pt+\epsilon)$, then the equation becomes

$$\nabla^4 w-\nu^4 w=0,$$

where $\nu^4=\frac{p^2}{c^4}$.

A solution of this differential equation is known to be

$$w=C_n\, J_n(\nu r)\sin n\theta.$$

Hence we shall take

$$w=C_n\, J_n(\nu r)\sin n\theta\cos pt. \tag{25}$$

The value of the constant $C_n$ can be obtained from the condition that $w$ and $\delta z$ must be continuous at the boundary. This gives

$$C_n\cos pt=-\frac{B_n}{n}\,\frac{a}{J_n(\nu a)}. \tag{26}$$

We can assume that $J_n(\lambda a)$ is very large, and consequently that the normal vibration of the disk is very small. The potential energy of deformation for a length $l$ of the cylinder is

$$V=\frac{4\pi\mu t^3}{3a}(n^2-1)^2\left[\frac{\lambda+\mu}{\lambda+2\mu}\,\frac{n^2l^2}{3a^2}+1\right]B_n^2 \tag{27}$$

The potential energy of vibration of the disk is given by

$$\frac{Et'^3}{3(1-\mu^2)}\int_0^a\int_0^{2\pi}\left[(\nabla^2 w)^2-2(1-\mu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\}\right]r\,d\theta\,dr,$$

where E = Youngs' modulus and $t'$ = thickness of the disk. The value of this integral can be easily obtained. But as we regard the vibration of the disk compared to that of the cylindrical surface to be very small, the value of this ex-

pression is also very small. We can denote this expression by $V_1 B_n^2$, since $w$ has been shown to be proportional to $B_n$.

If the volume density be $\sigma$, we get the expression for the kinetic energy in the form

$$T=\tfrac{1}{2}\pi\sigma t\, la\left[\tfrac{1}{3}l^2\,(1+n^2)+n^{-2}\,a^2\right]\left(\frac{dB_n}{dt}\right)^2$$

$$+\tfrac{1}{2}\pi\sigma t'\,\frac{a^2}{n^2[J_n\,(\nu a)]^2}\int_0^a [J_n\,(\nu r)]^2\, rdr\left(\frac{dB_n}{dt}\right)^2. \tag{28}$$

If the cylinder contain frictionless incompressible fluid, the motion of the fluid will depend upon a velocity potential $\Phi$ which satisfies the equation $\nabla^2\Phi=0$, or in cylindrical co-ordinates

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}=0.$$

The solution of this differential equation can be written in either of the forms.

$$\Phi=\alpha_n\, zr^n \sin n\theta \cos pt. \tag{29}$$

$$\Phi=\beta_n\, e^{-kz}\, J_n\,(kr)\,\sin n\theta\cos pt. \tag{30}$$

The boundary conditions to be satisfied by $\Phi$ are

$$\text{(i)}\quad \frac{\partial\Phi}{\partial r}=\frac{d\delta r}{dt},\ \text{when } r=a. \tag{31}$$

$$\text{(ii)}\quad \frac{\partial\Phi}{\partial z}=\frac{dw}{dt},\ \text{when } z=0.$$

$$\text{(iii)}\quad \Phi=0 \text{ at the free surface, } i.e., \text{ when } z=h.$$

We assume that

$$\Phi=\alpha_n\, zr^n \sin n\theta\cos pt$$
$$+\sum_k J_n\,(kr)\,[D_k\cosh kz+E_k\sinh kz]\sin n\theta\cos pt, \tag{32}$$

where the summation extends for all values of k which are roots of the equation

$$\frac{d}{da}J_n\,(ka)=0. \tag{33}$$

We at once get by condition (*i*),

$$\alpha_n \cos pt = -\frac{1}{a^{n-1}} \frac{dB_n}{dt}. \tag{34}$$

The condition (*ii*) gives

$$-\left[\frac{a}{n}\frac{J_n(\nu r)}{J_n(\nu a)} - \frac{r^n}{a^{n-1}}\right]\frac{1}{k}\frac{dB_n}{dt} = \sum_k E_k J_n(kr)\cos pt.$$

This equation must be satisfied for all values of $r$ between the limits $(0<r<a)$ and will give the value of the constant $E_k$

Now since

$$\int_0^a r^{n+1} J_n(kr)\,dr = \frac{na^n}{k^2} J_n(ka),$$

$$\int_0^a J_n(kr)\,J_n(\nu r)\;r dr = \frac{a\nu}{k^2-\nu^2} J_n(ka)\,J_n'(\nu a),$$

and
$$\int_0^a J_n^2(kr)\,r dr = \tfrac{1}{2}a^2\left(1-\frac{n^2}{k^2a^2}\right) J_n^2(ka),$$

we get

$$-\frac{1}{k}\frac{dB_n}{dt}\left[\frac{a}{nJ_n(\nu a)}\int_0^a J_n(kr)\,J_n(\nu r)\;r dr - \frac{1}{a^{n-1}}\int_0^a r^{n+1}J_n(kr)\,dr\right]$$

$$= E_k \cos pt \int_0^a J_n^2(kr)\,r dr,$$

and therefore

$$E_k \cos pt = -\frac{2a}{ka}\frac{dB_n}{dt}\frac{k^2a^2}{(k^2a^2-n^2)\,J_n(ka)}\left[\frac{\nu}{(k^2-\nu^2)}\frac{J_n'(\nu a)}{nJ_n(\nu a)} - \frac{na}{k^2a^2}\right]. \tag{35}$$

The condition (*iii*) gives

$$\alpha_n h r^n + \sum_k (D_k \cosh kh + E_k \sinh kh) J_n(kr) = 0.$$

for all values of $r$ between the limits $(0<r<a)$. Therefore we get

$$D_k \cosh kh + E_k \sinh kh = -\alpha_n \frac{2na^n h}{(k^2a^2-n^2) J_n(ka)}. \tag{36}$$

The equations (35) and (36) give the values of constants $D_k$ and $E_k$.

To calculate the kinetic energy we have to integrate $\Phi\frac{\partial\Phi}{\partial n}$ over the boundary of the shell. At the free surface $\Phi=0$. We have therefore only to consider the cylindrical surface and the bottom. The expression can be written in the form

$$\begin{aligned} T = \tfrac{1}{2}\pi\rho \cos^2 pt \Big[ & n\alpha_n^2 a^{2n} \frac{h^3}{3} + n\alpha_n a^n \Sigma J_n(ka) \Big\{ D_k \Big( \frac{h \sinh kh}{k} \\ & -\frac{1}{k^2}\cosh kh + \frac{1}{k^2}\Big) + E_k \Big( \frac{h\cosh kh}{k} - \frac{1}{k^2}\sinh kh \Big) \Big\} \\ & - \sum_k \Big\{ \alpha_n D_k \frac{na^n}{k^2} J_n(ka) + \tfrac{1}{2}ka^2 E_k D_k \Big( 1-\frac{n^2}{k^2a^2}\Big) J_n^2(ka) \Big\} \Big]. \end{aligned} \tag{37}$$

The constants $E_k$ and $D_k$ are very small compared to $\alpha_n$. If we neglect $E_k$ and $D_k$, the expression for the kinetic energy reduces to the simple form

$$T = \frac{\pi}{2} \rho n \alpha_n^2 a^{2n} \frac{h^3}{3} \cos^2 pt. \tag{38}$$

In this case the expression for $\Phi$ reduces to the form

$$\Phi = \alpha_n z r^n \sin n\theta \cos pt. \tag{39}$$

This expression represents the principal mode of vibration of the liquid and all the other coexistent modes are very small compared to this one. Since the expression for the velocity varies as $r^{n-1}$, the velocity is very marked near the margin of the vessel and is almost imperceptible near the centre.

The sum of the kinetic and potential energies of the solid and liquid together must be independent of the time. From this we easily obtain an expression for the frequency of vibrations in the most general case from the expressions for the kinetic and potential energies already given. If we neglect $E_k$ and $D_k$, the frequency equation takes a very simple form. The expression in this case is

$$\left[\sigma t l a\left\{\tfrac{1}{3}l^2(1+n^2)+n^{-2}a^2\right\}+\tfrac{1}{3}\rho n a^2 h^3\right]p^2$$

$$=\frac{8\mu t^3 l}{3a}(n^2-1)^2\left[\frac{\lambda+\mu}{\lambda+2\mu}\,\frac{n^2 l^2}{3a^2}+1\right]. \tag{40}$$

Thus we see that the law of variation of the frequency with the height of water in the vessel can be expressed in the form

$$p^2=\frac{1}{A+B\cdot\left(\frac{h}{l}\right)^3},$$

where A and B are two constants for the vessel.

For a glass cylinder whose dimensions are given by $l/a=4$, $t/a=\cdot02$ and which has the density $\sigma=2\cdot6$ and the elastic constants $\mu=1\cdot8$ and $\lambda=1\cdot53$, we easily find that the frequency $p_n$ is given by

$$\left[\cdot052\left\{(1+n^2)+\frac{\cdot1875}{n^2}\right\}+n\left(\frac{h}{l}\right)^3\right]p_n^2$$

$$=\frac{8\mu t^3}{3a^2 l^2}(n^2-1)^2\left[10\cdot4n^2+3\right].$$

For the three gravest tones given by $n=2$, $n=3$ and $n=4$, the values of the frequencies $p_2$, $p_3$ and $p_4$ with different quantities of water in the cylinder are shown in Table II.

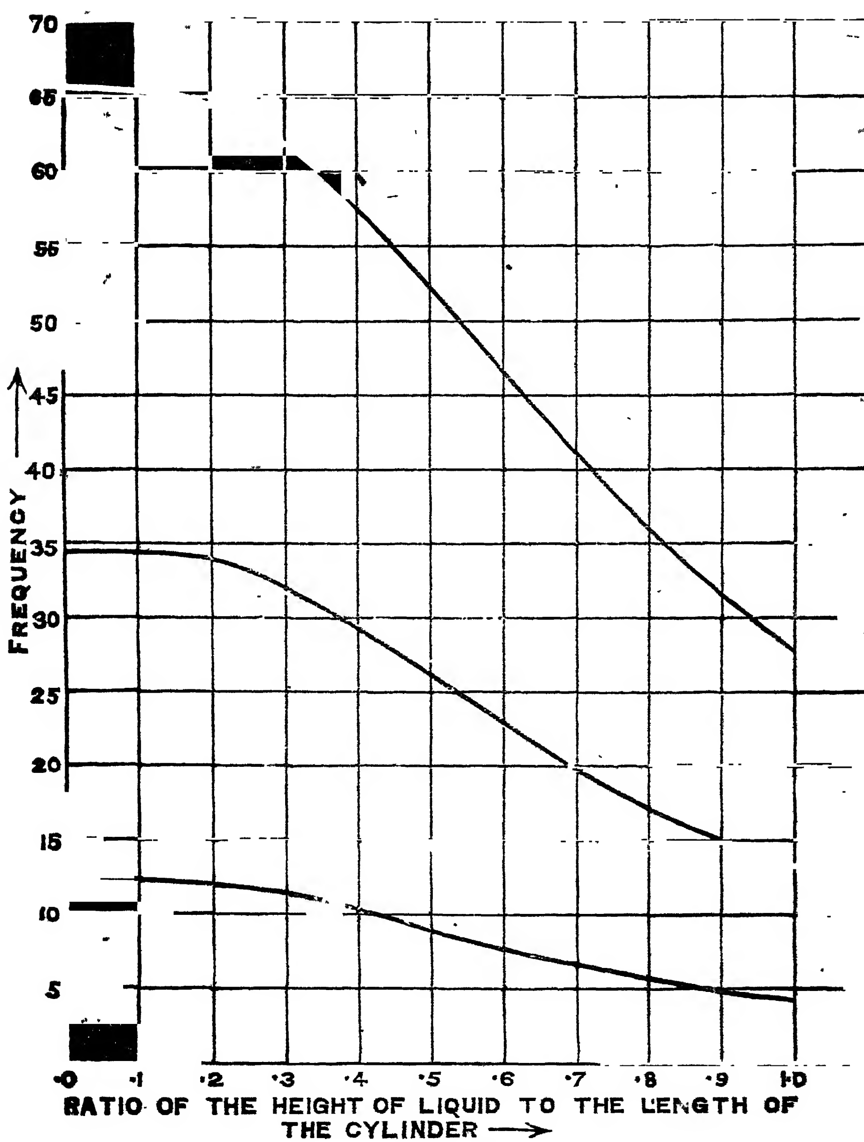
TABLE II.

| $h/l$ | $p_2 \times$ const. | $p_3 \times$ const. | $p_4 \times$ const. |
|---|---|---|---|
| 0 | 12·47 | 34·44 | 65 63 |
| ·1 | 12·33 | 34·40 | 65·48 |
| ·2 | 12·02 | 33·98 | 64·47 |
| ·3 | 11·29 | 32·05 | 61·95 |
| ·4 | 10·15 | 29·45 | 57·79 |
| ·5 | 8·85 | 26·26 | 52·45 |
| ·6 | 7·61 | 22·99 | 46 68 |
| ·7 | 6·56 | 19 97 | 41·09 |
| ·8 | 5·60 | 17·34 | 36·05 |
| 9 | 4·97 | 15·13 | 31·67 |
| 1·0 | 4·21 | 13·25 | 27·93 |

The values of the frequencies given in Table II have been plotted in fig. 2.

## § 4. *Conical Cups.*

It is shown in Lord Rayleigh's *Theory of Sound* in the article already referred to that if a cone for which $\rho = \tan\gamma . z$, $\gamma$ being the semi-vertical angle, executes flexural vibrations, the displacements $\delta\rho$, $\delta\phi$, $\delta z$ at any point whose cylindrical coordinates are $(\rho, \phi, z)$ are given by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RATIO OF THE HEIGHT OF LIQUID TO THE LENGTH OF THE CYLINDER

Fig. 2.

$$\delta\rho = n \tan\gamma \, (A_n z + B_n) \sin n\phi,$$
$$\delta\phi = (A_n + B_n z^{-1}) \cos n\phi,$$
$$\delta z = \tan^2\gamma \, [n^{-1} B_n - n \, (A_n z + B_n)] \sin n\phi.$$

If the cone be complete up to the vertex at $z=0$, then $B_n=0$, so that

$$\begin{aligned} \delta\rho &= n \tan\gamma . \, A_n \, z \sin n\phi, \\ \delta\phi &= A_n \cos n\phi, \\ \delta z &= -n \, A_n \tan^2\gamma . \, z \sin n\phi. \end{aligned} \tag{42}$$

If the displacements in polar coordinates $(r, \theta, \phi)$ be denoted by $\delta r$, $\delta\theta$, $\delta\phi$, we easily obtain

$$\delta\phi = A_n \cos n\phi,$$
$$\delta r = \delta\rho \sin\gamma + \delta z \cos\gamma = 0,$$
$$r\delta\theta = \delta\rho \cos\gamma - \delta z \sin\gamma = n \, A_n \tan\gamma . \, r \sin n\phi.$$

It is easy to see that the potential energy of deformation for a length $l$ of the cone

$$W = \frac{4\pi}{3}\mu t^3 \frac{\lambda+\mu}{\lambda+2\mu} A_n^2 \sin\gamma \left[ (-n^3 \frac{\tan\gamma}{\sin^2\gamma} + n \tan\gamma + n \cot\gamma)^2 + \cos^2\gamma \right] \log\frac{2l}{t}$$

where $t$ = thickness.*

The expression for the kinetic energy of vibration of the shell can be easily obtained in the form

$$T = \frac{\pi}{8} \sigma t l^4 \sin^3\gamma \, [n^2 \sec^2\gamma + 1] \left(\frac{dA_n}{dt}\right)^2. \tag{45}$$

---

* This equation can be readily deduced from a very general expression for the potential energy due to strain in curvilinear coordinates obtained by Prof. Love. (*Vide* his paper on "The small free vibrations and deformation of a thin Elastic Shell," *Phil. Trans.*, Vol. 179, 1888, A.) The expression has been criticised by Prof. Basset (*Phil. Trans*, Vol. 181, 1890, A) on the ground that Prof. Love has omitted several terms which involve the extension of the middle surface. As the inextensional vibrations only have been considered in this paper, this criticism does not affect us in any way.

If the cup contains frictionless incompressible fluid, the velocity potential of the fluid must satisfy Laplace's equation. Let us assume that the velocity potential is given by

$$\Phi = C_n r^2 \phi_n(\cos\theta) \sin n\phi . \cos pt, \tag{46}$$

where $\phi_n(\cos\theta)$ is a function of $\theta$ only.

Then it is easy to see by substitution in the differential equation

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{2}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\cot\theta}{r^2}\frac{\partial\Phi}{\partial\theta}+\frac{\operatorname{cosec}^2\theta}{r^2}\frac{\partial^2\Phi}{\partial\phi^2}=0,$$

that $\phi_n(\cos\theta)$ satisfies the equation

$$\frac{\partial^2\phi_n}{\partial\theta^2}+\cot\theta\frac{\partial\phi_n}{\partial\theta}+(6-n^2\operatorname{cosec}^2\theta)\,\phi_n=0.$$

A solution of this dlfferential equation can be easily obtained in the form

$$\phi_n(\cos\theta)=\tan^n\tfrac{1}{2}\theta\left[(1-n)(2-n)-6(2-n)\cos^2\frac{\theta}{2}+12\cos^4\frac{\theta}{2}\right]. \tag{47}$$

The relation between $C_n$ and $A_n$ can be easily obtained by equating the value of $\frac{\partial\Phi}{r\partial\theta}$, when $\theta=\gamma$, to $\frac{d(r\delta\theta)}{dt}$, both of which represent the normal velocity at the bounndary. We thus get

$$C_n\cos pt\frac{\partial}{\partial\gamma}\left[\tan^n\tfrac{1}{2}\gamma\left\{(1-n)(2-n)-6(2-n)\cos^2\frac{\gamma}{2}+12\cos^4\frac{\gamma}{2}\right\}\right]$$
$$=n\tan\gamma\frac{dA_n}{dt} \tag{48}$$

The principal mode of vibration of the liquid will therefore be expressed by (46) except for a small correction to be introduced on account of the existence of a free surface. At the free surface the condition to be satisfied is given by

$$\Phi=0 \text{ when } z=h,$$

where $h$ denotes the height of the liquid.

To satisfy this condition, we take

$$\Phi = C_n r^2 \phi_n (\cos\theta) \sin n\phi \cos pt + \sum_m D_m r^m P_m^n (\cos\theta) \sin n\phi \cos pt, \qquad (49)$$

where the summation extends for all va  es of m which are the roots of the equation

$$\frac{\partial}{\partial\gamma} P_m^n (\cos\gamma) = 0. \qquad (50)$$

The constants $D_m$'s have to be determined by means of the equation

$$C_n (h \sec\theta)^2 \phi_n (\cos\theta) + \sum_m D_m (h \sec\theta)^m P_m^n (\cos\theta) = 0, \qquad (51)$$

which must be satisfied for all values of $\theta$ between the limits $0<\theta<\gamma$. Approximate values of the constants $D_m$'s can be easily obtained from this equation. To get an idea of the magnitude of the constant $D_m$, we shall obtain its value in the particular case when the semi-vertical angle $\gamma$ of the cone is small and the height $h$ of the liquid is large compared to the radius of the cross-section of the cone by the free surface. In this case the free surface can be taken to be practically coincident with the surface of the sphere $r=h$. The equation for determining $D_m$ is then

$$C_n h^2 \phi_n (\cos\theta) + \Sigma D_m h^m P_m^n (\cos\theta) = 0.$$

Now, since

$$\int_{\cos\gamma}^{1} P_m^n (\cos\theta)\, P_{m'}^n (\cos\theta) \sin\theta d\theta = 0.$$

m, m′ being two different roots of the equation (50), and

$$\int_{\cos\gamma}^{1} [P_m^n (\cos\theta)]^2 \sin\theta\, d\theta = \frac{1-\cos^2\gamma}{2m+1} P_m^n (\cos\gamma) \frac{\partial^2}{\partial m\, \partial\cos\gamma} P_m^n (\cos\gamma),$$

we easily get

$$D_m = -\frac{2m+1}{1-\cos^2\gamma} \frac{C_n^{-} h^{2-m} \int_{\cos\gamma}^{1} \phi_n^{-}(\cos\theta)\, P_m^n(\cos\theta) \sin\theta d\theta}{P_m^n(\cos\gamma) \frac{\partial^2}{\partial m \partial \cos\gamma} P_m^n(\cos\gamma)}$$

Neglecting the small correction introduced by the free surface, we see that the kinetic energy of the fluid motion is

$$\frac{\pi}{2}\rho\, C_n^2 \cos^2 pt\, \phi_n(\cos\gamma) \frac{\partial\phi_n(\cos\gamma)}{\partial\gamma} \sin\gamma \frac{h^5}{5} \sec^5\gamma$$

Since the sum of the kinetic and potential energies of the solid and liquid together must be independent of the time, we easily obtain, on assuming that $A_n \propto \cos pt$, the frequency-equation in the form

$$\left[\frac{1}{4}\bar{\sigma} t l^4 \sin^3\gamma\, (n^2 \sec^2\gamma + 1) + \rho n^2 \tan^2\gamma \sin\gamma \frac{\phi_n(\cos\gamma)}{\frac{\partial}{\partial r}\phi_n(\cos\gamma)} \frac{H^5}{5}\right] p^2$$

$$= \frac{8}{3}\mu t^3 \frac{\lambda+\mu}{\lambda+2\mu} A_n^2 \sin\gamma \left[\left(-n^3\frac{\tan\gamma}{\sin^2\gamma} + n\tan\gamma + n\cot\gamma\right)^2 \right.$$

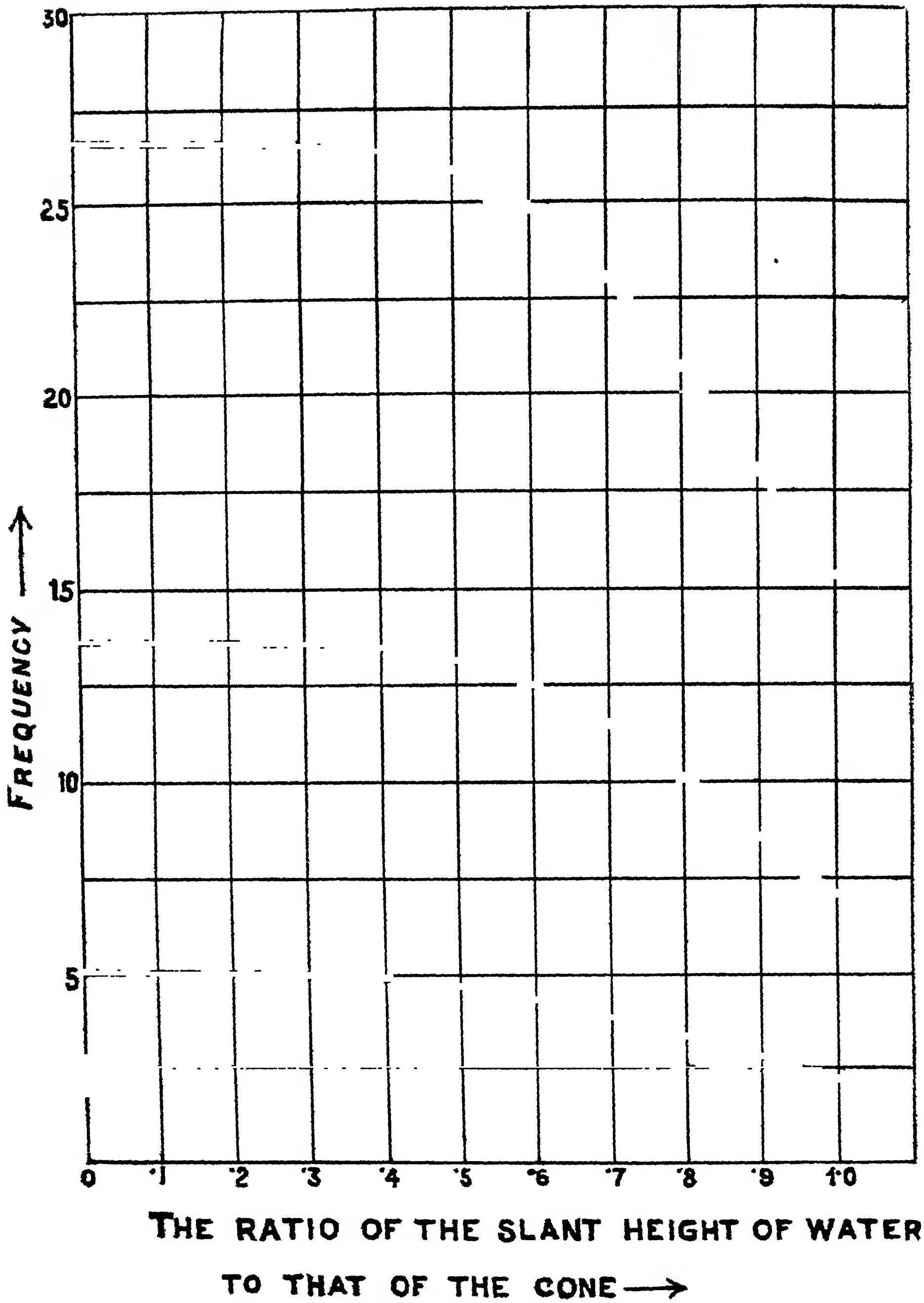
$$\left. + \cos^2\gamma\right] \log\frac{2t}{t},$$

where $H = h\sec\gamma$, $H$ being the slant height of the liquid.

In this case we see that the law of variation of frequency with the height of liquid can be expressed in the form

$$p^2 = \frac{1}{A + B\left(\frac{h}{l}\right)^5}$$

A and B being two constants for the particular shell.



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THE RATIO OF THE SLANT HEIGHT OF WATER
TO THAT OF THE CONE →


Fig. 3.

The frequencies $p_2$, $p_3$ and $p_4$ with different quantities of water for the three gravest modes of vibrations given by n=2, n=3 and n=4 have been calculated from this expression for a cone of semi-vertical angle $30^0$, the ratio of the thickness of the sides of the cone to the slant height being equal to ·02 and are shown in Table III.

Table III.

| $H/l$ | $p_2\times$ const. | $p_3\times$ const. | $p_4\times$ const. |
|---|---|---|---|
| 0 | 5·030 | 13·58 | 26 75 |
| ·1 | 5·030 | 13·58 | 26·75 |
| ·2 | 5 028 | 13·57 | 26 73 |
| ·3 | 5 008 | 13·54 | 26·69 |
| ·4 | 4 937 | 13·41 | 26·47 |
| ·5 | 4·761 | 13 08 | 25·94 |
| ·6 | 4 428 | 12·43 | 24 87 |
| ·7 | 3 942 | 11·64 | 23·13 |
| ·8 | 3·399 | 10 07 | 20·79 |
| ·9 | 2 808 | 8·63 | 18·07 |
| 1·0 | 2·310 | 7·26 | 15·41 |

The curves showing the fall of frequency for these three modes of vibrations of the cone when loaded with different quantites of water are plotted in Fig. 3.

CALCUTTA :
*The 20th July, 1918.*

PROCEEDINGS

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INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. IV. PART III.

# On the staining of Barite Celestite and Anhydrite.

By

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*Professor of Chemistry, Ripon College, Calcutta, April, 1918.*

For sometime past, I have been trying to distinguish between the members of group of minerals by means of stains — and two communications of mine on the subject have already been published (1). The present note contains descriptions and results of my experiments with barite, celestite, and anhydrite to which I have fixed stains by means of aniline black and cochineal and observed that these minerals can be easily distinguished by this means. So far as I know, no attempt has hitherto been made to distinguish between these minerals by means of stains.

The description of the experiments is as follows:—(*a*) *Experiments with aniline black*;—The powders of the minerals barite, celestite and anhydrite are taken in separate test tubes and approximately the same quantities of a solution of silver nitrate are added and the mixtures warmed when barite

(1) Proceedings of the Indian Association for the Cultivation of Science, Vol. II pp. (47-50) 1917. Proceedings of the Indian Association for the Cultivation of Science, Vol. III, part 4, pp. (89-93) 1917.

always assumed a very light straw colour. This though singles out barite from celestite and anhydrite, requires trained eye to identify the colour and thus is of not much value. However, the contents of the test tubes are washed with water and warmed in approximately the same quantities of a solution of aniline black and they are then washed with water when the powders are observed to have stains fixed on them.. The colour is blue black and is of the same intensity in each case. The stained powders are boiled in solutions of sodium carbonate when the blue black stain is removed and the minerals have on them brown tinge of varying intensity. This cannot be the means of identification. Now some liquid, (*i.e.*, the sodium carbonate solution) is poured off from each test tube such that there remains, approximately the same quantities of sodium carbonate solution in the test tubes. The powders are not washed this time with water. To the above are added approximately the same quantities of a solution of aniline black and the mixtures warmed and subsequently washed by means of water. The minerals are observed to have distinguishing stains fixed on them—the order being always constant. These stains being of very marked difference in colour and intensity, and for the fact that each particular stain being always taken by each particular mineral, can be used in identification with much value. These stained minerals now being boiled with the solution of sodium carbonate and washed with water, will show some change in the fixed colours and this is of such a character that it will bring out all the more clearly the distinction between the minerals. This phenomenon takes place in every set of such experiments. Thus it can be utilised as confirmatory tests after the fixing of distinguishing stains as stated above

(*b*) *Experiments with cochineal*:—(A) The powders of the above minerals are taken in separate test tubes as before

and approximately same quantities of cochineal solution are added and the mixtures warmed when in the case of anhydrite a gray or blackish gray precipitate is formed and the mineral itself tends to assume gray colour which remains even if the mineral is washed with water but the other two minerals under the same conditions do not give any coloured precipitate neither assume any stain. Thus anhydrite can be identified.

(B) The powders of the minerals are taken again in separate test tubes and approximately same quantities of a solution of sodium carbonate are added and then cochineal solution is poured quantity added being approximately the same in each case. Next the mixtures are warmed and the powders washed with water when they are observed to have distinguishing stains on them. In this case, anhydrite tends to assume deeper colour than celestite when barite always takes lightest stain. Thus after the performance of the experiment (A) and hence the detection of anhydrite, barite and celestite can be identified and anhydrite confirmed by this method.

(C) The powders of the minerals are taken again in separate test tubes and warmed in approximately the same quantities of silver nitrate solution. The contents of the test tubes are washed with water and then treated with solutions of sodium carbonate and cochineal as described in the experiment (B), when the minerals assume distinguishing stains. There is some difficulty, in this case, to fix colours on celestite and anhydrite, on the first occasion, and thus to differentiate the minerals, when the powders—not freshly taken but already treated in this experiment—are treated twice or thrice with solutions of sodium carbonate and cochineal, as noted before, until they assume distinguishing stains. Thus this experiment can be utilised as confirmatory tests, after the performance of (A) and (B).

In all the foregoing experiments the solution especially of aniline black and cochineal used are always strong but the quantities taken to experiment are always small. The above experiments were performed on chemically prepared pure sulphates of barium, strontium and calcium and the same results as described before or hereafter were obtained.

TABLE I.

| | (1) | (2) |
|---|---|---|
| | STAINS. | STAINS. |
| Names of Minerals. | The powders of the minerals are first warmed in $A_gNo_3$—afterwards washed with water; then warmed in aniline black solution—then again washed with water then warmed in solution of $Na_2\ Co_3$—some of the liquid being poured off, warmed after all in aniline black solution. | The stained minerals from the column (1) of this table are boiled in Sodium carbonate solution and washed with water. |
| Barite ... | Brown with light blue black ... | Brown. |
| Celestite ... | Deep dark blue black ... ... | Blue black. |
| Anhydrite | Blue black always lighter than in celestite. | White practically. |

TABLE II.

| | (1) | (2) | (3) |
|---|---|---|---|
| | STAINS. | STAINS. | STAINS. |
| Names of Minerals. | The powders of the minerals are warmed in Cochineal solution. | The powders of the minerals are warmed in solutions of Sodium Carbonate and Cochineal and washed with water. | The powders of the minerals are warmed in $A_g$ $No_3$ solution and washed by means of water and then two or three times if required treated in solutions of sodium carbonate and cochineal and washed by means of water until stains are fixed. |
| Barite ... | *Nil.* | *Nil* (practically) a very light shade of violet. | Brown. |
| Celestite .. | *Nil.* | Rich violet ... | Gray with violet Often ad mixture o brownish tinge. |
| Anhydrite ... | Gray precipitate some times blackish gray precipitate. The mineral tends to assume gray stain. | Violet with admixture of gray. | Gray with violet. Often admixture of brownish tinge. |

The following points noted during the experiments :—

(1) The minerals when treated alone with aniline black solution assume blue black colour of same intensity. These stained minerals if boiled afterwards in $Na_2$ $Co_3$ solution become white, *i.e.*, the colouring matter is removed.

(2) The minerals after being treated with the solution of $Na_2$ $Co_3$ and cochineal as described in Table II column (2) if after being washed with water, treated with $A_g$ $No_3$ solu-

tion, become gray, the time of discharge of colour being different in three cases. Deeper stains take more time to be removed.

(3) As regards the treatment with aniline black and cochineal, the properties exhibited by gypsum are same as those of anhydrite.

(4) The stains of the minerals are best seen under water.

# On the Diffraction of Light by an Obliquely held Cylinder.*

By

T. K. CHINMAYANANDAM, M.A.

## § 1. *Introduction.*

The problem of the Diffraction of Light by a transparent cylinder, on which plane waves are ***normally*** incident, has been treated by many writers† both with respect to the classical wave theory and from the fundamental Electro-magnetic equations, the problem having special application to the theory of the rain-bow. The writer has observed some very interesting phenomena which are exhibited when the light falls ***obliquely*** on the cylinder, and which do not seem to have been previously described. It is proposed in this paper to give an account of the phenomena, and to attempt a theoretical explanation.

If light falls on a transparent cylinder normally, the rays that emerge after refraction and one internal reflection form, as is well known, two parallel caustics whose directions make a definite angle, which, for glass, is about 45°. If the incidence of light on the cylinder is made oblique,

* Reprinted from the *Physical Review*, Oct. 1918.

† For detailed references to the earlier work by Airy, Pernter, Miller, Pulfrich, Mascart and others, see Aichi and Tanakdate's paper on the "Theory of the Rain-bow due to a circular source of light" *Journ. of the Coll. of Sc., Imp. Univ. of Tokyo,* Vol. xxi., Art. 3, (1906). Among recent references, may be mentioned Debye—*Phys. Zeitschr*, 9, Nov. 1908; also *Science Abstracts*, 1909, p. 88, and Mobius—"On the Theory of the Rain-bow," *Annal der Phy.*, 33, (1910) and 40, (1913). See also Potzger—on "Diffraction in the Ultra-microscope," *Annal der Phy.*, 30, Oct. 1909 and *Science Abstracts*, 1909.

the two caustics are found to approach each other more and more closely. At a certain angle of incidence (about 50° for glass), the two caustics of the emergent rays coalesce, and absolutely cease to exist for greater angles of incidence. At and near the stage where the caustics and the supernumerary fringes which accompany them coalesce, the phenomena presented are of particular interest, and may be studied by mounting the cylinder horizontally on the table of a spectrometer. The collimator slit should also be made horizontal, and may be illuminated by the electric arc or by a Cooper-Hewitt lamp. With this arrangement, we can observe visually or photograph the phenomena corresponding to varying angles of incidence simultaneously. In Plate I, is reproduced a photograph of the effects thus observed in mono-chromatic light. If part of the length of the horizontal slit of the collimator be covered up so as to leave only a short opening, the phenomena corresponding to any particular angle of incidence are observed in the field. In the photograph shown, they would correspond to a narrow strip perpendicular to the bisector of the angle between the two bright caustic lines.

The interesting feature of the diffraction pattern, which distinguishes it from the fringes in the rain-bow, is that the two systems of caustic fringes, which are far apart in the case of normal incidence, overlap and interfere with each other when the incidence is oblique, giving rise to a beautiful structure in the fringe-system, especially near its termination where the caustics meet and vanish. As the angular separation of the caustics increases, the overlapping of the fringes becomes less and less perceptible in white light, but a few fine interference fringes which form practically a separate system may be seen midway between the caustics; with the spectrometer, these are visible even at an incidence nearly normal to the cylinder, if its diameter is about 0·5 mm.

or less. When the diameter of the cylinder is of the order of 0·01 mm., these fringes become broad and bright. In this case the caustics and the supernumeraries accompanying them are practically achromatic, but the system of interference fringes seen midway between the caustics is strongly coloured, except of course the central fringe, which is white. Figs. *a*, *b*, and *c*, on Plate II represent the phenomena observed with a glass fibre of that diameter, and correspond respectively to three different angles of incidence.

## § 2. *Geometrical Theory.*

Let the axis of the cylinder be the z-axis, the incident rays being parallel to the x z plane. The direction cosines of the incident ray may be taken to be $l, o, n$, and the point of incidence of any ray on the cylinder may be represented by the cylindrical coordinates $(a \cos\phi, a \sin\phi, z)$, $a$ being the radius of the cylinder. Also, let the direction cosines of the refracted ray, the internally reflected ray, and the emergent ray be respectively $(l,' m,' n')$, $(l,'' m,'' n'')$ and $(\lambda, \mu, \nu)$. Then we shall have, for the first refracted ray, from the fundamental laws of refraction,

$$\begin{vmatrix} l, & o, & n \\ \cos\phi & \sin\phi, & o \\ l', & m', & n' \end{vmatrix} = o \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

$$\text{and} \quad 1 - l^2 \cos^2\phi = \mu^2 \left\{1 - (l' \cos\phi + m' \sin\phi)^2\right\} \qquad \ldots \quad (2)$$

and we shall have two similar pairs of equations for the other two rays. Solving equations (1) and (2), we get

$$\left.\begin{aligned} l' &= -K \cos\phi + \frac{l}{\mu} \sin^2\phi \\ m' &= -K \sin\phi - \frac{l}{\mu} \sin\phi \cos\phi \\ n' &= \frac{n}{\mu} \end{aligned}\right\} \ldots \qquad \ldots \qquad \ldots \quad (A)$$

$$\text{where } K^2=1-\frac{1}{\mu^2}(1-l^2\cos^2\phi). \qquad \ldots \qquad \ldots \quad (3)$$

If this refracted ray meets the surface of the cylinder at a point $\phi'$, we have

$$\frac{\cos\phi'-\cos\phi}{l'}=\frac{\sin\phi'-\sin\phi}{m'} \qquad \ldots \qquad \ldots \qquad \ldots \quad (4)$$

from which, using equations (A), we deduce the relation

$$\tan\frac{\phi-\phi'}{2}=\frac{K\mu}{l\sin\phi}. \qquad \ldots \qquad \ldots \qquad \ldots \quad (5)$$

Similarly, if the point of emergence of the ray from the surface of the cylinder has the co-ordinate $\phi''$, it can be shown that

$$\tan\frac{\phi'-\phi''}{2}=\frac{K\mu}{l\sin\phi}=\tan\frac{\phi-\phi'}{2}. \qquad \ldots \qquad \ldots \quad (6)$$

and also that

$$\left.\begin{aligned} l''&=-K\cos\phi'+\frac{l}{\mu}\sin\phi\sin\phi' \\ m''&=-K\sin\phi'-\frac{l}{\mu}\sin\phi\cos\phi' \\ n''&=\frac{n}{\mu} \end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad \ldots \quad (B)$$

and

$$\left.\begin{aligned} \lambda&=l\cos(\phi+\phi'')=l\cos 2\phi' \\ \mu&=l\sin(\phi+\phi'')=l\sin 2\phi' \\ \nu&=n \end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad \ldots \quad (C)$$

Equations (C) determine the direction of the emergent rays If the radius of the cylinder is very small, we can regard, as an approximation. that the emergent rays all start from the origin O itself. Then, the emergent rays will lie on the cone

$$x^2+y^2=\frac{l^2}{n^2}z^2 \qquad \ldots \qquad \ldots \quad (7)$$

They would be parallel when

$$\frac{d\lambda}{d\phi}=0, \text{ and } \frac{d\mu}{d\phi}=0$$

which reduce to the condition that $d\phi'/d\phi=0$.

Now, from (4), we have

$$\cos(\phi-\phi')=\frac{l^2\sin^2\phi-\mu^2K^2}{\mu^2-1+l^2}$$

$$\sin(\phi-\phi')=\frac{4\,l^2\sin\phi\cos\phi}{\mu^2-1+l^2},\quad \text{since } \frac{d\phi'}{d\phi}=0$$

$$=\frac{2\mu Kl\sin\phi}{\mu^2-1+l^2}\qquad \text{by (4).}$$

Hence $\cos\phi=\dfrac{\mu k}{2l}$

or by (3) $\cos^2\phi=\dfrac{\mu^2-1}{3l^2}$. ... ... ... (8)

If the incidence is normal, $l^2=1$ and equation ($b$) reduces to

$$\cos^2\phi=\frac{\mu^2-1}{3}\quad \ldots \quad \ldots \quad \ldots \quad (9)$$

which is the condition for crowding of the emergent rays from a raindrop. It is also seen from (8) that $\phi$ will be imaginary, if $l^2<\dfrac{\mu^2-1}{3}$ which happens for glass if the angle of incidence is greater than about 50°. The caustics cease to exist, therefore, at very oblique incidence.

The actual direction of the emergent rays where they are parallel, *i.e.* the direction of the caustics, may now be determined.

$$\cos 2\phi'=\cos\{2(\phi-\phi')-2\phi\}$$

$$=\frac{1}{(\mu^2-1+l^2)^2}\left\{12l^2s+3s^2-l^4+\frac{2s^3}{l^2}\right\},$$

S being written for $\dfrac{\mu^2-1}{3}$ ... ... (10)

from equations (4) and (8). The directions of the two caustics are thus given by

$$\left.\begin{array}{l}\lambda=l\cos 2\phi'\\ \mu=\pm l\sin 2\phi'\\ \nu=n\end{array}\right\}$$

where $\phi'$ is determined by (10). The angle between these two directions is given by

$$\cos \epsilon = 1 - 2\, l^2 \sin^2 2\phi'.$$

The results of a few measurements of the angular separation of the caustics at varying angles of incidence are given below with the corresponding values calculated from theory. The experimental arrangement used was simply to have the cylinder mounted on the table of a spectrometer, making the collimator slit short and narrow. The electric arc was used with a green ray filter for illuminating it. The angle of incidence was measured as usual, by setting the cross-wire of the telescope on the reflected image of the slit and then taking the direct reading. The angle between the caustics was determined by reading off the linear separation of the caustics in cms. on a screen held at a distance of one metre from the cylinder and dividing it by 100. The refractive index of the cylinder was measured by dipping it in a rectangular glass cell, illuminated by green light and containing Thoulet's solution.* The liquid, whose refractive index could be continuously varied by mixing water, was diluted until the shadow of the cylinder became practically invisible. The chief difficulty in the experiment was the selection of the cylinder itself. Most of the large number of glass threads that were examined, gave somewhere irregular effects, a slight rotation of the cylinder about its own axis changing the angle between the emergent caustics very much.† The irregularities are apparently due to the cylinder being not quite circular, i.e. to slight ellipticity in its cros-section.

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* R. W. Cheshire—*Phil Mag.*, Oct., 1916.

† Some very good glass cylinders were drawn for me by Mr. Damodar Kini of the Madras Presidency College, to whom my best thanks are due for the same.

TABLE I.

| Angle of insidence. | Angular Separation of the Caustics. | |
|---|---|---|
| | Obsd. | Calcd. |
| 32° 15′ | 17° 12′ | 17° 48′ |
| 35° 15′ | 13° 42′ | 13° 47′ |
| 38° 15′ | 10° 30′ | 10° 0′ |
| 41° 15′ | 6° 54′ | 7° 18′ |
| 44° 15′ | 3° 42′ | 3° 12′ |
| 47° 15′ | 1° 12′ | 1° 50′ |
| 49° 39′ | 0° | 0° |
| 52° | — | — |

($\mu$=1° 51).

Considering the fact that even a slight ellipticity in the cross-section of the cylinder affects the results appreciably, the agreement between the observed and calculated values is good. As was anticipated from theory from equation (8), the caustics cease to exist after a critical angle of incidence, which for glass is about 50.°

### § 3. *Form of the Emergent Wave-surface.*

Passing on now to consider the phenomena in the light of the wave-theory, we have first to determine the form of the wave on emergence from the cylinder. Figs. 1 and 2. represent the projection of the wave front on the plane passing through the two caustics, fig. 1. illustrating the case of normal incidence, and fig. 2., the case when the incidence is about 50,° the angle between the caustics being then almost zero; they have been drawn by plotting out a number

of emergent rays with data calculated from the formulæ in § 2 above, and drawing curves normal to them. It will be

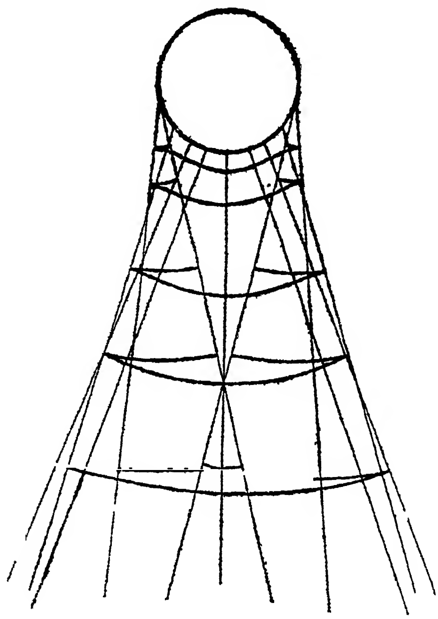

Fig. 1.

seen that the section of the emergent wave has two points of inflexion on either side of the central line, and at a great distance from the cylinder, the disturbance at any point may be regarded as due to the interference of three sets of wave-trains to which the emergent cusped wave gives rise. In a direction close inside either of the caustics, the effect of one of the wave-trains is negligible in comparison with that of the other two, and the problem of determining the illumination in any direction is then the same as in the case of the rainbow, which has been worked out by Airy, Mascart and others.* But the approximation suggested is not valid in directions nearly midway between the caustics, even when the latter are widely separated, and is totally inapplicable when the caustics make only a small angle with each other. This is evident on an inspection of the photograph on Plate I. At

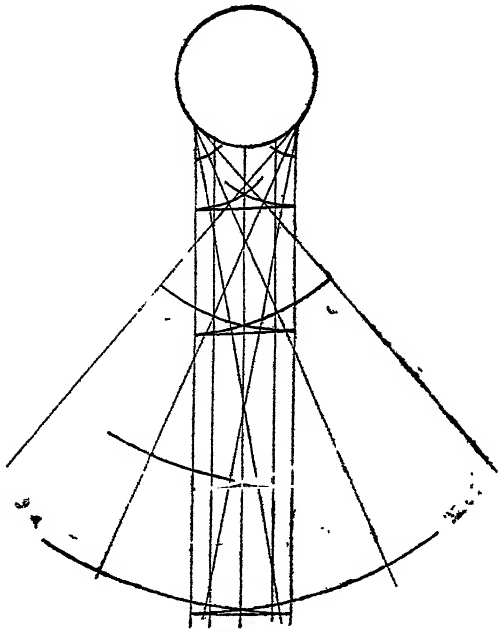

Fig. 2.

a sufficient distance from the "tail" of the pattern, where the caustics are widely separated, the fringe system is much like the supernumerary fringes in the rain-bow, except near the central line where, as already explained, some interference fringes are observed. But near the tail, a peculiar criss-cross structure is observed in the fringe system.

## § 4. *Calculation of the Intensity of Illumination.*

It follows from equation (5) in § 2 above that the emergent wave may be regarded approximately as spreading on the surface of a cone whose axis coincides with that of the cylinder. But, if the angular separation of the caustics is small, we may make a further approximation and assume that it spreads entirely in planes parallel to that passing through the two caustics.

The section of the wave-surface by that plane may, in effect, be considered to be a symmetrical curve, of the form

ODO′ shown in fig. 3, with two inflection points O and O′, the normals at those two points indicating the directions of the emergent rays, where they are parallel. In any direction inclined at a small angle $\theta$ to one of the normals, it is apparent from the figure that there are three 'rays' originating at three different points *a*, *b*, *c*, on the emerging wave. We can consider the effect at any point in the direction $\theta$ as due to the interference of these three rays emerging from the wave-front in that direction. We can for simplicity regard the amplitude of the disturbance to be the same at all points on the emerging wave-surface itself. Then the amplitudes of the three rays which proceed in the direction $\theta$, will at a great distance from the cylinder be

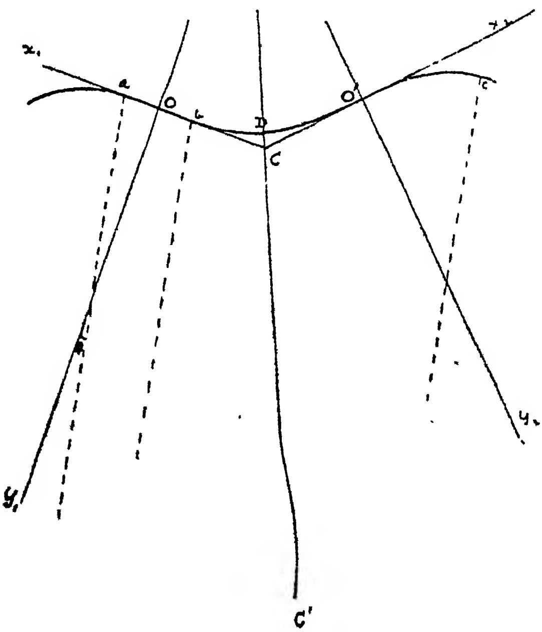


Fig. 3.

respectively propotional to the square roots of the radius of curvature of the wave ODO′ at the points *a*, *b*, *c* from which they originate. We have next to calculate the path difference

of each pair of these rays, and then the illumination at a point can be easily calculated.

A simple method of analytically representing the curve ODO′ suggests itself. The curve may be regarded as two curves OD, DO′ fitted up to form one continuous curve, the equation to the former being

$$y_1 = Ax_1^3$$

with respect to O, the inflection point, as origin, and the inflectional tangent OX and normal OY as axes of reference and the equation to the latter being

$$y_2 = Ax_2^3$$

with respect to O′ as origin and the inflectional tangent and normal at that point as axes of reference. The curves are considered as extending to infinity on the positive side of $x$ but are obviously limited on the negative side by the central line CC′.

Consider first the rays that start out from the points $a$ and $b$ (fig. 3). The amplitudes of the disturbances due to them are, by symmetry, equal and will be proportional to $a_1$ where

$$a_1^2 = 1 \Big/ \frac{d^2y}{dx^2} = 1 \Big/ 6Ax = \pm 1 \Big/ 6A\left(\frac{\theta}{3A}\right)^{\frac{1}{2}}$$

Taking into account the gain in phase of $\pi/2$ (indicated by the negative sign) of one of the rays which passes through a focus, we find that the amplitude of either ray is

$$a_1 = (12\ A\theta)^{-\frac{1}{4}} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

The phase difference between the two rays can be shown to be equal to

$$\frac{2\pi}{\lambda}\left[\frac{4}{3}\theta\left(\frac{\theta}{3A}\right)^{\frac{1}{2}}\right] - \frac{\pi}{2} \quad \ldots \quad \ldots \quad (12)$$

The amplitude of the resultant of these two, will be

$$a_2 = 2\ (12A\theta)^{-\frac{1}{4}} \cos\left\{\frac{\pi}{\lambda} \cdot \frac{4}{3}\theta\left(\frac{\theta}{3A}\right)^{\frac{1}{2}} - \frac{\pi}{4}\right\} \quad \ldots \quad (13)$$

and its phase will be $-\frac{\pi}{4}$ with respect to the ray from the centre O, in the same direction. We have now to determine the amplitude $a_3$ of the ray from the point $c$, and its phase $\delta$ with respect to the resultant of the first two rays. If the angle between $Oy_1$ and $O'y_2$ be denoted by $2\alpha$,

$$a_3 = \{12\,A\,(2\alpha-\theta)\}^{-\frac{1}{4}} \quad \ldots \quad \ldots \quad (14)$$

The perpendicular from O on the tangent to the curve DO′ at the point $c$ can be shown to be

$$\frac{1}{\{1+(2\alpha-\theta)^2\}^{\frac{1}{2}}}\left[2\,(2\alpha-\theta)\left(\frac{\alpha}{3A}\right)^{\frac{1}{2}}-\alpha\left(\frac{\alpha}{3A}\right)^{\frac{1}{2}}+\tfrac{2}{3}(2\alpha-\theta)\left(\frac{2\alpha-\theta}{3A}\right)^{\frac{1}{2}}\right]$$

$$=\left(\frac{\alpha}{3A}\right)^{\frac{1}{2}}(3\alpha-2\theta)+\tfrac{2}{3}\,(2\alpha-\theta)\left(\frac{2\alpha-\theta}{3A}\right)^{\frac{1}{2}} \text{ (approx.)}$$

Hence, allowing for the gain in phase of $\frac{\pi}{2}$ of the third ray in passing through a focus, we get

$$\delta=\frac{2\pi}{\lambda}\left[\left(\frac{\alpha}{3A}\right)^{\frac{1}{2}}(3\alpha-2\theta)+\tfrac{2}{3}\,(2\alpha-\theta)\left(\frac{2\alpha-\theta}{3A}\right)^{\frac{1}{2}}\right]-\frac{\pi}{4} \quad \ldots \quad (15)$$

The illumination in the direction $\theta$ is thus given by

$$I=a_2^2+a_3^2+2a_2\,a_3\cos\delta \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$a_2$, $a_3$ and $\delta$ being determined by equations (13), (14) and (13).

### § 5. *Outside the caustics.*

It is seen from (13) that $a_2$ becomes infinite when $\theta$ is equal to zero, and also that the above theory is inapplicable when $\theta$ becomes negative. For, outside the caustics, it is apparent that there is only one ray which emerges in any given direction. But since the emergent cone of light is abruptly terminated in the direction $\theta=0$, we must also take into account a diffraction effect similar to that within the geometrical shadow of a straight edge. According to Sommerfeld's well-known investigation,* this effect can be

* Sommerfeld—On the Math. Theory of Diffraction. *Math. Annalan*, Vol. XLVII, p. 317, (1895).

represented by that due to a single radiating source placed at the edge, *i.e.* at O (fig. 3) in our problem. Since the whole effect is small, we can approximately consider the diffracted ray to be of the same amplitude as the ray which emerges in the same direction from the other half of the wave-front. The illumination curvės outside the caustics' have been actually drawn on this assumption, the curves being rounded off at the point corresponding to $\theta=0$.

The value of the constant A can be calculated approximetely by equating the length OD (fig. 3) to ($a \sin \phi''$), where $\phi''$ corresponds to the point of emergence of the caustics from the surface of the cylinder. Thus, we shall have

$$\left(\frac{\alpha}{3A}\right)^{\frac{1}{2}}=a \sin \phi'' \text{ (approx.)}. \quad \ldots \quad \ldots \quad (17)$$

The intensity of Illumination has been calculated, on the above theory, for different values of $\theta$, and the illumination curves for the cases when the angle between the caustics ($2\alpha$ approximately) is (1) $0^\circ\ 56'$ (2) $1^\circ\ 52'$ are shown in Fig. 4.* A comparison of the curves with the photograph on Plate I shows that the theory is capable of explaining the facts very closely. To make the comparison quantitative, a particular section (perpendicular to the bisector of the angle between the caustic lines) was chosen, so that the general form of the Illumination curve was practically the same as that obtained by calculation, and then the positions of the successive maxima with respect to the central direction CC', were measured (fig. 3). The calculated and observed values are tabulated here for comparison, for the two cases for which values have been calculated.

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* The abscissæ are on the same scale in the two diagrams.

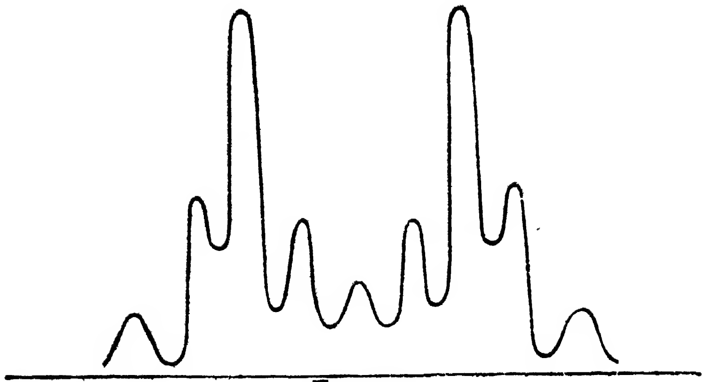


FIG. A.

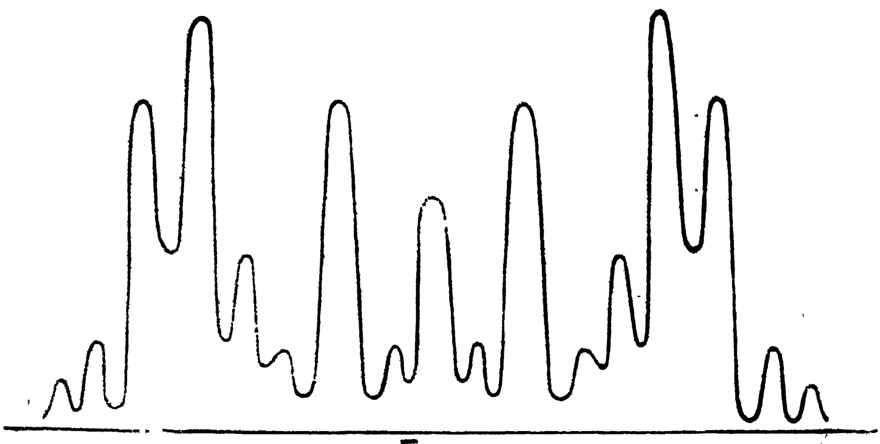


FIG. B.

Fig. 4.

TABLE II.

| Serial No. of maximum (reckoned from the central direction). | Separation of castics 0°56′. | | | 1°52′. | | |
|---|---|---|---|---|---|---|
| | Calculated Intensity. | Position. | | Calculated Intensity. | Position. | |
| | | Obsd. | Calcd. | | Obsd. | Calcd. |
| 1 | 1·6 | 0·10′ | 0·11′ | 0·5 | 0·8′ | 0·8′ |
| 2 | 4·0 | 0·21′ | 0·21′ | 1·7 | 0·17′ | 0·18′ |
| 3 | 2·0 | 0·31′ | 0·30′ | 0·4 | 0·26′ | 0·29′ |
| 4 | 0·7 | 0·41′ | 0·41′ | 0·9 | 0·33′ | 0·36 |
| 5 | | | | 2·2 | 0·43′ | 0·43 |
| 6 | | | | 1·7 | 0·53′ | 0·52′ |
| 7 | | | | 0·5 | 1·4′ | 1·3′ |
| 8 | | | | 0·3 | 1·13′ | 1·10′ |

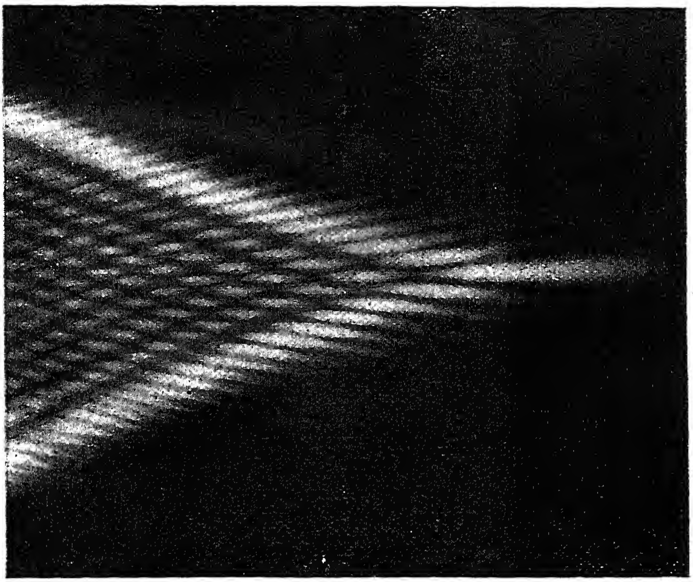
Radius of the cylinder=0·0184 cm; $\lambda=5{\cdot}5.\ 10^{-5}$ cm.

The agreement between the observed and calculated values is fairly close. It is also noteworthy that the maxima and minima seem to be ranged at roughly equal intervals.

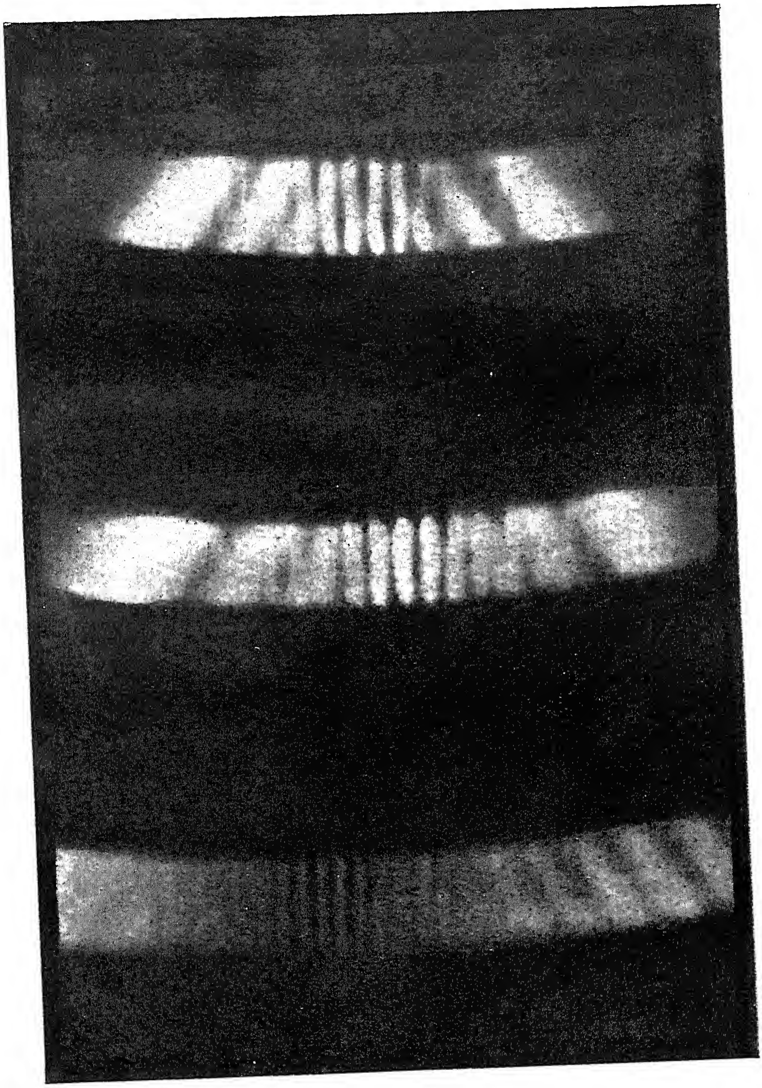
Some explanation may now be given of the peculiarity of the phenomena, described in § 1, observed when the diameter of the cylinder is very small. In a direction midway between the caustics it may be noted that the rays emerging from the points (*a*) and (*c*) (fig. 3) have nearly equal paths, while the central ray from (*b*) differs very much in path from either of the other two. Hence we may regard the phenomena in white light as due simply to the superposition of a practically uniform illumination (due to the ray from *b*) on fringe system due to interference of light from the two point sources *a* and *c*. It would be seen from the photographs on Plate II that the central fringes are very

distinct and bright, and this would indicate that the effect due to the ray from *b* is much smaller than that due to those from *a* and *c*. This seems to be what actually happens, for, as a reference to figs. 1 and 2 will show, there is indeed a greater concentration of the rays near the points *a* and *c* than at the centre of the wave front. The achromatism of the caustics and its supernumeraries* is due to the fact that the effect of dispersion in the glass is compensated by that of diffraction of the emergent wave. In conclusion the writer wishes to acknowledge the interest taken in this investigation by Prof. C. V. Raman.

* *See* Mascart—*Ann. de Chim. et de Phys.*, (6) 26 (1892).



Diffraction of light by a glass cylinder held obliquely.



Diffraction by a thin glass fibre held obliquely.

# On the Theory of Superposed Diffraction-Fringes.

By Prof. Chandi Prasad, M.A.

In the Proceedings of the American Philosophical Society for 1913 (pp. 276-282), C. F. Brush[1] has published various interesting observations of diffraction phenomena which he had attempted to group together under a common explanation by regarding them as 'superposed fringes.' Among the effects observed by him were those due to the diffraction of light by a cylindrical edge which have recently been further investigated and explained in an entirely different manner by N. Basu[2]. In the present paper, I am concerned with what were perhaps the most interesting and original observations recorded by Brush, that is, the diffraction effects produced by a row of straight edges placed in echelon order. The arrangement adopted by Brush is shown diagrammatically in fig. 1. A row of Gillete razor blades (sometimes as many as 24) were clamped together, so that their edges lay as nearly as possible in one plane which was placed very obliquely in the path of the train of light waves. The phenomena in the immediate neighbourhood of the system was observed through a microscope. Brush noticed that the fringes showed contrast between the maxima and minima of illumination which were much more marked than in the diffraction-fringes of the Fresnel type due to a single straight edge and explained this as due to the superposition of the fringe systems produced by the successive edges. I propose in the present paper to show how this principle of superposition suggested by Brush may be formulated mathematically

1. See also *Science Abstracts* (1913). No. 1810

2. *Phil Mag.*, Jan. 1918, page 79.

and its validity tested in experiment. The value of the principle is that it simplifies the treatment of the problem of diffraction by a succession of edges which would otherwise be very laborious[3]. Incidentally I also describe some observations of my own which appear to be of interest from the point of view of the general theory of diffraction.

Sommerfeld has shown in his well-known investigation on the mathametical theory of diffraction[4] that the diffraction-fringes due to a semi-infinite screen may be regarded as due to the interference of a system of a series of cylindrical waves emitted by the edge of the screen with the incident plane waves. In the case of a number of semi-infinite screens placed parallel to each other in echelon order, it would obviously not be correct to assume that in the region to the rear of the system, each of the edges emits cylindrical waves, the amplitude of which, in any direction is the same

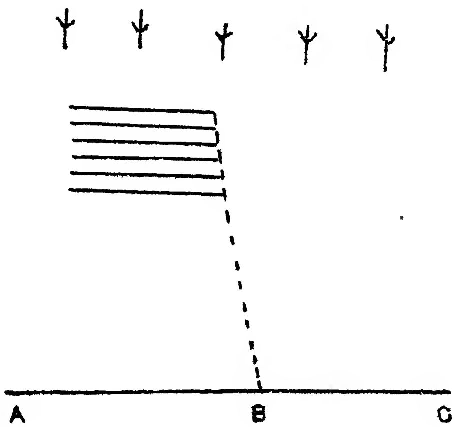



FIG 1

as if the other screens were absent. The assumption would certainly be incorrect in respect of the region lying on one side of the plane containing the edges (AB in fig. 1); for

3. On this point see Mascart's *Traite de Optique*, Vol. I, p. 287.

4. *Math. Annalen*, Vol. XLVII, 1896, p. 317.

in this region each of the screens would obviously intercept and cut off the radiations emitted by the edges in front of it. But in the region lying on the other side (BC in fig. 1) an assumption of the kind stated above would be justified as a first approximation, provided each edge in the echelon is sufficiently in advance of the preceeding edges in relation to its distance from it.

For, the illumination in the region at and near each edge would then practically be the same as that due to the incident waves alone, and the edge of each screen therefore emits a radiation, the amplitude of which in the region considered is practically the same as in the absence of the other screens. By superposing the effects of the cylindrical waves emitted by the edges upon that due to the incident waves, a mathematical treatment of the phenomena observed by Brush would be possible.

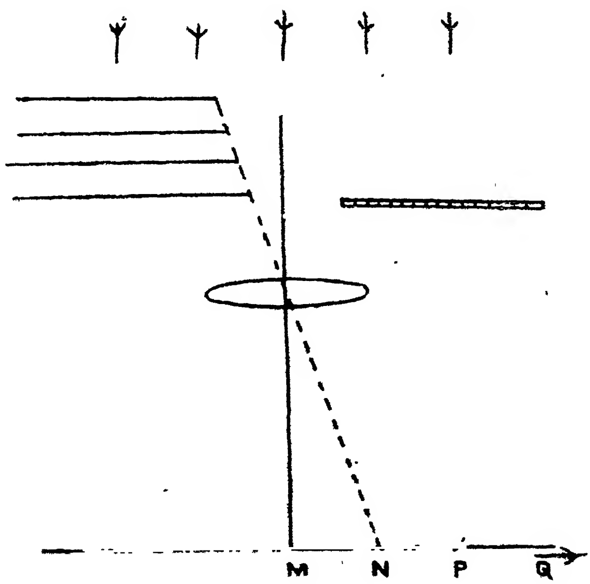



FIG 2

In order to test the preceding views and to obtain a definite confirmation, I have devised the experiment shown diagramatically in fig. 2 in which the effects due to the echelon of edges may be examined separately from that due to the incident waves.

A row of razor-edges is placed on an optical bench, the distances between successive edges being about 10 cm. The edges are brought carefully into parallelism with each other and placed in echelon order. The incident plane waves of light are diffracted by the edges and pass through the lens L, the observations being made in the focal plane. The superfluous light was cut off by the screen E. The lens may be dispensed with, if the fringes are examined at a great distance behind the system. Some interesting effects are then observed. First, we have the direct light which comes to a focus at or near the point M, and on either side of which we have the diffracted light due to the limitation of aperture of the lens. In the region NPQ, we have another system of fringes which are due to the superposition of the light diffracted by the series of edges. These fringes appear only to the right of the plane containing the edges, ON being the line drawn through the centre of the lens parallel to this plane. The centre of the system of fringes (as seen in white light) is at P where MN = NP. Fig. 3 *(a)* and fig. 3 *(b)* reproduce photographs of the system of fringes observed with an echelon of two edges only, the second edge being further in advance of the first edge in the latter case than in the former. It will be seen that the fringe widths are assymetrical, that is, the fringe width decreases continuously as we move from one side of the system to the other. The positions of the fringes are approximately the same as that due to the interference of two cylindrical sources of light placed respectively at the two diffracting edges. This is shown by

the following measurements of the widths of the fringes on either side of the centre P.

| Positions of the maxima read on a micrometer scale | | Fringe widths | |
|---|---|---|---|
| | | Observed | Calculated |
| 1st Maxima | ... 3·283 | ·125 | ·124 |
| 2nd ,, | ... 3·158 | ·105 | ·098 |
| 3rd ,, | ... 3·053 | ·094 | ·084 |
| 4th ,, | ... 2·959 | ·071 | ·075 |
| 5th ,, | ... 2·888 | ·069 | ·068 |
| 6th ,, | ... 2·819—the central P. | ·062 | ·063 |
| 7th ,, | ... 2·757 | ·056 | ·058 |
| 8th ,, | ... 2·701 | ·051 | ·054 |
| 9th ,, | ... 2·650 | ·051 | ·052 |
| 10th ,, | ... 2·599 | ·045 | ·052 |
| 11th ,, | ... 2·554 | ·045 | ·048 |
| 12th ,, | ... 2·509 | | |

The figures in the last column were calculated from the formula

$$a\ (\cos\theta - \cos\psi) = \pm n\lambda$$

where $a$ is the distance between the two edges, $\theta$, $\psi$ are the small angles made by the incident and diffracted rays respectively with the plane containing the edges.

Fig. 3 *(c)* reproduces a photograph of the fringes obtained with an echelon of three edges and fig. 3 *(d)* reproduces one obtained with four edges. It will be seen that in the former case, we have one faint secondary maxima between each pair of primary maxima, and in the latter case, we have two secondary maxima between the two principal maxima. In

fact, the results are somewhat, though not completely, analogous to those due to a diffraction grating composed of a small number of elements. The difference in this case is that the light is diffracted by simple edges, while in an ordinary diffraction grating we are concerned with either a corrugated surface or the strips of a plane surface. Photographs of the diffraction spectra formed by a very obliquely held surface consisting of 2 or 3 reflecting strips in a plane have recently been published by Mitra[5], who has observed that the corresponding bands on either side of the pattern are of unequal brightness, the wider fringes on one side being of very feeble intensity in comparison with the narrow fringes on the other. The photographs of diffraction effects produced by a row of straight edges published in the present paper (figs. 3*a* to 3*d*) do not show this effect, thus, indicating an essential difference between the two cases. In fact, in the diffraction spectra due to a row of straight edges, the broader fringes on one side are, if anything, actually brighter than the narrow fringes on the other.

With a view to further study of the effects noticed by Brush (in which the radiations from the edges are superposed on the transmitted waves), I have also measured and photographed the diffraction fringes of the Fresnel type due to two edges lying in nearly the same line as the direction of propagation of the incident light. Four photographs are reproduced in figs. 3 (*e* to *h*) the second edge being in advance of the first edge by the different distances in the four pictures. The diffraction-fringes due to a single edge are shown in fig. 3 (*i*) for comparison. Some of the minima of illumination due to the superposition of the effects of the two edges are seen in the photographs to be much darker than the minima in the fringes of the Fresnel type due to a single edge. If we

5. *Phil. Mag.*, Jan. 1918, Plate V.

assume as a first approximation that the effects due to the two edges are practically additive, the illumination at any point in the fringe system may be calculated mathematically as follows.

The problem of the diffraction of light by a semi-infinite screen has been solved by Sommerfeld whose solution of the equation

$$\frac{\partial^2 s}{\partial t^2}=a^2\left(\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}\right)$$

is

$$s=\frac{e^{i\left(nt+\frac{\pi}{4}\right)}}{\sqrt{\pi}}\left[e^{ikr\cos(\phi-\phi')}\int_{-\infty}^{\sqrt{2kr}\cos\frac{\phi-\phi'}{2}}e^{-iT^2}\,dT\mp e^{ikr\cos(\phi+\phi')}\int_{-\infty}^{\sqrt{2kr}\cos\frac{\phi+\phi'}{2}}e^{-iT^2}\,dT\right]$$

This solution may be simplified by expanding the integrals in a semi-convergent series as shown by Sommerfeld. In the region of light where $\pi-\phi'<\phi<\pi+\phi'$ and provided $\sqrt{2kr}\cos\frac{\phi-\phi'}{2}>1$, we have denoting the real part of $s$ by S

$$\mathrm{S}=\cos\left(kr\cos\overline{\phi-\phi'}+nt\right)$$
$$+\frac{1}{4\pi}\sqrt{\frac{\lambda}{r}}\cos\left(kr-nt+\frac{\pi}{4}\right)\left\{\pm\frac{1}{\cos\frac{\phi+\phi'}{2}}-\frac{1}{\cos\frac{\phi-\phi'}{2}}\right\}$$

In the case of normal incidence $\phi'=\frac{\pi}{2}$ and writing $\phi'=\frac{3\pi}{2}-\delta$, where $\delta$ is small, we find on reduction and simplification

$$\mathrm{S}=\cos(kd-nt)-\frac{\sqrt{\lambda d}}{2\pi x}\cos\left(kd-nt+\frac{kx^2}{2d}+\frac{\pi}{4}\right)$$

which gives the usual maxima and minima when

$$\frac{k^2}{2d}+\frac{\pi}{4}=2n\pi \text{ or } (2n+1)\,\pi \text{ respectively.}$$

Now, if another edge were placed at a distance $a$ cm., and projecting $b$ cm. from the line of the geometrical shadow of the first edge and its effect is added to that

$$S=\cos(kd-nt)-\frac{\sqrt{\lambda d}}{2\pi x}\cos\left(kd-nt+\frac{kx^2}{2d}+\frac{\pi}{4}\right)$$

$$-\frac{\sqrt{\lambda(d-a)}}{2\pi(x-b)}\cos\left\{kd-nt+\frac{k(x-b)^2}{2(d-a)}+\frac{\pi}{4}\right\}$$

This gives for the intensity of illumination at any point

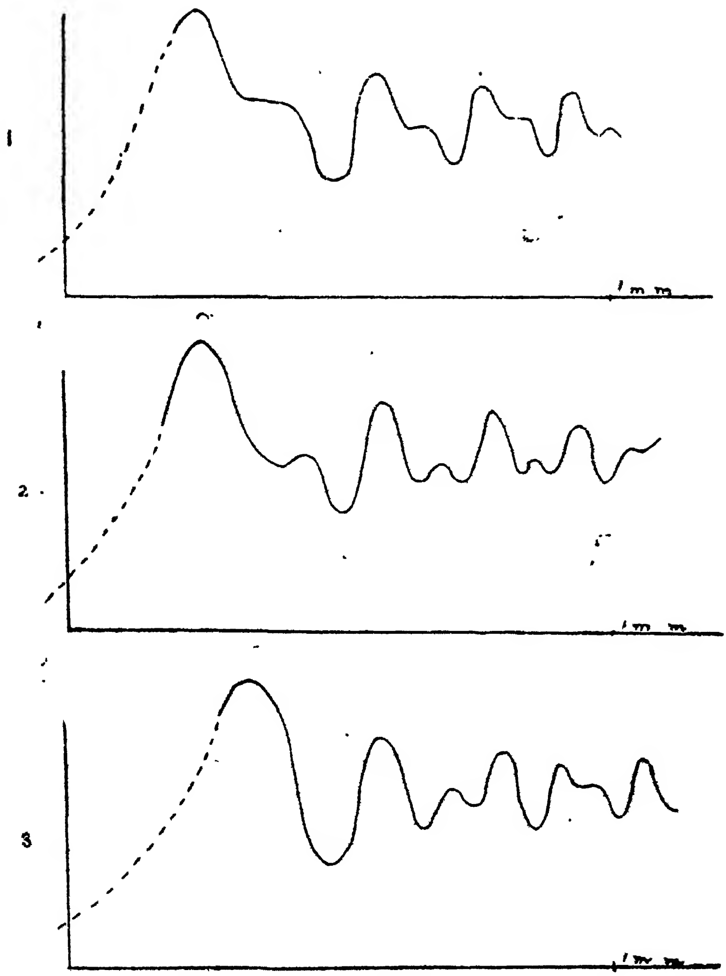
$$J=1+\frac{\lambda d}{4\pi^2x^2}+\frac{\lambda(d-a)}{4\pi^2(x-b)^2}-\frac{\sqrt{\lambda d}}{\pi x}\cos\left(\frac{kx^2}{2d}+\frac{\pi}{4}\right)$$

$$-\frac{\sqrt{\lambda(d-a)}}{\pi(x-b)}\cos\left\{\frac{k(x-b)^2}{2(d-a)}+\frac{\pi}{4}\right\}+\frac{\lambda\sqrt{d(d-a)}}{2\pi^2x(x-b)}\cos\frac{k}{2}\left(\frac{x^2}{d}-\frac{(x-b)^2}{d-a}\right)$$

Of these 6 terms, the 2nd, 3rd and 6th are small compared with the 1st, 3rd and 4th and may, in calculating the position of the maxima and minima, be left out of account.

To test the results obtained from the formula, measurements have been made of the maxima and minima in several cases, the experimental data and the theoretical values are shown for comparison in the table, the latter being taken from the illumination curves plotted from the formula. Fig. 4 shows three illumination curves and illustrates the general effect of the superposition of the fringes due to two edges.

Positions of the maxima and minima of illumination are measured from the geometrical edge of the shadow of the edge nearest the plane of observation.




FIG

ILLUMINATION CURVES FOR TWO EDGES

1. $b$ = ·000 cm.
2. $b$ = ·002 cm
3. $b$ = ·014 cm

I

$d=20$ cm. $a=10$ cm. $b=\cdot002$ cm.

| Bands | | | | Observed | Calculated |
|---|---|---|---|---|---|
| 1st | Maximum | . | .. | ·024 | ·025 |
| „ | Minimum | .. | ... | ·040 | ·040 |
| 2nd | Maximum | ... | ... | ·046 | ·044 |
| „ | Minimum | ... | ... | ·052 | ·050 |
| 3rd | Maximum | ... | ... | ·058 | ·058 |
| „ | Minimum | ... | ... | ·065 | ·065 |
| 4th | „ | ... | ... | ·073 | ·073 |
| 5th | „ | ... | ... | ·083 | ·083 |
| 6th | „ | ... | ... | ... | ·089 |
| 7th | „ | ... | ... | ·096 | ·098 |
| 8th | „ | ... | ... | ·108 | ·105 |

## II

$d = 20$ cm. $a = 10$ cm. $b = {\cdot}014$ cm.

| Bands | | Observed | Calculated |
|---|---|---|---|
| 1st | Minimum ... ... | ·0378 | ·034 |
| 2nd | " ... ... | ·0526 | ·051 |
| 3rd | " ... ... | ·0617 | ·060 |
| 4th | " ... ... | ·0712 | ·072 |
| 5th | " ... ... | ... | ·081 |
| 6th | " ... ... | ·0892 | ·088 |
| 7th | " ... ... | ·102 | ... |

## III

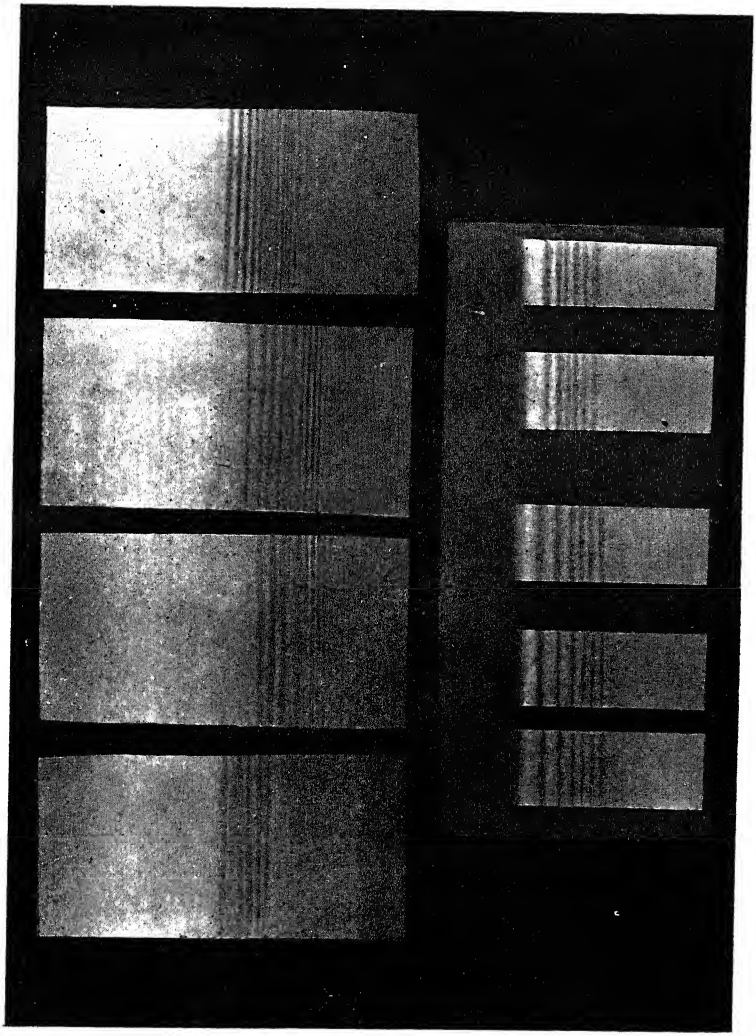
$d = 20$ cm. $a = 10$ cm. $b = {\cdot}022$ cm.

| Bands | | Observed | Calculated |
|---|---|---|---|
| 1st | Minimum ... ... | ·021 | ·019 |
| 2nd | " ... ... | ·032 | ·032 |
| 3rd | " ... ... | ·042 | ·042 |
| 4th | " ... ... | ·051 | ·050 |
| 5th | " ... ... | ·059 | ·059 |

The agreement is fairly satisfactory.

I would like to express my indebtedness to Prof. C. V. Raman for the help and encouragement he gave me during the course of this investigation, which was carried on last summer in the laboratory of the Indian Association for the Cultivation of Science, Calcutta.

QUEEN'S COLLEGE, BENARES, INDIA.
*28th March, 1918.*



Superposed Diffraction Fringes.

PROCEEDINGS

OF THE

INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE.

Vol. IV. PART IV.

# On Three New Species of Opalina, Purk. et. Val.

BY EKENDRA NATH GHOSH, M.D., M.Sc.

The genus *Opalina* may be diagnosed by the following characters :—

Flattened or cylindrical body without mouth and without C.V.

Two to numerous nuclei, without differentiation into macro- and micro-nuclei and with distinct nuclear membrane.

The ordinary forms are asexual generations which alternate with sexual ones with the formation of gamates (macro- and micro-gamates).

Up to the year 1915, 23 distinct species have been described; to these are to be added, the three new species described below :—

1. *Opalina triangularis, n. sp.*

Body flattened, leaf-like, twice as long as broad or less, widest in the anterior body half; one side nearly straight, and the other strongly convex giving the appearance of two curved sides meeting at the widest part of the body; anterior end rounded and in the same line with the straight side; posterior end tapering and rounded; numerous nuclei. In the lower part of the intestine and upper part of the rectum of *Bufo melanostictus*.

2. *Opalina plicata, n. sp.*

Body broadly or elongately ovate (slightly longer than broad), tapering and rounded anteriorly, wide and rounded posteriorly; dorsal surface with four ridges, two of which are nearly parallel and extend from near the anterior end down to each side posteriorly: sometimes in broader forms, two ridges of one side may be absent; numerous nuclei. In the lower part of the intestine and upper part of the rectum of *Bufo melanostictus.*

3. *Opalina sealprifarmis, n. sp.*

Body elongated, about 4 to 5 times as long as broad, being flattened at the anterior end with a truncate edge, four-sided (in transverse section) in the anterior half or so, and cylindrical and slightly tapering to a blunt point posterioriy; the four ridges in the anterior portion of the body run in a slightly spiral curve posteriorly, so that the animal appears twisted round the long axis; numerous nuclei. In the lower part of the intestine and upper part of the rectum of *Bufo melanostictus.*

The known species of *Opalina* may now be tabulated in a synopsis as follows :—

*a.* Nuclei one to five in number.

*a*¹. Only one nucleus, of two lobes joined by a thread, each half with prominent chromosomes; body elongated, rounded anteriorly and tapering posteriorly; cilia arranged in furrows.

O. primordialis, Awerinz.

*b*¹. Nuclei two in number.

*a*². Body narrow and much elongated, about 10 times as long as broad, may be twisted; anterior nucleus towards the anterior end,

the other in the middle or a little posterior to it.

O. tenuis, Raff.

$b^2$. Body not so.

$a^3$. Body flattened.

$a^4$. Body much flattened.

$a^5$. Body broadly triangular or oval in shape, sometimes pointed posteriorly ; ciliary striæ spiral; nuclei rather central, chromosomes in mitosis of two forms, massive and granular.

O. antilliensis, Metcalf.

$b^5$. Body spindleshaped, rounded and bent to one side, posterior end tapering and pointed; dumb-bell-shaped nuclei with distinct chromosomes resting in the "late anaphase" stage.

O. mitotica, Metalf.

$b^4$. Body flattened dorsally and convex ventrally, ending in a tail posteriorly; interval between the nuclei less than the body height ; posterior portion very long and thinned out.

O. acuta, Raff.

$c^4$. Body moderately flattened, being oval in transverse section.

$a^5$. Body elongately oval, posterior end narrower than the wide and rounded anterior end ; a depression on the right margin anteriorly; nuclei rounded with 2 to 3 thick chromatin masses on the surface.

O. macronucleata, Bezzen.

$b^5$ Body oval; prosterior end wider than the more bluntly pointed anterior end; chromatin scattered without any definite arrangement; surface with irregular longitudinal ridges.

O. binucleata, Raff.

$d^4$. Body broadly oval or circular in transverse section.

$a^5$. Body broadly oval in transverse section; elongated, with anterior end very wide and slightly flattened, and tapering to a point posteriorly; surface with corkscrew like folds; nuclei in the anterior half of the body.

O. dorsalis, Raff.

$b^5$. Body circular in transverse section.

$a^6$. Body with a non-ciliate posterior end; body elongated and cylindrical, anterior end rounded and curved to one side; posterior end tapering to a point; nuclei close to each other in the anterior body half.

O. satunnalis, Leg. & Dubosq.

$b^6$. No non-ciliate posterior end.

$a^7$. Body elongately oval and of enormous size; anterior end rounded, posterior end sightly rounded and not tapering; nuclei widely apart.

O. hyalarum, Raff.

$b^7$. Body spindle-shaped with a tapering and pointed posterior end.

$a^8$. Body elongately spindle-shaped, anterior end rounded and slightly bent to the

right; nuclei in the anterior half of the body.

O. intestinalis, Stein.

*b*[8]. Body broadly spindle-shaped, anterior end rounded and slightly bent to the right; posterior end rather abruptly pointed; nuclei in the middle of the body.

O. caudata, Zeller.

*c*[1]. Nuclei 4-5 in number; body lanceolate, rounded anteriorly and tapering posteriorly; chromation in compact masses in the surface of the nuclei; a thick layer of ectoplasm.

O. lanceolata, Bezzen.

*b*. Numerous nuclei.

*a*[1]. Body flattened and leaf like.

*a*[2]. Anterior end of the body narrower than the posterior.

*a*[3]. Surface of the body smooth.

*a*[4]. Body rounded posteriorly.

*a*[5]. Body twice as long as broad.

*a*[6]. Ectoplasm thin; nuclei small, and few or none in the process of division; body irregularly oval, with a notch on the right side in its posterior half.

O. ranarum, Purk.

*b*[6]. Ectoplasm thick; most of the comparatively large (9 $\mu$) nuclei in the process of division; body evenly and broadly oval.

O. cincta, Collin.

*b*[5]. Body not twice as long as broad; body irregularly oval with a straight left margin and a notch on the right margin anteriorly.

O. lata, Bezzen.

*b*⁴. Body with the posterior end terminating in a sharp point bent on one side; body evenly oval.

O. coracoidea, Bezzen.

*b*³. Surface of the body with 2 or 4 longitudinal ridges extending form anterior to the posterior end; body ovate in shape.

O. plicata, n. sp.

*b*². Anterior portion of the body wider than the posterior.

*a*³. Body comma-shaped.

*a*⁴. Anterior end of the body not swollen; posterior end tapering and pointed; body twice as long as broad.

O. obtrigona, Stein.

*b*⁴. Anterior end of the body swollen on one side; posterior end rounded and not tapering; body thrice or more as long as broad.

O. vergula, Dobell.

*b*³. Body triangular, elongately or broadly oval; one side straight and other of two curved sides meeting at the widest part of the body, which lies in the anterior body half; tapering and rounded posteriorly.

O. triangularis, n. sp.

*c*³. Body very slightly or not flattened.

*a*⁴. Body oval or circular in transverse section.

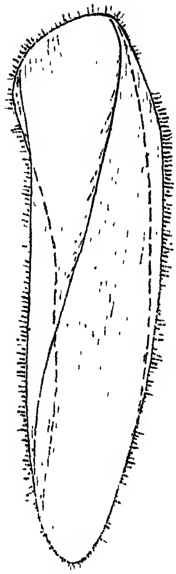
*a*⁵. Body very slightly flattened, pyriform; ciliary stripes spiral.

O. flava, Stokes.

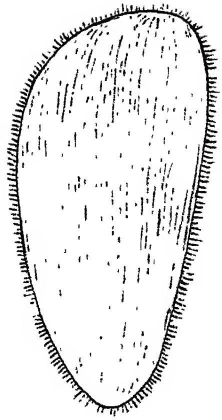
*b*⁵. Body not flattened.

*a*⁶. Body elongated and cylindrical.

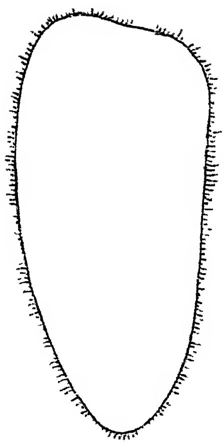
E. N. Ghosh. Plate I.



1.



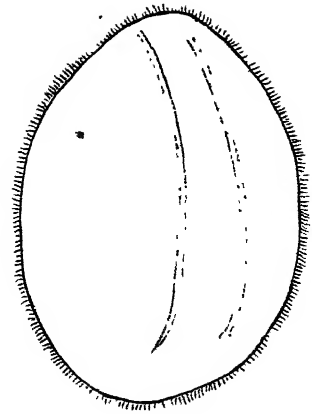
2.



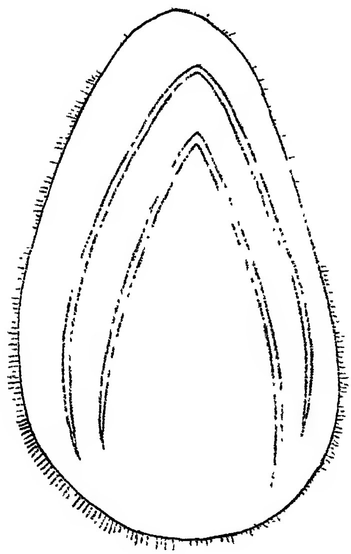
3.

**1.** Opalina Scalpriformis, sp. nov.

**2,3.** Opalina triangularis, sp. nov.



4.



5.

4,5. *Opalina plicata*, sp. nov.

*a*[7]. Body slender, length = 10 breadth; rounded anteriorly and tapering to a point posteriorly.

O. longa, Bezzen.

*b*[7]. Body stout and cylindrical; length = 2 breadth; body folded posteriorly with deep grooves between the folds.

O. Zelleri, Metcalf.

*b*[6]. Body spindle shaped, wide and rounded anteriorly and and tapering to a point posteriorly.

O. dimediata, Stein.

*b*[4]. Body quadrangular in transverse secton, elongated and twisted; flattened at the anterior end (with a truncate edge) and cylindrical and slightly tapering posteriorly.

O. scalpriformes, sp. n.

# Descriptions of Fungi in Bengal. (Agaricaceœ and Polyporaceœ).

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## Introduction.

The following Agaricaceœ and Polyporaceœ were collected in Calcutta, Hooghly and neighbouring places, some of them are very common and grow abundantly every year with the approach of rains, a few are cosmopolitan ;

As these appearing in planes are different from the British species as well as those appearing in the hilly tracts of Darjeeling, Himalaya, Sikkim etc., which were collected by Dr. J. D. Hooker in 1850 and described by Berkeley in Hooker's Journal of Botany and Kew Garden Miscellany, Vols. II, III, IV and VI, I propose to give detailed description along with their plates in each case, so that they can be easily made out. Descriptions of Polyporaceœ in Bengal will appear in regular series. in four or five issues every year and I hope to complete the whole series in course of next two or three years.

It only remains to add here that any suggestions or criticisms in respect of my paper from workers interested in Indian Mycology will be most thankfully received.

## Agaricaceœ.

### 1. *Schizophyllum commune.*

*Habitat*:—It is found growing abundantly in small dense clusters on bamboos, dead wood and timber yards in moist weather.

*Cap*:—Sessile, more or less fanshaped, thin, dry, coriaceous, tapering downwards into a stem-like base, $\frac{1}{2}$ inch across, mostly lobed, divided longitudinally at the edge, external face of the cap covered with numerous minute soft whitish

grey hairs, the edge revolute, colour varies from white to brownish-grey.

*Gills* :—On the undersurface of the cap, radiating from the point of attachment like parallel divergent veins of a palmate leaf. Each gill bifid at the edge, the two lips of the split edge are commonly revolute, colour deep grey, in some cases brownish, the divided surface villous.

*Spores* :—White, roundish.

It is a cosmopolitan species, very common in all parts of Bengal.

For figure See Engler and Prantle Vol. 1 teil, abt 1 1900 Page 223, Fig. 112 A & B.

2. *Lentinns praerigidus*,

*Habitat* :—It is found growing in small clusters on dead wood and trunks of trees; of the specimens the bigger ones were collected from Daltangonge, Behar; the smaller, from the interior of the Hooghly district, they were found growing on a Railway slipper (B. P. Ry.).

*Cap* :—With a short stalk, in dry state coriaceous, surface white dotted with blackish small hairy scales which can be removed partially after hard rubbing, breadth of smaller ones being 2 inches and that of the larger ones being 4-5 inches, excentric, very rigid, thin depressed at the centre, slightly funnel-shaped, often split at the margin when dry, margin incurved and in smaller ones finely divided.

*Stem* :—short scarcely exceeding one inch and half in height, solid, very hard, $\frac{1}{4}$ inch thick, studded with small minute black scales; not rooting at the base.

*Gills* :—Decurrent, edge minutely toothed, not anastomosing, not branched, thin, in dry state dark-brown.

*Spores* :—White.

It is a large species, bearing close resemblance to Lentinus Tigrinus (the spotted variety).

3. *Lentinus caespitosus.* (*Currey*).

The specimen seems to agree with Currey's description in all essentials.

*Habitat* :—Growing on dead wood and trunks in dense clusters.

*Caespitose* :—Growing in tufts, very small plants scarcely exceeding an inch in height.

*Cap* :—Distinctly infundubliform, with short stalks ($\frac{1}{2}$ – 1 inch), broad, the plant is white when fresh, in the dried state it is of brownish colour, surface covered with darkish scales, thin coriaceous, margins slightly incurved (involute), and marginal outline finely divided.

*Stem* :—Very short scarcely exceeding $\frac{1}{2}$ inch in length, covered all over with similar scales, central.

*Gills* :—Decurrent, irregularly serrated and torn at the margin, somewhat darker than the pileus.

*Spores* :—White.

4. *Lentinus irregularis* (*Currey*).

Closely allied to British species Lentinus fimbriatus.

*Habitat* :—Growing on dead wood and trees, common in tropical countries, very small plants scarcely exceeding one inch in height.

*Cap* :—Infundubliform, 1 inch broad, dark-colour, surface smooth, thin, coriaceous, margin distinctly involute, often split at the margin when dry.

*Stem* :—Medium, $\frac{1}{2}$ inch long, central, surface smooth, dark.

*Gills* :—Descending, but they terminate abruptly and break off in a way very similar to what is seen in the British species Lentinus fimbriatus. Gills are dark.

*Spores* :—White.

5. *Lepiota ermineus.*

Plant with ring fixed on the stem.

*Habitat* :—Found in grassy places.

*Cap* :—1 – 2 inches broad, whitish, soft and fleshy, at first campanulate umbonate, then flattened with revolute (turned up) margin, surface smooth.

*Stem* :—1 – 2 inches long, hollow, smooth, the membraneous ring on the top at length torn and fugacious.

*Gills* :—Broad, free, distant, very obtuse at both ends turn a little blackish in course of time.

The specimen agrees with L. ermineus in all essentials though it varies from the type in some particulars.

6. *Collybia mimicus.*

*Habitat* :—Very delicate plants growing on broken pieces of wood in grassy fields.

*Cap* :—1 – 2 inches across, brown, pale tan-colour, surface smooth, soft, general outline crenate, broken at places, convex-explanate with margin revolute, outer skin broken at the periphery.

*Stem* :—Almost central, 1-2 inches long, thin, tapering downwards, base narrower, of the same colour as the cap, almost naked with few minute hairs.

*Gill* :—Very broad, somewhat distant, brownish, free.

*Spores* :—White, elliptic.

*N. B.*—The stem can hardly be called fibrillose, in other respects it agrees with Coll. mimicus.

7. *Collybia ambustus.*

*Habitat* :—Growing on burnt ground.

*Cap* :—1½ inches broad, smooth soft slightly striate, colour blackish central portion darker than the peripheral one, convex-plane with a small umbo at the centre, marginal

outline rarely circular often turned into rounded corners, broken at places.

*Stem* :—1 inch long, $\frac{1}{8}$th of an inch thick, almost central, cartilaginus, naked, black, base thicker, outer surface channeled.

*Gills* :—Crowded, very narrow, free, white brownish, finely toothed.

*Spores* :—White.

The specimen comes very near to Coll. ambustus though it varies in having a channeled stem.

## Polyporaceœ.

### 8. *Daedalea quercina.*

*Habitat* :—Growing on dead wood and split bamboos, very common.

*Cap* :—Sessile, 2-4 inches across, corky, pale wood colour partaking of the substance on which it grows, irregularly rounded, upper surface uneven, closely applied to the substratum, wholly resupinate, the free-surface bearing the hymenium formed of thick tough plates, the wavy pores becoming variously branched form a labyrinth, the margin thin.

It is at once marked out from other species by the peculiar character of labyrinthine pores on the free surface.

### 9. *Favolus Scaber.*

(Hexagonia Scabra).

*Habitat* :—Growing on dead rotten wood in fields, sometimes in clusters.

*Cap* :—With a very short stalk, more or less reniform, fleshy, pliant, surface smooth, yellowish-white coloured, annual, 2-5 inches across, sometimes the marginal outline broken into two or three rounded corners (crenate margin).

*Hymenium* :—Of lighter colour, alveolate, pores angular (with five angles), very distinct, in the form of small depressions like shallow honeycomb, very beautiful to look at.

*Margin* :—Thinning out.

*Sporcs* ;—White.

*Stalk* :—Very short with a few minute hairs.

10. *Polysticuts sanguinus.*

*Habitat* :—Beautifully red plants growing on trees,

*Cap*:—2-3 inches across, thin, smooth, lower surface bearing some marks of concentric zones, more or less reniform, sessile or sometimes with a short stalk attached by a reduced base to the tree on which it grows.

Hymenium on the upper surface consisting of innumerable very minute pores, the upper surface also equally red and similarly concentrically zoned, in some cases there are no zones. The colour fades in time. It is the common red species of the tropical world.

12. *Hexagonia sub-tenius (discopoda).*

*Habitat*:—Hard species growing on mango trees and dried up branches of common fruit trees, 1-2 inches across,

*Cap.*:—Sessile, attached by broad base, sometimes circular, sometimes reniform, thin, hard, lower surface smooth marked by distinct concentric zones, colour, greyish brown.

Hymenial surface bearing prominent medium-sized pores which are distinctly yellowish grey.

The plant is very common in our fruit trees, it is very hard and resisting.

# Iodination by Means of Nitrogen Iodide. Synthesis of Substituted Nitrogen Haloids.

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## § 1. *Introduction.*

In continuation of the study on the iodination of organic bodies by means of nitrogen iodide, the iodination of benzophenone and piperazine has been taken up. Benzophenone condenses with a molecule of nitrogen iodide with the formation of 3-iodacridone $CO\left<\begin{array}{c}C_6H_4\\C_6H_4\end{array}\right>NI$ which has been found to be hydrolysed by chlorine, benzophenone being regenerated. Piperazine forms a substituted nitrogen iodide under the influence of nitrogen iodide, *viz.*, piperazo-di-iodhydrazine $NH\left<\begin{array}{c}CH_2-CH_2\\CH_2-CH_2\end{array}\right>N.N.\left<\begin{array}{c}I\\I\end{array}\right.$. This compound on treatment with chlorine forms tetrachloropiperazine, $Cl.N\left<\begin{array}{c}CH_2.CH_2\\CH_2.CH_2\end{array}\right>NCl.Cl_2$ all the chlorine atoms being united with nitrogen. The same compound has also been prepared by the direct chlorination of piperazine.

## § 2. *The action of iodine in the presence of ammonia on benzophenone. Formation of 3-Iodacridone.*

By the action of iodine and ammonia on benzophenone, a condensation with nitrogen iodide takes place with the formation 3-iodacridone. The reaction that takes place is represented thus—

$$\mathrm{CO}\left\langle\begin{matrix}\mathrm{C_6H_5}\\ \mathrm{C_6H_5}\end{matrix}\right. + \left.\begin{matrix}\mathrm{I}\\ \mathrm{I}\end{matrix}\right\rangle\mathrm{N.I} = \mathrm{CO}\left\langle\begin{matrix}\mathrm{C_6H_4}\\ \mathrm{C_6H_4}\end{matrix}\right\rangle\mathrm{NI} + 2\mathrm{HI}$$

The nitrogen iodide that is formed in the nascent state reacts upon benzophenone with the elimination of two molecules of hydriodic acid. The reaction becomes possible by the fact that benzophenone has a great tendency for the formation of ring compounds, the hydrogens of the para-positions of the phenyl groups taking part in the reaction. The reaction is also rendered possible by the neutralisation by ammonia of the hydriodic acid which is formed as a result of the condensation since it has been found that no condensation takes place when benzophenone is reacted upon with pure nitrogen iodide.

*Experimental.*

To an aqueous solution of ammonia, benzophenone is added and the mixture is heated on the water when the benzophenone melts and remains at the bottom of the liquid without dissolving. To this iodine solution is added gradually with vigorous shaking, the mixture being kept on the water-bath all the time. When no more iodine is taken up which is the case when the solution turns permanently brown, the reaction is stopped. The mixture is kept on the water-bath for a further period of half-hour and then allowed to cool. When the mixture has completely cooled, the melted lilquid at the bottom gets to a hard mass consisting of shining magenta-like crystals. This is separated, powdered and freed from adhearing mother liquor. After repeated recrystallisations from glacial acetic acid it yields a pure magenta-like crystalline substance melting at 87°C.

0·0692 gave 0·1229 $CO_2$ and 0·0274 $H_2O$; C=48·43; H=4·39
0·0429 gave 0·0770 $CO_2$ and 0·0216 $H_2O$; C=48·97; H=5·06
0·0814 gave 3·4 c.c. $N_2$ at 28°C· and 760 mm·; N=4·63
0·0750 gave 0·0531 AgI; I=38·25

$$\text{Calc. for } C_6H_4{<}\begin{matrix}CO\\NI\end{matrix}{>}C_6H_4\text{; } C=48{\cdot}59\text{; } H=2{\cdot}4\text{; } N=4{\cdot}36\text{; } I=39{\cdot}56$$

*Action of Chlorine.*—When a current of chlorine is passed through an aqueous suspension of the substance, it gradually turns yellow and finally a colourless oil separates at the bottom. At this stage the chlorination is stopped. After allowing this to cool it solidifies to a mass of crystals. This on recrystallisation from alcohol gave a pure product melting at 46°-47° and is identified to be pure benzophenone.

(Found C=85·58 ; H=6·44 ; Calc. C=85·71 ; H=5·47).

The reaction that takes place is one of hydrolysis and is represented thus—

$$CO{<}\begin{matrix}C_6H_4\\C_6H_4\end{matrix}{>}NI + Cl + H_2O = CO{<}\begin{matrix}C_6H_5\\C_6H_5\end{matrix} + ICl + NO$$

§ 3. *The Action of Nitrogen Iodide on Piperarine. Formation of Piperazo-Diiodo-Hydrazinc.*

By the action of iodine on an ammoniacal solution of piperazine, per-iodides are obtained. But if piperazine is treated with pure nitrogen ioide, a condensation product is the result viz. piperazo-di-iodo-hydrazine.

$$NH{<}\begin{matrix}CH_2.CH_2\\CH_2.CH_2\end{matrix}{>}NH + I.N{<}\begin{matrix}I\\I\end{matrix}$$

$$= NH{<}\begin{matrix}CH_2.CH_2\\CH_2.CH_2\end{matrix}{>}N.N{<}\begin{matrix}I\\I\end{matrix} + HI$$

The substance decomposes rapidly when it is kept and is slightly explosive which is due to its being a substitued nitrogen iodide and must have its explosive property though in a milder form being stabilised by piperazine.

Great care has to be exercised in the preaparation of this compound. First pure nitrogen iodide is prepared by

adding ammonia solution to a potassium iodide solution of iodine as a jet black precipitate. The precipitate is thoroughly washed with cold water by decantation to make it free from all potassium iodide and free ammonia. This moist nitrogen iodide is added at once to an excess of a dilute solution of piperazine when on shaking the whole of the nitrogen iodide turns into a greenish yellow precipitate. This is rapidly filtered, washed once or twice and is dried rapidly by means of a porous plate. The substance is found to explode at 75°C.

0·0565 gave 0·0279 $CO_2$ and 0·0133 $H_2O$ ; C=13·46 ; H=2·61
0·0317 gave 3 c.c. $N_2$ at 27°C. and 760 mm.; N=11·09
0·0394 gave 0·0532 AgI ; I=72·97

Calc. for $NH\begin{array}{c}\diagup CH_2.CH_2 \diagdown\\ \diagdown CH_2.CH_2 \diagup\end{array}N.NI_2$ ; C=13·59; H=2·54 ; I=71·95 ; N=11·89

*Action of Chlorine.*—On passing a current of chlorine through the substance suspended in water, the solid first dissolves completely after which the liquid turns milky and a yellow oil gradually separated. When no more haziness is produced and a good quantity of the oil has collected, the reaction is stopped.

The yellow oil is separated, dried over calcium chloride. The yield is very small. The oil consists of tetrachloropiperazine, all the chlorine atoms being united to the two nitrogen atoms.

0·1557 gave by Carius' Method 0·3964 AgCl ; Cl=62·98
0·0948 ,, ,, ,, 0·2395 AgCl ; Cl=62·50

Calc. for $Cl.N\begin{array}{c}\diagup CH_2.CH_2 \diagdown\\ \diagdown CH_2.CH_2 \diagup\end{array}NCl.Cl_2$ ; Cl=63·35

The substance is a pungent smelling irritating liquid having an odour similar to substituted nitrogen chlorides.

The vapours bring tears to the eyes. On slight heating it decomposes rapidly.

The same oil is also formed by the direct action of chlorine on piperazine.

The oil is estimated for NCl by means of thiosulphate :

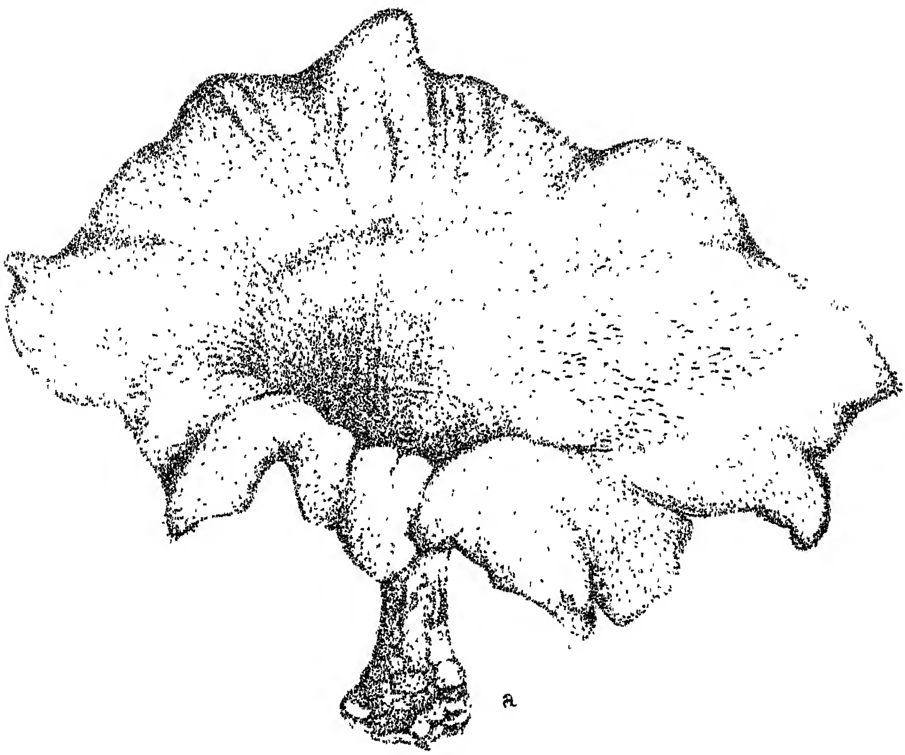
Sample I Chlorine as : NCl ; Cl = 57·04

Sample II ,, ,, Cl = 55·73

From this it appears that all the chlorine is united to nitrogen. A little deficit to that as obtained by Carius' Method may be due to some secondary reaction.

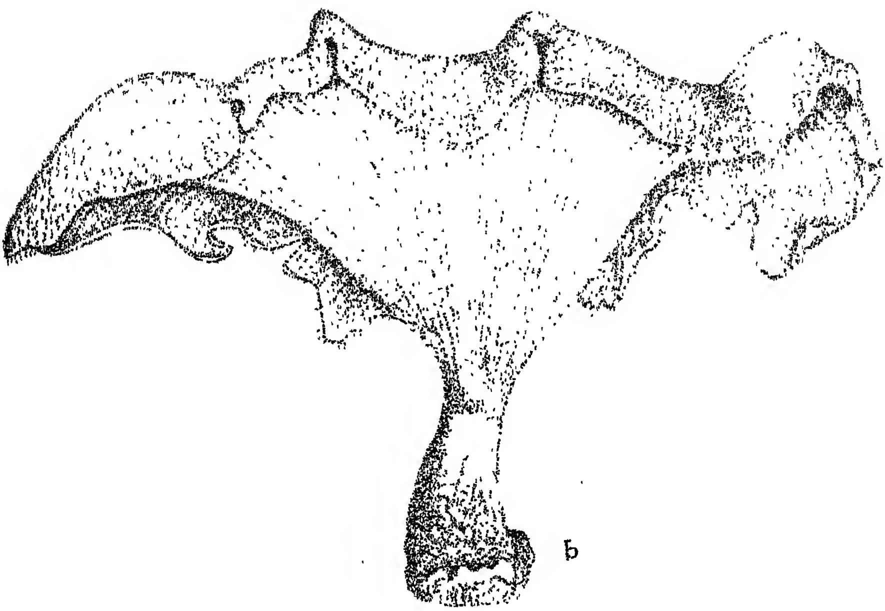
Hence the constitutional formula of the substance is

$$Cl.N\begin{array}{l}\diagup CH_2.CH_2 \diagdown \\ \diagdown CH_2.CH_2 \diagup\end{array}NCl.Cl_2$$



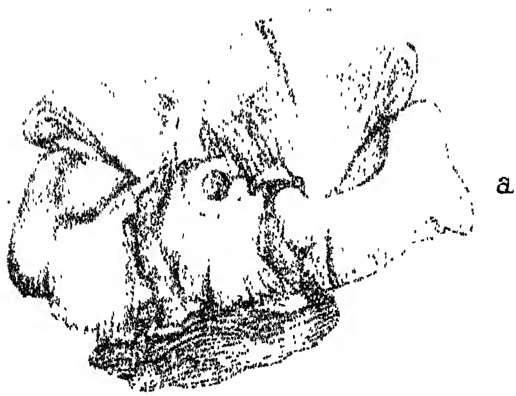

Lentinus prærigidus.

*(a)* Upper surface.

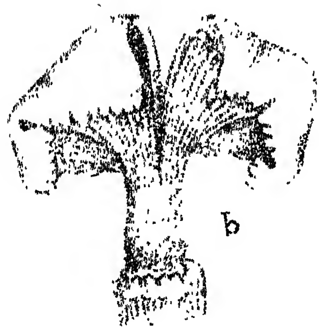



Lentinns prærigidus.

(*b*) Lower surface.




*(a)* Upper surface.




*(b)* Lower surface.

Lentinus caespitosus.

a

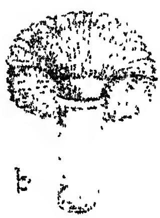
b

c

*(a) (b) (c)* Lentinus irregularis.

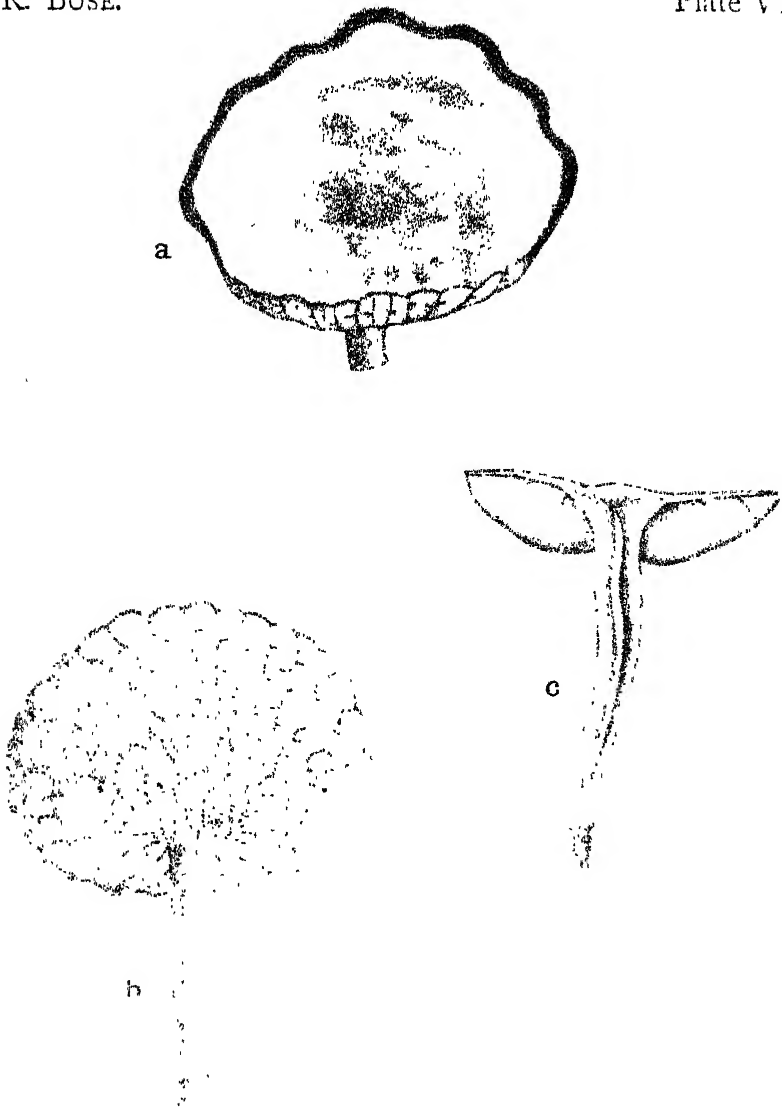
Upper and Lower surfaces.


*(a)* Upper surface.




*(b)* Lower surface.

Lepiota ermineus.



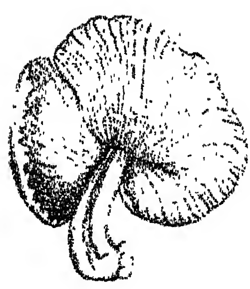

*(a)* Upper surface.

*(b)* Lower surface.

*(c)* A section across median plane.

Collybia mimicus.

a

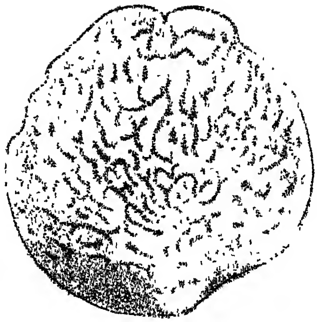


b

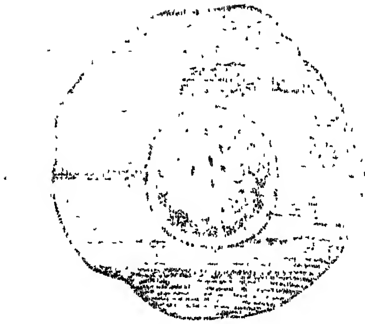
*(a)* Upper surface.

*(b)* Lower surface.

Collybia ambustus.



a

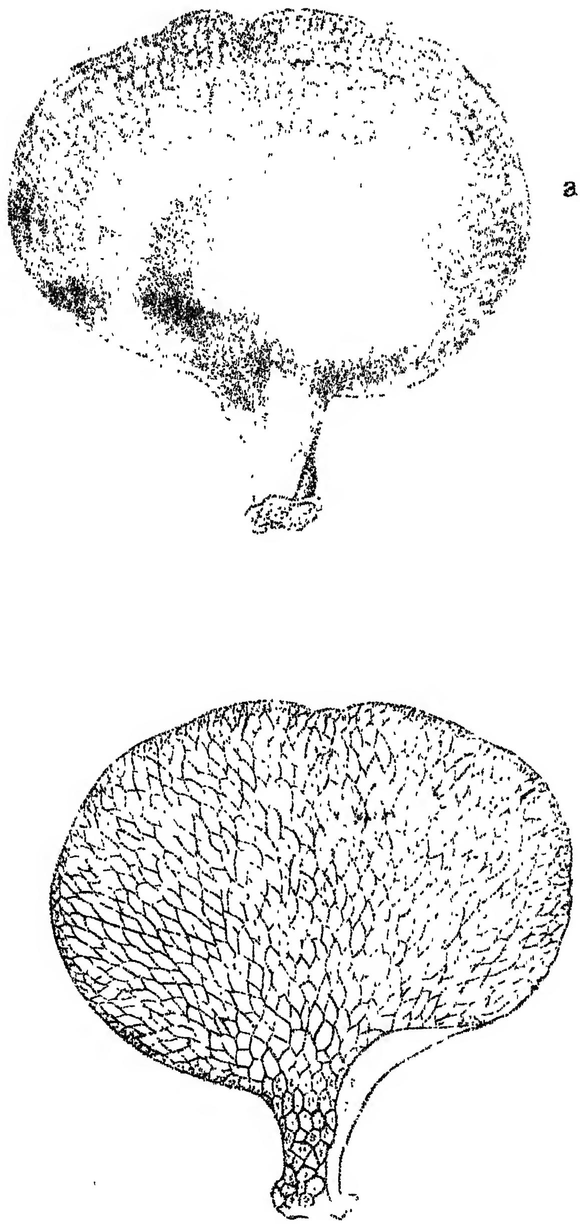


b

*(a)* Upper surface.

*(b)* Lower surface.

Daedalea quercina.



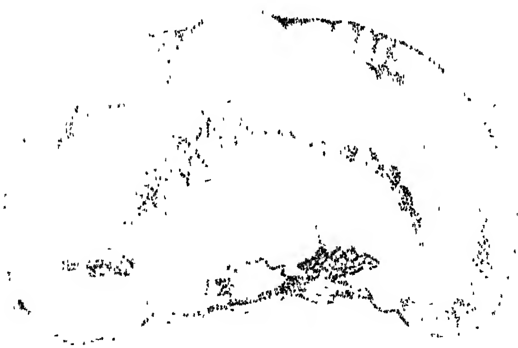
a

b

(a) Upper surface.

(b) Lower surface.

a



b

*(a)* Upper surface.

*(b)* Lower surface.

Polystictus sanguinus.